QUESTION PAPER CODE 65/2/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. $\{8, 27\}$ 2. π 3. $\begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix}$

4. 0 5. k = 27 6. tan x – cot x + c 7. $\log \sqrt{2}$ or $\frac{1}{2} \log 2$

8. 1 9. $6\hat{i} - 9\hat{j} + 18\hat{k}$ 10. $\cos^{-1}\left(\frac{19}{21}\right)$

SECTION - B

11. Let $x, y \in W$

If x and y both are even, f (x) = f (y) $\Rightarrow$ x + 1 = y + 1 $\Rightarrow$ x = y

If x and y both are odd, f (x) = f (y) $\Rightarrow$ x – 1 = y – 1 $\Rightarrow$ x = y

If x is odd and y is even i.e. $x \neq y$, (x – 1) is even, (y + 1) is odd

$$x \neq y \quad \Rightarrow \quad f(x) \neq f(y)$$

Similarly for x is even and y is odd.

f is one – one 1½ m

Range of f = $\{f(0), f(1), f(2) \ldots\ldots\} = \{1, 0, 3, 2, \ldots\ldots\}$

= w = codomain

f is onto, 1½ m

Hence f is invertible

$f^{-1} : W \to W \quad f^{-1}(x) = \begin{cases} x - 1, & x \text{ is odd} \\ x + 1, & x \text{ is even} \end{cases}$ 1 m

12. $\cos\left\{\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right\} = \sin\left\{\sin^{-1}\left(\frac{4}{5}\right)\right\}$ 2 m

$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$ 1 m

$\Rightarrow \; 1+x^2 = \frac{25}{16} \;\Rightarrow\; x = \frac{3}{4}, \frac{-3}{4}$ ½ m

$x = \frac{-3}{4}$ does not satisfy so $x = \frac{3}{4}$ ½ m

OR

$\text{L.H.S.} = \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{18}\right)$ 1 m

$$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{56}}\right) + \tan^{-1}\left(\frac{1}{18}\right)$$ 1 m

$$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18}$$ 1/2 m

$$= \tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{1}{11\times 18}}\right) = \tan^{-1}\left(\frac{1}{3}\right) = \cot^{-1} 3$$ 1½ m

13. $\text{L.H.S.} = \begin{vmatrix} a+x+y+z & y & z \\ a+x+y+z & a+y & z \\ a+x+y+z & y & a+z \end{vmatrix}$ operating $c_1 \to c_1 + c_2 + c_3$

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 1 & a+y & z \\ 1 & y & a+z \end{vmatrix}$$ 1 m

$$= (a+x+y+z)\begin{vmatrix} 1 & y & z \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} \quad \begin{array}{l} \text{operating} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$ 2 m

Expanding along c_1, $(a + x + y + z)(a^2 – 0 – 0) = a^2 (a + x + y + z)$ 1 m

14. $\frac{dx}{d\theta} = - a \sin\theta + b\cos\theta, \ \frac{dy}{d\theta} = a\cos\theta + b\sin\theta$ 1½ m

$$\frac{dy}{dx} = - \frac{a\cos\theta + b\sin\theta}{a\sin\theta - b\cos\theta} = - \frac{x}{y}$$ 1 m

$$\frac{d^2y}{dx^2} = - \frac{\left(y - x\frac{dy}{dx}\right)}{y^2} \Rightarrow \frac{y^2\, d^2y}{dx^2} - \frac{x\,dy}{dx} + y = 0$$ 1½ m

15. Taking log on both sedes

$m \log x + n \log y = (m + n) \log (x + y)$ ½ m

$$\Rightarrow \frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{m+n}{x+y}\left(1+\frac{dy}{dx}\right)$$ 1½ m

$$\Rightarrow \frac{dy}{dx}\left(\frac{n}{y} - \frac{m+n}{x+y}\right) = \frac{m+n}{x+y} - \frac{m}{x}$$ 1 m

$$\Rightarrow \frac{dy}{dx}\left\{\frac{nx-my}{y(x+y)}\right\} = \frac{nx-my}{x(x+y)} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$ 1 m

16. $f^1(x) = 6x + 5,$ let $x = 3, \ \Delta x = 0.02$ 1 m

$$f^1(x) \cong \frac{f(x+\Delta x) - f(x)}{\Delta x} \Rightarrow f(x+\Delta x) = (\Delta x) f^1(x) + f(x)$$ 1½ m

$\therefore \quad f(3.02) = (0.02)\ f^1(3) + f(3)$

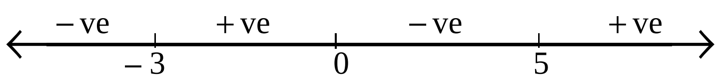
$= (0.02)(23) + 45$

$= 45.46$ 1½ m

OR

$f^1(x) = 6x^3 - 12x^2 - 90x = 6x(x-5)(x+3)$ 1+1 m

$f^1(x) = 0 \Rightarrow x = -3, x = 0, r = 5$ ½ m



$f^1(x) > 0,\ \forall\, x \in (-3, 0) \cup (5, \infty) \Rightarrow$ Strictly increasing

$f^1(x) < 0,\ \forall\, x \in (-\infty, -3) \cup (0, 5) \Rightarrow$ Strictly decreasing 1+½ m

17. Put $x = \cos\theta \quad dx = -\sin\theta\, d\theta$ 1 m

$I = \int \frac{\theta \cos\theta}{\sin\theta} (-\sin\theta)\, d\theta = -\int \theta \cos\theta\, d\theta$ 1 m

$I = -\left\{\theta \sin\theta - \int 1 \cdot \sin\theta\, d\theta\right.$

$\Rightarrow \quad I = -\left\{\theta \sin\theta - \int 1.\sin\theta\, d\theta\right\} = -\theta\sin\theta - \cos\theta + c$ 1 m

$\Rightarrow \quad I = -\sqrt{1-x^2} \cdot \cos^{-1}x - x + c$ 1

OR

$= \int (3x-2)\sqrt{x^2+x+1}\, dx = \int \left\{\frac{3}{2}(2x+1) - \frac{7}{2}\right\} \sqrt{x^2+x+1}\, dx$ 1 m

$$= \frac{3}{2}\int (2x+1)\sqrt{x^2+x+1}\,dx - \frac{7}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\,dx$$ 1 m

$$= \left(x^2+x+1\right)^{3/2} - \frac{7}{2}\left(\frac{2x+1}{4}\sqrt{x^2+x+1}+\frac{3}{8}\log\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|\right)+c$$ 1 + 1 m

18. $x^2(1-y)\,dy = -y^2(1+x^2)\,dx$ ½ m

$$\therefore \int\frac{1-y}{y^2}\,dy = -\int\frac{x^2+1}{x^2}\,dx$$ 1 m

$$\Rightarrow \int\left(\frac{-1}{y^2}+\frac{1}{y}\right)dy = \int\left(1+\frac{1}{x^2}\right)dx$$

$$\Rightarrow \frac{1}{y}+\log|y| = x-\frac{1}{x}+c$$ 1½ m

Putting x = 1, y = 1 we get, c = 1 ½ m

$$\Rightarrow \frac{1}{y}+\log|y| = x-\frac{1}{x}+1$$ ½ m

19. Integrating factor $= e^{\int \cot x dx} = e^{\log \sin x} = \sin x$ 1 m

Solution is $y.\sin x = \int 2\cos x \sin x + c$ 1 m

$$\Rightarrow y\sin x = \int \sin 2x\,dx + c$$

$$\Rightarrow y\sin x = -\frac{\cos 2x}{2}+c$$ 1 m

Here $y=0,\ x=\frac{\pi}{2} \Rightarrow c=-\frac{1}{2}$ ½ m

Solution is $y\sin x = -\frac{\cos 2x}{2}-\frac{1}{2}$ ½ m

20 Here $\left(\vec{a}+\vec{b}\right), \left(\vec{b}+\vec{c}\right), \left(\vec{c}+\vec{a}\right)$ are coplanar, 1 m

$\left(\vec{a}+\vec{b}\right)\cdot\left\{\left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\right\} = 0$ 1 m

$\Rightarrow \left(\vec{a}+\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right) = 0$ 1 m

$\Rightarrow \vec{a}\cdot\left(\vec{b}\times\vec{c}\right)+\vec{a}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{a}\cdot\left(\vec{c}\times\vec{a}\right)+\vec{b}\cdot\left(\vec{b}\times\vec{c}\right)$

$+\vec{b}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{b}\cdot\left(\vec{c}\times\vec{a}\right) = 0$ ½ m

$\Rightarrow 2\left\{\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)\right\} = 0$ **Q** $\vec{b}\cdot\left(\vec{b}\times\vec{c}\right) = 0$ ½ m

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplaner

Similarly converse part can also be proved.

OR

$\vec{a}+\vec{b}=\left(2\hat{i}+3\hat{j}+4\hat{k}\right), \vec{a}-\vec{b}=-\hat{j}-2\hat{k}$ 1+½ m

Let $\vec{c} = \left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}$ 1½ m

$\Rightarrow \vec{c} = -2\hat{i}+4\hat{j}-2\hat{k}$

$\Rightarrow \hat{c} = -\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$ 1 m

21. $\vec{a_1}=\hat{i}+\hat{j}, \vec{b_1}=2\hat{i}-\hat{j}+\hat{k}$

$\vec{a_2}=2\hat{i}+\hat{j}-\hat{k}, \vec{b_2}=3\hat{i}-5\hat{j}+2\hat{k}$

$\vec{a_2}-\vec{a_1}=\hat{i}-\hat{k}$ 1 m

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$$ 1 m

$$\left| \vec{b_1} \times \vec{b_2} \right| = \sqrt{9+1+49} = \sqrt{59}$$ ½ m

$$\left(\vec{a_2} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right) = 3+7=10$$ ½ m

$$S \cdot D = \left| \frac{\left(\vec{a_2} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right)}{\left| \vec{b_1} \times \vec{b_2} \right|} \right| = \frac{10}{\sqrt{59}}$$ 1 m

22. $X \rightarrow$ be the number of red cards drawn

$X = 0\ 1,\ 2,\ 3$ ½ m

$$P(X=0) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2}{17}$$

$$P(X=1) = \frac{{}^{26}C_2 \cdot {}^{26}C_1}{{}^{52}C_3} = \frac{13}{34}$$

$$P(X=2) = \frac{{}^{26}C_2 \cdot {}^{26}C_1}{{}^{52}C_3} = \frac{13}{34}$$

$$P(X=3) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2}{17}$$ 2 m

$$\left.\begin{aligned} \text{Mean} &= \sum pi\ xi = 0 \times \frac{2}{17} + 1 \times \frac{13}{34} + 2 \cdot \frac{13}{34} + 3 \times \frac{2}{17} \\ &= \frac{51}{34} = \frac{3}{2} \text{ or } 1.5 \end{aligned}\right\}$$ 1½ m

SECTION C

23. Here $3x + 2y + z = 2200$

$4x + y + 3z = 3100$ 1½ m

$x + y + z = 1200$

$$\therefore \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix} \text{ or } AX = B$$

$|A| = 3(-2) - 2(1) + 1(3) = -5 \neq 0 \therefore X = A^{-1} B$ ½ m

cofactors are :

$A_{11} = -2$	$A_{12} = -1$	$A_{31} = 3$
$A_{21} = -1$	$A_{22} = 2$	$A_{32} = -1$
$A_{31} = 5$	$A_{23} = -5$	$A_{33} = -5$

1½ m

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$\therefore$ $x = 300$, $y = 400$, $z = 500$ 1½ m

One more value like punctuality, honesty etc 1 m

24. Let V $\rightarrow$ volume, S $\rightarrow$ Total surface area

r $\rightarrow$ radius, h $\rightarrow$ height

$V = \pi r^2 h \Rightarrow h = \frac{v}{\pi r^2}$ ....(i) 1 m

$S = \pi r^2 + 2\pi rh$ ½ m

$$S = \pi r^2 + 2\pi r \cdot \frac{v}{\pi r^2} = \pi r^2 + \frac{2v}{r}$$ 1 m

$$\frac{ds}{dr} = 2\pi r - \frac{2v}{r^2}$$ ½ m

$$\frac{ds}{dr} = 0 \Rightarrow \quad r = \left(\frac{v}{\pi}\right)^{1/3}$$ 1 m

$$\frac{d^2 s}{dr^2} = 2\pi + \frac{4v}{\pi^3} = 2\pi + 4\pi = 6\pi > 0$$ 1 m

at $r = \left(\frac{v}{\pi}\right)^{1/3}$, Total surface area is minimum ½ m

Putting $V = \pi r^3$ in (i)

$$\pi r^3 = \pi r^2 h \quad \Rightarrow \quad r = h$$ ½ m

25. $$I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx \Rightarrow \int_0^{\pi} \frac{(\pi - x) \tan x}{\sec x + \tan x}$$ 1 m

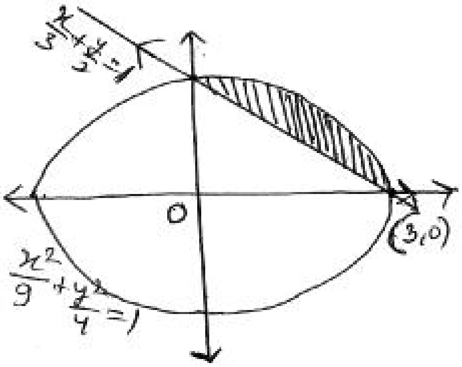
$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} dx \Rightarrow 2I = 2\pi \int_0^{\pi/2} \frac{\tan x}{\sec x + \tan x} dx$$ 1½ m

$$\Rightarrow I = \pi \int_0^{\pi/2} \frac{\sin x (1 - \sin x)}{\cos^2 x} dx = \pi \int_0^{\pi/2} (\sec x \tan x - \tan^2 x) dx$$ ½ m

$$\Rightarrow I = \pi \int_0^{\pi/2} (\sec x \tan x - \sec^2 x + 1) dx = \pi (\sec x - \tan x + x)_0^{\pi/2}$$ 1 m

$$\Rightarrow I = \pi \left(\frac{1 - \sin x}{\cos x} + x\right)_0^{\pi/2} = \pi \left(\frac{\cos x}{1 + \sin x} + x\right)_0^{\pi/2}$$ 1 m

$$\Rightarrow I = \pi \left[\left(0 + \frac{\pi}{2}\right) - (1 + 0)\right] = \frac{\pi}{2} (\pi - 2)$$ 1 m

26. 

Correct figure 1 m

Area of shaded region $= \int_0^3 \left\{ \frac{2}{3}\sqrt{9-x^2} - \frac{2}{3}(3-x) \right\} dx$ 2 m

$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + \frac{(3-x)^2}{2} \right]_0^3$ 2 m

$= \frac{2}{3}\left[\left(0 + \frac{9}{2}\cdot\frac{\pi}{2} + 0\right) - \left(0 + 0 + \frac{9}{2}\right) \right]$

$= \frac{2}{3}\left(9\frac{\pi}{4} - \frac{9}{2}\right) = 3\left(\frac{\pi}{2} - 1\right)$ sq. units 1 m

27. Let equation of plane through (1, –1, 2) with dr's of perpendicular as a, b and c is

$a(x-1) + b(y+1) + c(z-2) = 0$ 1 m

The plane is $\perp$ to $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$

$\therefore \quad 2a + 3b - 2c = 0$ and $a + 2b - 3c = 0$ 1½ m

$\frac{a}{-5} = \frac{b}{4} = \frac{c}{1} = k \Rightarrow a = -5k,\ b = 4k,\ c = k$ 1½ m

Equation of the plane is

$-5k(x-1) + 4k(y+1) + k(z-2) = 0 \Rightarrow -5x + 4y + z + 7 = 0$ 1 m

Distance of plane from (– 2, 5, 5) is

$$d = \left| \frac{10+20+5+7}{\sqrt{25+16+1}} \right| = \frac{42}{\sqrt{42}} = \sqrt{42}$$ 1 m

OR

Line through A (2, –1, 2) and B (5, 3, 4) is

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda$$ 1½ m

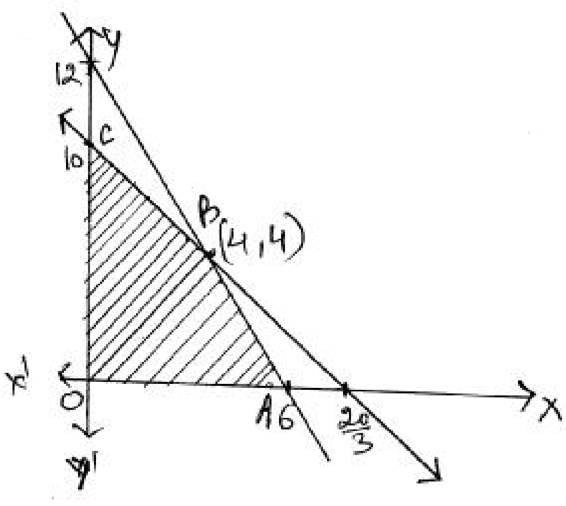
General point on the line is $(3\lambda+2,\ 4\lambda-1,\ 2\lambda+2)$ 1

$\therefore\ 3\lambda+2-(4\lambda-1)+2\lambda+2 = 5 \Rightarrow \lambda=0$ 1½ m

Point of intersection is (2, –1, 2) 1 m

$$d = \sqrt{(3)^2+(4)^2+(12)^2} = \sqrt{169} = 13$$ 1 m

28. Let the number of lamps and shades manufactured be x and y respectively



$\therefore$ L.P.P. is Maximise $Z = 25x + 15y$ ½ m

Subject to $2x + y \le 12$

$3x + 2y \le 20$

$x \ge 0,\ y \ge 0$ 2 m

For correct graph 2 m

Vertices of feasible

region are 0(0, 0), A(6, 0), B(4, 4) C(0, 10)

P(A) = 150, P (B) = 160, P(C) = 150 ½ m

For max Prof no. of lamps = 4

No. of shades = 4 1 m

Maximum Profit = Rs. 160

29. Let E_1 : Scooter driver is chosen

E_2 : Car driver is chosen

E_3 : Truck driver is chosen ½ m

A : Person meets with an accident

$$P(E_1) = \frac{1}{6},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{2}$$ 1 m

$$P\left(A/E_1\right) = 0.01,\ P\left(A/E_2\right) = 0.03,\ P\left(A/E_3\right) = 0.15$$ 1 m

$$P\left(E_3/A\right) = \frac{\frac{1}{2}\times(0.15)}{\frac{1}{6}\times(0.01)+\frac{1}{3}\times(0.03)+\frac{1}{2}\times(0.15)} = \frac{45}{52}$$ 1+1 m

$$P\left(E_1/A \text{ or } E_2/A\right) = 1-\left(E_3/A\right)$$ 1 m

$$= 1-\frac{45}{52} = \frac{7}{52}$$ ½ m

OR

Let E be the event drawing a diamond card

$$n=5,\ p = \frac{1}{4},\ q = \frac{3}{4}$$ 1½ m

$$P(E) = \frac{13}{52} = \frac{1}{4}$$

$$P(\overline{E}) = \frac{3}{4}$$

(i) $P(5) = {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0 = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$ 1½ m

(ii) $P(3) = {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 = \frac{45}{512}$ 1½ m

(iii) $P(0) = {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5 \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$ 1½ m