QUESTION PAPER CODE 65/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. {1, 2, 3} 2. 1 3. – I 4. 3

5. – 2 6. x sin x 7. $\frac{1}{2}(\log 17 - \log 5)$ or $\frac{1}{2}\log\left(\frac{17}{5}\right)$

8. $p = -\frac{1}{3}$ 9. – 10 10. $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$ 1×10 = 10 m

SECTION - B

11. getting $fog(x) = f\left(\frac{x}{x-1}\right) = \left(\frac{x}{x-1}\right)^2 + 2$ 1½ m

$fog(2) = 6$ ½ m

getting $g\,of(x) = g(x^2 + 2) = \frac{x^2 + 2}{x^2 + 1}$ 1½ m

$g\,of(-3) = \frac{11}{10}$ ½ m

12. Putting $x = \cos\theta$ in LHS, We get

$$LHS = \tan^{-1}\left[\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right]$$ 1 m

$$= \tan^{-1}\left[\frac{\sqrt{2}\cos\theta/2 - \sqrt{2}\sin\theta/2}{\sqrt{2}\cos\theta/2 + \sqrt{2}\sin\theta/2}\right]$$ 1 m

$$= \tan^{-1}\left[\frac{1-\tan\theta/2}{1+\tan\theta/2}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta/2\right)\right]$$ ½+1 m

$$= \frac{\pi}{4} - \frac{1}{2}\theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x = \text{R.H.S}$$ ½ m

OR

Given equation can be written as

$$\tan^{-1}\left(\frac{x-2}{x-4}\right) = \tan^{-1} 1 - \tan^{-1}\left(\frac{x+2}{x+4}\right)$$ ½ m

$$= \tan^{-1}\left(\frac{1-\frac{x+2}{x+4}}{1+\frac{x+2}{x+4}}\right) = \tan^{-1}\left(\frac{2}{2x+6}\right)$$ 1+½ m

$$\therefore \quad \frac{x-2}{x-4} = \frac{1}{x+3}$$ ½ m

$$\Rightarrow \quad x^2 + x - 6 = x - 4 \quad \text{or} \quad x^2 = 2 \;\therefore\; x = \pm\sqrt{2}$$ ½+1 m

13. Operating $R_2 \to R_2 - 4R_1$ and $R_3 \to R_3 - 8R_1$, we get

$$\text{LHS} = \begin{vmatrix} x+y & x & x \\ x & 0 & -2x \\ 2x & 0 & -5x \end{vmatrix}$$ 2 m

Expanding along C_2, we get

$$-x\,(-5x^2 + 4x^2) = x^3$$ 1+1 m

14. $$\frac{dx}{d\theta} = a\,e^{\theta}(\sin\theta - \cos\theta) + a\,e^{\theta}(\cos\theta + \sin\theta)$$ 1 m

$$= 2\,a\,e^{\theta}\sin\theta$$ ½ m

$$\frac{dy}{d\theta} = a\,e^{\theta}(\sin\theta + \cos\theta) + a\,e^{\theta}(\cos\theta - \sin\theta) = 2\,a\,e^{\theta}\cos\theta$$ 1 m

$$\frac{dy}{dx} = \frac{2\,a\,e^{\theta}\cos\theta}{2\,a\,e^{\theta}\sin\theta} = \cot\theta$$ 1 m

$$\left.\frac{dy}{dx}\right|_{at\ \theta=\pi/4} = \cot\pi/4 = 1$$ ½ m

15. $y = P\,e^{ax} + Q\,e^{bx} \Rightarrow \frac{dy}{dx} = a\,P\,e^{ax} + b\,Q\,e^{bx}$ 1 m

$$\frac{d^2y}{dx^2} = a^2\,P\,e^{ax} + b^2\,Q\,e^{bx}$$ 1 m

$$\therefore\ \text{LHS} = \frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby$$

$$= a^2\,P\,e^{ax} + b^2\,Q\,e^{bx} - (a+b)\left\{a\,P\,e^{ax} + b\,Q\,e^{bx}\right\} + ab\left\{P\,e^{ax} + Q\,e^{bx}\right\}$$ 1 m

$$= P\,e^{ax}\left\{a^2 - a^2 - ab + ab\right\} + Q\,e^{bx}\left\{b^2 - ab - b^2 + ab\right\}$$ 1 m

$= 0 + 0 = 0. =$ R.H.S.

16. $y = [x\,(x-2)]^2 = \left[x^2 - 2x\right]^2 \therefore \frac{dy}{dx} = 2\left(x^2 - 2x\right)(2x-2)$ 1 m

$$\Rightarrow \frac{dy}{dx} = 4\,x\,(x-1)(x-2)$$ 1 m

$$\frac{dy}{dx} = 0 \Rightarrow x=0,\ x=1,\ x=2$$ ½ m

$\therefore$ Intervals are $(-\infty, 0),\ (0, 1),\ (1, 2),\ (2, \infty)$ ½ m

since $\frac{dy}{dx} > 0$ in $(0, 1)$ or $(2, \infty)$

$\therefore$ f(x) is increasing in $(0, 1)$ **U** $(2, \infty)$ 1 m

OR

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{b^2x}{a^2y}$$ 1 m

slope of tangent at $\left(\sqrt{2}a, b\right) = \frac{\sqrt{2}b}{a}$ ½ m

slope of normal at $\left(\sqrt{2}a, b\right) = -\frac{a}{\sqrt{2}b}$ ½ m

Equation of tangent is $y - b = \frac{\sqrt{2}\,b}{a}\left(x - \sqrt{2}\,a\right)$ ½ m

i.e. $\sqrt{2}\,bx - ay = ab$ ½ m

and equation of normal is $y - b = -\frac{a}{\sqrt{2}\,b}\left(x - \sqrt{2}\,a\right)$ ½ m

i.e. $ax + \sqrt{2}\,by = \sqrt{2}\,(a^2 + b^2)$ ½ m

17. Let $I = \int_0^{\pi} \frac{4x \sin x}{1 + \cos^2 x} dx$

$x \to (\pi - x)$ gives $I = \int_0^{\pi} \frac{4(\pi - x)\sin(\pi - x)}{1 + \cos^2(\pi - x)} dx = \int_0^{\pi} \frac{4(\pi - x)\sin x}{1 + \cos^2 x} dx$ 1 m

$\therefore \quad 2I = 4\pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$ ½ m

Put $\cos x = t$

$\therefore \quad \sin x\, dx = -dt$ ½ m

$\therefore \quad I = 2\pi \int_1^{-1} \frac{-dt}{1 + t^2}$ or $2\pi \int_{-1}^{1} \frac{dt}{1 + t^2}$ 1 m

$= 2\pi \left[\tan^{-1} t\right]_{-1}^{1} = 2\pi \left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \pi^2$ 1 m

OR

$I = \int \frac{x + 2}{\sqrt{x^2 + 5x + 6}} dx = \int \frac{\frac{1}{2}(2x + 5) - \frac{1}{2}}{\sqrt{x^2 + 5x + 6}} dx$ 1 m

$$= \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}\,dx \;-\; \frac{1}{2}\int \frac{dx}{\sqrt{\left(x+5/2\right)^2-\left(\frac{1}{2}\right)^2}}$$ ½+½ m

$$= \sqrt{x^2+5x+6} \;-\; \frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right| + c$$ 1+1 m

18. $\frac{dy}{dx} = 1 + x + y + xy = (1 + x)\,(1+y)$ ½ m

$\therefore \quad \int \frac{dy}{1+y} = \int (1+x)\,dx$ 1 m

$\log|1+y| = x + \frac{x^2}{2} + c$ ½+1 m

$x = 1, \quad y = 0 \quad \Rightarrow \quad c = -\frac{3}{2}$ ½ m

$\therefore$ solution is $\log|1+y| = x + \frac{x^2}{2} - \frac{3}{2}$ ½ m

19. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{1}{1+x^2}\cdot y = \frac{1}{1+x^2}\cdot e^{\tan^{-1}x}$$ 1 m

Integrating factor $= e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$ 1 m

$\therefore$ solution is, $y \cdot e^{\tan^{-1}x} = \int \frac{1}{1+x^2}\; e^{2\tan^{-1}x}\,dx$ 1 m

$\Rightarrow \quad y \cdot e^{\tan^{-1}x} = \frac{1}{2}\; e^{2\tan^{-1}x} + c$ 1 m

or $\quad y = \frac{1}{2}\; e^{\tan^{-1}x} + c\; e^{-\tan^{-1}x}$

20. A, B, C, D are coplaner, if $\overrightarrow{AB}\cdot\overrightarrow{AC}\times\overrightarrow{AD}=0$ 1 m

$\overrightarrow{AB} = -\ 4\hat{i}-6\hat{j}-2\hat{k},\ \overrightarrow{AC} = -\hat{i}+4\hat{j}+3\hat{k},\ \overrightarrow{AD} = -8\hat{i}-\hat{j}+3\hat{k}$ 1½ m

$$\overrightarrow{AB}\cdot\overrightarrow{AC}\times\overrightarrow{AD} = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$ ½ m

$$= -4\,(15)+6\,(21)-2\,(33) = 0$$ 1 m

OR

Given that $\vec{a}\cdot\dfrac{\vec{b}+\vec{c}}{\left|\vec{b}+\vec{c}\right|}=1$ ½ m

or $\quad \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c} = \left|\vec{b}+\vec{c}\right|$ ½ m

$(\hat{i}+\hat{j}+\hat{k})\cdot(2\hat{i}+4\hat{j}-5\hat{k})+(\hat{i}+\hat{j}+\hat{k})\cdot(\lambda\hat{i}+2\hat{j}+3\hat{k}) = \left|(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}\right|$ ½ m

$\Rightarrow \quad (2+4-5)+(\lambda+2+3) = \sqrt{(\lambda+2)^2+36+4}$ 1 m

$\therefore \quad (\lambda+6)^2 = (\lambda+2)^2+40 \quad \Rightarrow \quad \lambda=1$ ½ m

Hence $\quad \dfrac{\vec{b}+\vec{c}}{\left|\vec{b}+\vec{c}\right|} = \dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{7} \quad$ or $\quad \dfrac{3}{7}\hat{i}+\dfrac{6}{7}\hat{j}-\dfrac{2}{7}\hat{k}$ 1 m

21. The direction perpendicular to the given lines is given by

$(2\hat{i}-2\hat{j}+\hat{k})\times(\hat{i}+2\hat{j}+2\hat{k})$ 1 m

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = -6\hat{i}-3\hat{j}+6\hat{k}$$
or $2\hat{i}+\hat{j}-2\hat{k}$ 1 m

$\therefore$ Vector equation of required line is

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$$ 1 m

and the cartesian form is

$$\frac{x-2}{2} = \frac{y+1}{1} = \frac{z-3}{-2}$$ 1 m

22. Let probability of success be p and that of failure be q

$\therefore$ p = 3 q, and p + q = 1

$\therefore$ $p = \frac{3}{4}$ and $q = \frac{1}{4}$ 1 m

P (atleast 3 successes) = P($r \geq 3$) = P(3) + P(4) + P(5) ½ m

$$= {}^5C_3\left(\frac{1}{4}\right)^2 \cdot \left(\frac{3}{4}\right)^3 + {}^5C_4\left(\frac{1}{4}\right)^1 \cdot \left(\frac{3}{4}\right)^4 + {}^5C_5\left(\frac{3}{4}\right)^5$$ 1½ m

$$= \frac{10.27}{1024} + \frac{5.81}{1024} + \frac{243}{1024} = \frac{918}{1024} \text{ or } \frac{459}{512}$$ 1 m

SECTION - C

23. Here

$$\begin{aligned} 3x + 2y + z &= 1600 \\ 4x + y + 3z &= 2300 \\ x + y + z &= 900 \end{aligned}$$ 1½

$$\therefore \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix} \text{ or } AX = B$$

$|A| = 3(-2) - 2(1) + 1(3) = -5 \neq 0 \therefore X = A^{-1}B$ ½ m

Cofactors are :

$$\begin{aligned} &A_{11} = -2, \quad A_{12} = -1, \quad A_{13} = 3 \\ &A_{21} = -1, \quad A_{22} = 2, \quad A_{23} = -1 \\ &A_{31} = 5, \quad A_{32} = -5, \quad A_{33} = -5 \end{aligned}$$ 1½ m

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix}\begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix}$$

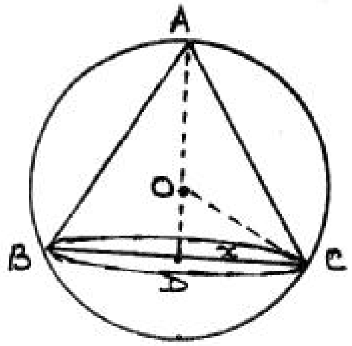
$\therefore$ x = 200, y = 300, z = 400 — 1½ m

i.e. Rs 200 for sincerity, Rs 300 for truthfulness and

Rs 400 for helpfulness

One more value like, honesty, kindness etc. — 1 m

24.



For correct fiqure — ½ m

let radius of cone be x and its height be h.

$\therefore$ OD = (h – r) — ½ m

Volume of cone (v) = $\frac{1}{3}\pi\ x^2h$ ....(i) — ½ m

In Δ OCD, $x^2 + (h-r)^2 = r^2$ or $x^2 = r^2 - (h-r)^2$

$\therefore \quad V = \frac{1}{3}\pi h\ \{r^2 - (h-r)^2\} = \frac{1}{3}\pi\ (-h^3 + 2h^2r)$ — 1 m

$\frac{dv}{dn} = \frac{\pi}{3}\ (-3h^2 + 4hr)$ — 1 m

$\therefore \quad \frac{dv}{dh} = 0 \quad \Rightarrow \quad h = \frac{4r}{3}$ — ½ m

$\frac{d^2v}{dh^2} = \frac{\pi}{3}\ (-6h + 4r) = \frac{\pi}{3}\left(-6\left(\frac{4r}{3}\right) + 4r\right) = -\frac{4\pi r}{3} < 0$ — 1 m

$\therefore$ at $h = \frac{4r}{3}$, Volume is maximum

Maximum volume = $\frac{1}{3}\pi \cdot \left\{-\left(\frac{4r}{3}\right)^2 + 2\left(\frac{4r}{3}\right)^3 r\right\} = \frac{8}{27}\cdot\left(\frac{4}{3}\pi r^3\right)$ — 1 m

$= \frac{8}{27}$ (volume of sphere)

25. $I = \int \frac{1}{\cos^4 x + \sin^4 x}\, dx$

dividing numerator and denominator by $\cos^4 x$

$$= \int \frac{\sec^4 x}{1 + \tan^4 x}\, dx = \int \frac{(1 + \tan^2 x)\sec^2 x}{1 + \tan^4 x}\, dx$$ 1+½ m

Putting $\tan x = t$

$\therefore\ \sec^2 x\, dx = dt$

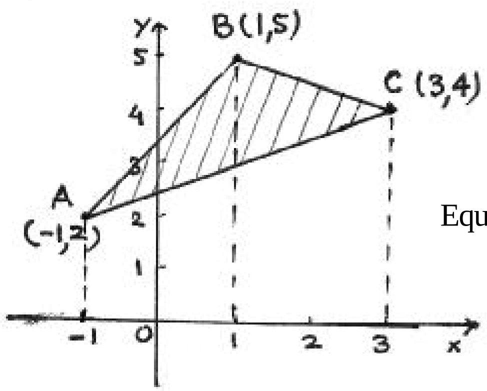
$$= \int \frac{(t^2 + 1)\, dt}{t^4 + 1} = \int \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2}}\, dt \qquad \{\text{dividing by } t^2\}$$ 1+½ m

$$= \int \frac{dz}{z^2 + (\sqrt{2})^2} \quad \text{where} \quad t - \frac{1}{t} = z$$ 1 m

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + c$$ 1 m

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t^2 - 1}{\sqrt{2}\, t}\right) + c = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2} \tan x}\right) + c$$ 1 m

26.



Correct fiqure 1 m

Equation of
- AB is : $y = \frac{1}{2}(3x + 7)$ ½ m
- BC is : $y = \frac{1}{2}(11 - x)$ ½ m
- AC is : $y = \frac{1}{2}(x + 5)$ ½ m

$$\text{Required area} = \frac{1}{2}\int_{-1}^{1}(3x + 7)\, dx + \frac{1}{2}\int_{1}^{3}(11 - x)\, dx - \frac{1}{2}\int_{-1}^{3}(x + 5)\, dx$$ 1 m

$$= \left[\frac{1}{12}(3x+7)^2\right]_{-1}^{1} - \frac{1}{4}\left[(11-x)^2\right]_1^3 - \frac{1}{4}\left[(x+5)^2\right]_{-1}^{3}$$ 1½

$$= 7 + 9 - 12 = 4 \text{ sq. units}$$ 1 m

27. Equation of plane through the intersection of given two planes is :

$$x + y + z - 1 + \lambda\ (2x + 3y + 4z - 5) = 0$$ 1 m

or $(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z - 1 - 5\lambda = 0$ .... (i) ½ m

Plane (i) is perpendicular to the plane $x - y + z = 0$,

so, $1(1+2\lambda) - 1(1+3\lambda) + 1(1+4\lambda) = 0$ 1½ m

$$\Rightarrow \quad 3\lambda = -1 \quad \therefore \quad \lambda = -\frac{1}{3}$$ ½ m

$\therefore$ Equation of plane is $\left(1-\frac{2}{3}\right)x + (1-1)y + \left(1-\frac{4}{3}\right)z - 1 + \frac{5}{3} = 0$ ½ m

i.e $x - z + 2 = 0$ 1 m

Distance of above plane from origin $= \frac{2}{\sqrt{2}} = \sqrt{2}$ units 1 m

OR

Any point on the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ is

$$(2 + 3\lambda)\hat{i} + (-4 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}$$ 1½ m

For the line to intersect the plane, the above point must satisfy the equation of plane, for some value of λ

$\therefore$ $\left\{(2 + 3\lambda)\hat{i} + (-4 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}\right\} \cdot \left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0$ 1 m

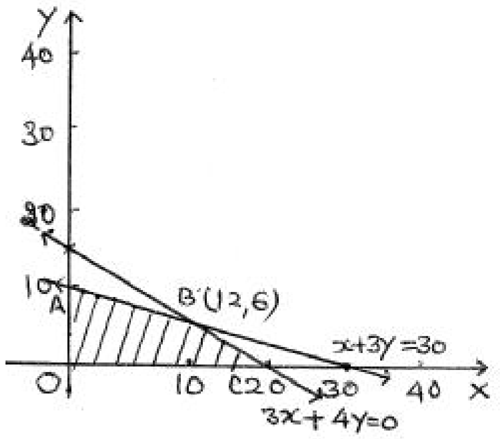
$\Rightarrow$ $2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0 \Rightarrow \lambda = 4$ 1½ m

$\therefore$ The point of intersection is $14\hat{i} + 12\hat{j} + 10\hat{k}$ 1 m

Required distance $= \sqrt{12^2 + 0^2 + 5^2} = 13$ units 1 m

28. Let number of pieces of type A and type B, manufactured per week be x and y respectivily

$\therefore$ L.P.P. is Maximise $P = 80x + 120y$ ½ m



subject to $9x + 12y \leq 180$ or $3x + 4y \leq 60$

$x + 3y \leq 30$

$x \geq 0 \quad y \geq 0$ 2 m

For correct graph : 2 m

Vertices of feasible region are

A (0, 10), B (12, 6), C (20, 0)

P(A) = 1200, P(B) = 1680, P(C) = 1600

$\therefore$ For Max. P, No. of type A = 12

No. of type B = 6 1 m

Maximum Profit = Rs. 1680 ½ m

29. Let event E_1 : choosing first (two headed) coin

E_2 : choosing 2nd (biased) coin

E_3 : choosing 3rd (biased) coin ½ m

$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ 1 m

A : The coin showing heads.

$\therefore \quad P(A/E_1) = 1,\ P(A/E_2) = \frac{75}{100} = \frac{3}{4},\ P(A/E_3) = \frac{60}{100} = \frac{3}{5}$ 1½ m

$$P(E_1/A) = \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{3}{5}}$$ 1 + 1 m

$$= \frac{20}{47}$$ 1 m

OR

Total number of ways of selecting two numbers = ${}^{6}C_{2} = 15$ ½ m

Values of x (larger of the two) can be 2, 3, 4, 5, 6 1 m

$P(x = 2) = \frac{1}{15}, \quad P(x = 3) = \frac{2}{15}, \quad P(x = 4) = \frac{3}{15}$

2½

$P(x = 5) = \frac{4}{15} \quad \text{and} \quad P(x = 6) = \frac{5}{15}$

$\therefore$ Distribution can be written as

x :	2	3	4	5	6
P(x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$
x P(x) :	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{12}{15}$	$\frac{20}{15}$	$\frac{30}{15}$

1 m

$\text{Mean} = \sum x\, P(x) = \frac{70}{15} = \frac{14}{3}$ 1 m