Marking Scheme

Q.No.	Value Points/Solution	Marks.

SECTION-A

1-10

1. $\frac{11\pi}{12}$ **2.** $\frac{5}{12}$ **3.** $a = 1$ **4.** $x = 2$

5. $\begin{bmatrix} 8 & -3 & 5 \\ -2 & -3 & -6 \end{bmatrix}$ **6.** 2 **7.** zero

8. $\frac{\pi}{3}$ **9.** $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$

10. 30. 255, (i) Pollution control in the environment (ii) or, any other vlaue suggested with just fication)

SECTION-B

11. Let x_1, x_2, be the elements of A, then $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{4x_1+3}{6x_1-4} = \frac{4x_2+3}{6x_2-4}$$

$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one 1½

Let $y = \frac{4x+3}{6x-4} \Rightarrow 6xy - 4y = 4x + 3$

$$\Rightarrow x = \frac{4y+3}{6y-4}$$ 1

$\Rightarrow$ For each $y \in$ R $-$ {2/3},

there exists $x = \frac{4y+3}{6y-4} \in R - \left\{\frac{2}{3}\right\}$ such that $f(x) = y$ ½

$\therefore f$ is an onto function 1

Also $f^{-1}(x) = \frac{4x+3}{6x-4}$

12. $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right] = \tan\frac{1}{2}\left[2\tan^{-1}x+2\tan^{-1}y\right]$ 1½

$= \tan[\tan^{-1}x+\tan^{-1}y]=\tan\left[\tan^{-1}\frac{x+y}{1-xy}\right]$ 1½

$= \frac{x+y}{1-xy}$ 1

OR

LHS $= \tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{8}\right)$ 1+1

$= \tan^{-1}\left[\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\times\frac{1}{5}}\right]+\tan^{-1}\frac{1}{8}=\tan^{-1}\left[\frac{7}{9}\right]+\tan^{-1}\left(\frac{1}{8}\right)$ 1

$= \tan^{-1}\left[\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{72}}\right]=\tan^{-1}\left(\frac{65/72}{65/72}\right)=\tan^{-1}1$ 1

$= \frac{\pi}{4}$ = RHS 1

13. Using $R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$ 1

$$= (1+x+x^2)\begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$: using $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ 1

$$\Delta = (1+x+x^2)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & x^2-x & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$$ 1

$$= (1+x+x^2)(1-x)(1-x)\begin{vmatrix} 1+x & x \\ -x & 1 \end{vmatrix}$$ 1

$= (1-x)^2(1+x+x^2)^2$

$= (1-x^3)^2$ 1

14. $y = (\log x)^x + x^{\log x} = u + v(\text{say}) \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ½

$\log u = x \log(\log x) \Rightarrow \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right]$ 1½

$\log v = \log x.\log x = (\log x)^2 \Rightarrow \frac{dv}{dx} = x^{\log x}\left[\frac{2}{x}\log x\right]$ 1

$\therefore \ \frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x)\right] + x^{\log x}\left[\frac{x}{2}\log x\right]$ 1

15. $y = \log[x + \sqrt{x^2+a^2}] \Rightarrow \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left[1 + \frac{\not{2}x}{\not{2}\sqrt{x^2+a^2}}\right]$ 1

$$= \frac{1}{x\sqrt{x^2+a^2}}\left[\frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}\right]$$ 1

$\therefore \ \sqrt{x^2+a^2}\,\frac{dy}{dx} = 1$ ½

Diff. again, $\sqrt{x^2+a^2}\,\frac{d^2y}{dx^2} + \frac{\not{2}x}{\not{2}\sqrt{x^2+a^2}}\,\frac{dy}{dx} = 0$ 1

or $(x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$ ½

16. $f(x) = |x - 3| = (x - 3)$ if $x \geq 3$
$(3 - x)$ if $x < 3$ 1

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(3-x) = 0$ 1

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(x-3) = 0$ and $f(3) = 0$ ½

LHD = $\lim_{h\to 0}\frac{f(3-h)-f(3)}{(3-h)-3} = ...\lim_{h\to 0}\frac{[3-(3-h)]-0}{(3-h)-3} = -1$ 1

RHD = $\lim_{h\to 0}\frac{f(3+h)-f(3)}{(3+h)-3} = \frac{(h+3-3)-0}{3+h-3} = 1$ ½

As LHD ≠ f is not differentiable at $x = 3$. ½

OR

$x = a \sin t \Rightarrow \frac{dx}{dt} = a\cos t$ 1

$y = a(\cos t + \log \tan t/2) \Rightarrow \frac{dy}{dt} = a\left(-\sin t + \frac{1}{2}.\frac{\sec^2 t/2}{\tan t/2}\right)$ 1

$= a\left(-\sin t + \frac{1}{\sin t}\right) = a\frac{\cos^2 t}{\sin t}$ 1½

$\therefore \frac{dy}{dx} = \frac{a\cos^2 t}{\sin t}.\frac{1}{a\cos t} = \cot t$ ½

$\therefore \frac{d^2y}{dx^2} = -\text{cosec}^2 t.\frac{dt}{dx} = -\frac{\text{cosec}^2 t}{a\cos t}$ 1

17. $I = \int \frac{\sin(x-a)}{\sin(x+a)}dx = \int \frac{\sin\{(x+a)-2a\}}{\sin(x+a)}dx$ 1

$= \cos 2a\int dx - \int \cot(x+a).\sin 2a\, dx$ 1

$= x\cos 2a - \sin 2a \log|\sin(x+a)| + c$ 1

OR

$I = \int \frac{5x-2}{1+2x+3x^2}dx$

$5x-2 = A(6x+2)+B \Rightarrow A = \frac{5}{6}, B = \frac{-11}{3}$ ½+½

$\therefore \quad I = \frac{5}{6}\int \frac{6x+2}{1+2x+3x^2}dx - \frac{11}{3x3}\int \frac{1}{\left(x+\frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2}$ 1

$= \frac{5}{6}\log\left|1+2x+3x^2\right| - \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + C$ 1+1

18. $\frac{x^2}{(x^2+4)(x^2+9)}$: Let $x^2 = t$

$\therefore$ Given expression can be written as

$\frac{t}{(t+4)(t+9)} = \frac{A}{t+4} + \frac{B}{t+9}$ ½

On solving to get $A = -\frac{4}{25}$ and $B = \frac{9}{5}$ 1

Replacing t by x^2, we get

$\Rightarrow I = \int \frac{x^2}{(x^2+4)(x^2+9)}dx = \int \frac{-4}{5}\frac{dx}{x^2+4} + \int \frac{9}{5}\frac{dx}{x^2+9}$ 1

$= \frac{-2}{5}\tan^{-1}\frac{x}{2}+\frac{3}{5}\tan^{-1}\frac{x}{3}+C$ ½

19. $I = \int_0^4 (|x|+|x-2|+|x-4|)dx$ ½

$= \int_0^4 x dx+\int_0^2 (2-x)dx+\int_2^4 (x-2)dx+\int_0^4 (4-x)dx$ 1½

$= \left|\frac{x^2}{2}\right|_0^4+\left|2x-\frac{x^2}{2}\right|_0^2+\left|\frac{x^2}{2}-2x\right|_2^4+\left|4x-\frac{x^2}{2}\right|_0^4$ 1½

$= 8+(4-2)+[(8-8)-(2-4)]+[(16-8)]$ 1

$= 8 + 2 + 2 + 8$

$= 20$

20. $|\vec{a}+\vec{b}|=|\vec{a}| \Rightarrow |\vec{a}+\vec{b}|^2=|\vec{a}|^2$ 1

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}=|\vec{a}|^2$ 1

$\Rightarrow 2\vec{a}.\vec{b}+|\vec{b}|=0$ 1

or $(2\vec{a}+\vec{b}).\vec{b}=0$ 1½

which gives $2\vec{a}+\vec{b}$ is $\perp$ $\vec{b}$ ½

21. A general point on the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ is ...(i) 1

$3\lambda + 2,\ 4\lambda - 1,\ 2\lambda + 2$

If this point lies on the plane $x - y + z - 5 = 0$, ...(ii)

it should satisfy it

$\Rightarrow (3\lambda+2)-(4\lambda-1)+(2\lambda+2)-5=0$ 1

$\Rightarrow \lambda = 0$

$\therefore$ The point of intersection of line (i) and plane (ii) is (2, –1, 2) ½

Angle between line (i) and plane (ii) is given by

$$\sin\theta = \left|\frac{3.1+4(-1)+1(1)}{\sqrt{3^2+4^2+2^2}\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{87}}$$ 1

or $\theta = \sin^{-1}\left(\frac{1}{\sqrt{87}}\right)$ ½

OR

Vector equation of a plane containing line of intersection of planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and

$\vec{r}.(2\hat{i}+\hat{j}-\hat{k}+5)]=0$ is $[\vec{r}.(\hat{i}+2\hat{j}+3\hat{k}-4)+\lambda[\vec{r}.(2\hat{i}+\hat{j}-\hat{k}+5]=0$

$\Rightarrow\ \vec{r}.[(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}]=4-5\lambda$ ...(i) 1½

(i) is given $\perp$ to the plane $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$

$\therefore\ (1+2\lambda)5+(2+\lambda)3+(3-\lambda)(-6)=0$

$19\lambda = 7$ or $\lambda = \frac{7}{19}$ 1

$\therefore$ Regd equation of plane is

$$\vec{r}.\left[\left(1+\frac{14}{19}\right)\hat{i}+\left(2+\frac{7}{19}\right)\hat{j}+\left(3-\frac{7}{19}\right)\hat{j}\right]=4-\frac{35}{19}$$ 1

or $r.[33\hat{i}+45\hat{j}+50\hat{k}]=41$ ½

22. Let the events be defined as below :

E : A speak the truth

F : B speaks the truth

$P(E) = \frac{60}{100} = 0.6,\quad P(F) = \frac{9}{100} = 0.9$

$P(\bar{E}) = 0.4$, P $P(\bar{F}) = 0.1$

$\therefore$ Required probability

$= P(E\cap\bar{F})+P(\bar{E}\cap F)$

$= 0.6 \times 0.1 + 0.4 \times 0.9$

$= 0.42$ 1

$\therefore$ In 42% of the cases, A and B are likely to contradict each other.

Any value suggested with justificaion may be accepted. 1

SECTION-C

23. Let the values honesty, regularity and bard work be denotes by x, y and z respectively ½

From the question

(i) $x+y+z=6000$ (ii) $3z+x=11000$ (iii) $x+z=2y$ or $x+2-2y=0$ 1

In matrix from, the system of = x can be written as

$$\begin{pmatrix}1 & 1 & 1\\ 1 & 0 & 3\\ 1 & -2 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}6000\\ 11000\\ 0\end{pmatrix}$$

In the for AX = B, where A^{-1} exists of $|A| \neq 0$ 1

$|A| = 1\times 6 - 1(-2) + 1(-2) = 6 \neq 0 \Rightarrow A^{-1}$ exists

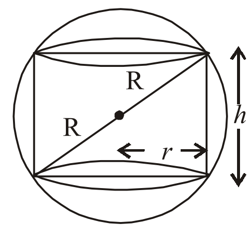
Adj A = $\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$, $\therefore$ $A^{-1} = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$ 1½

$$x = A^{-1}B = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix} = \begin{bmatrix} 500 \\ 2000 \\ 3500 \end{bmatrix}$$ 1

$\therefore$ Award money fo (i) Honest = Rs 500 (ii) Regularity = Rs. 2000 (iii) Hard work = Rs. 3500

Any other value suggested with full justification may be accepted. 1

24. From the figure, $h^2 + 4r^2 = 4R^2$ ....(i)



$v = \pi r^2 h = \pi \frac{[4R^2 - h^2]h}{4}$ [using (i)] 1

$\frac{dv}{dh} = \frac{\pi}{4}[4R^2 - 3h^2]$ 1½

$\frac{dv}{dh} = 0 \Rightarrow h = \frac{2R}{\sqrt{3}}$ 1½

Showing $\frac{d^2v}{dh^2}$ is negative when $h = \frac{2R}{\sqrt{3}}$ 1

$\therefore$ V is maximum at $h = \frac{2R}{\sqrt{3}}$ 1½

and maximum volume = $\frac{\pi}{4}\left[\frac{8R^3}{\sqrt{3}} - \frac{8R^3}{3\sqrt{3}}\right] = \frac{4\pi R^3}{3\sqrt{3}}$ 1

OR

The equation of curve is $y = \frac{x^2}{4} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$ 1½

$\therefore$ Slope of normal is $\frac{-2}{x}$, at (x_1, y_1)

$\therefore$ Equation of normal to the curve at (x_1, y_1) is

$$y - y_1 = -\frac{2}{x_1}(x - x_1) \quad ....(i)$$ 1

(i) passes through (1, 2) and using $y_1 = \frac{x_1^2}{4}$, we get

$x_1 = 2,\ y_1 = 1$ 1½

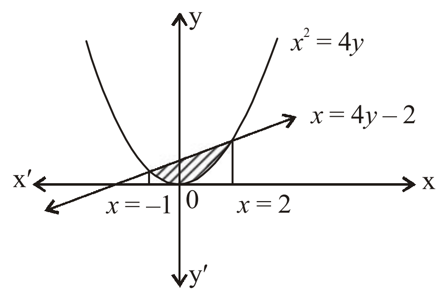
$\therefore$ Equation of normal is $y - 1 = -1\ (x - 2) \Rightarrow x + y = 3$ 1

Equation of corresponding tangent is

$y - 1 = 1(x - 2) \Rightarrow x - y = 1$ 1

25. Curves are intersecting figure 1

at $x = 2, -1$ 1½



Regq. area $= \int_{-1}^{2} \frac{x+2}{4} dx - \int_{-1}^{2} \frac{x^2}{4} dx$ 2

$= \frac{1}{4}\left[\left(\frac{x^2}{2} + 2x\right) - \left(\frac{x^3}{3}\right)\right]_{-1}^{2}$ 1

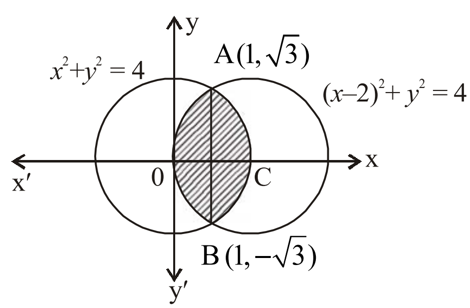
$= \frac{9}{8}$ sq. units. ½

OR

Points of intersection of two curres are $(1, \pm\sqrt{3})$ 1

The area is symmetrical about AB

$\therefore$ Reqd. area



$= 4\left[\int_{1}^{2} y dx\right] = 4\left[\int_{1}^{2} \sqrt{4x^2} dx\right]$ 2

$= 4\left[\frac{1}{2} x\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_1^2$ 2

$= \frac{8\pi}{3} - 2\sqrt{3}$

26. The given diff. equation can be written as

$$\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}}}{2ye^{\frac{x}{y}}} = \frac{2\frac{x}{y}e^{\frac{x}{y}}}{2e^{x/y}} \quad ...(i)$$

(i) is a homogenous function of $\frac{x}{y}$ 2

$\therefore$ we put $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ ½

(i) can be written as $v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$ 1

$\therefore \ y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{-1}{2e^v}$ ½

$\Rightarrow \ 2e^v dv - \frac{dv}{y}$

$\Rightarrow \ 2e^v = -\log y + c$

or $2e^{\frac{x}{y}} = C - \log y$

when $x = 0$, $y = 1 \Rightarrow c = z$ ½

$\therefore$ The particular solution of the given diff equation is

$2e^{\frac{x}{y}} + \log y = 2$ ½

27. Let A = $\hat{i} + \hat{j} - 2\hat{k}$, $B = 2\hat{i} - \hat{j} + \hat{k}$ and $C = \hat{i} + 2\hat{j} + \hat{k}$ 1

$\therefore \ \overrightarrow{AB} = \hat{i} - 2\hat{j} + 3\hat{k}, \overrightarrow{AC} = \hat{j} + 3\hat{k}$ 1

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = -9\hat{i} - 3\hat{j} + \hat{k}$$ 1

Equation of plane is $\vec{r}.\vec{n} = \vec{a}.\vec{n}$

or $\vec{r}.(-9\hat{i} - 3\hat{j} + \hat{k}) = -9 - 3 - 2 = -14$ 1

or $\vec{r}.(9\hat{i} + 3\hat{j} + \hat{k}) = 14$ 1

The equation of line is $\frac{x-3}{2} = \frac{y+1}{-2} = \frac{z+1}{1}$ ....(i)

A general point on (i) is $2\lambda + 3, -2\lambda - 1, \lambda - 1$

The equation of plane is $9x + 3y - z - 14 = 0$

The point should satify plane if it lies on it

$\Rightarrow \lambda = -1$ 1

$\therefore$ The point is (1, 1, – 2) 1

28. Let crop A be grown on x hectares of land and crop B be grown on y hectares of land

Profit function = $10500\, x + 9000\, y = p$ ½

Subject to constraints

(i) $x + y \leq 50$

(ii) $20x + 10y \leq 800$ or $2x + y \leq 80$

$x \geq 0,\ y \geq 0$

Corneas of feaible region are

A(0, 50), C(40, 0), O(0, 0), B(30, 20) 1½ + 1½

P(A) = 450000

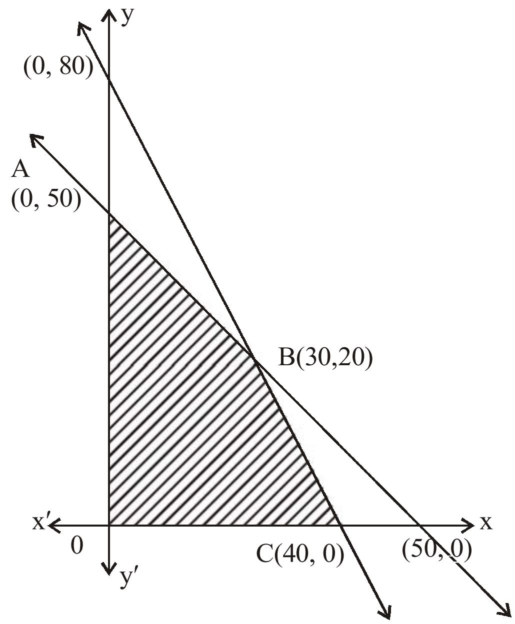
P(B) = 315000 + 180000 = 495000

P(C) = 420000

∴ For maximum profit 1

Crop A : 30 hectares

Crop B : 20 hectares



Value : (i) yes or no (any response written) or (ii) Any other value suggested withful justification may also accepted.

29. E_1 : The patient follows a course of meditation and yoga

E_2 : The patient takes certain drugs; A : The patient suffers a heart attack 2

$P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$, $P(A/E_1) = \frac{70}{100}\times\frac{40}{100}$ of $P(A/E_2) = \frac{75}{100}\times\frac{40}{100}$

Reqd. probability = $P(E_1/A) = \dfrac{P(E_1)\,.\,P(A/E_1)}{\sum_{i=1}^{2} P(E_i).P(A/E_i)}$ 1

$$= \frac{\frac{1}{2}\times\frac{70}{100}\times\frac{40}{100}}{\frac{1}{2}\times\frac{70}{100}\times\frac{40}{100}+\frac{1}{2}\times\frac{75}{100}\times\frac{40}{100}} = \frac{14}{29}$$ 2

∴ Meditation and yoga is very important and beneficial for human values.