Question Paper - Foreign (2012)

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three Sections A, B and C, Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

SECTION-A

Questions numbers 1 to 10 carry 1 mark each.

Q1. If the binary operation * on the set Z of integers is defined by $a * b = a + b - 5$, then write the identity element for the operation * in Z.

Q2. Write the value of $\cot(\tan^{-1} a + \cot^{-1} a)$.

Q3. If A is a square matrix such that $A^2 = A$, then write the value of $(I + A)^2 - 3A$.

Q4. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, write the value of x.

Q5. Write the value of the following determinant :

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

Q6. If $\int \left(\frac{x-1}{x^2}\right) e^x dx = f(x)e^x + c$, then write the value of $f(x)$.

Q7. If $\int_0^a 3x^2 dx = 8$, write the value of 'a'.

Q8. Write the value of $(\hat{i} \times \hat{j}) \cdot \hat{k} + (\hat{j} \times \hat{k}) \cdot \hat{i}$.

Q9. Write the value of the area of the parallelogram determined by the vectors $2\hat{i}$ and $3\hat{j}$.

Q10. Write the direction cosines of a line parallel to z-axis.

SECTION-B

Questions numbers 11 to 22 carry 4 mark each.

Q11. If $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$, show that fof($x$) = x for all $x\neq\frac{2}{3}$. What is the inverse of f ?

Q12. Prove that : $\sin^{-1}\left(\frac{63}{65}\right)=\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)$

OR

Sovle for x :

$$2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x), x\neq\frac{\pi}{2}$$

Q13. Using properties of determinants, prove that

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}=a^3$$

Q14. If $x^m y^n=(x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

Q15. If $y=e^{a\cos^{-1}x}, -1\leq x\leq 1$ show that

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0.$$

OR

If $x\sqrt{1+y}+y\sqrt{1+x}=0,\ -1<x<1, x\neq y$, then prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.

Q16. Show that $y=\log(1+x)-\frac{2x}{2+x}, x>-1$, is an increasing function of x throughout its domain.

OR

Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

Q17. Evaluate : $\int x^2\tan^{-1}x\,dx$

OR

Evaluate : $\int\frac{3x-1}{(x+2)^2}dx$

Q18. Solve the following differential equation :

$$\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right]\frac{dx}{dy}=1, x\neq 0.$$

Q19. Solve the following differential equation :

$$3e^x \tan y dx + (2 - e^x) \sec^2 y\, dy = 0$$, given that when $x = 0$, $y = \frac{\pi}{4}$.

Q20. If $\vec{\alpha} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 4\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicualr to $\vec{\alpha}$

Q21. Find the vector and cartesian equations of the line passing through the point P(1, 2, 3) and parallel to the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

Q22. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

SECTION-C

Questions numbers 23 to 29 carry 6 mark each.

Q23. Using matrices, solve the following system of equations :

$$x - y + z = 4;\ 2x + y - 3z = 0;\ x + y + z = 2$$

OR

If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Q24. Show that the altutude of the right circular cone of maximum volume that can be inscribed in a sphere of radius R is $\frac{4R}{3}$.

Q25. Find the area of the region in the first quadrant enclosed by x-axis, the line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

Q26. Evaluate : $\int_1^3 (x^2 + x)dx$ as a limit of a sum.

OR

Evaluate : $\int_0^{\pi/4} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} dx$

Q27. Find the vector equation of the plane passing through the points (2, 1, – 1) and (–1, 3, 4) and perpendicular to the plane $x - 2y + 4z = 10$. Also show that the plane thus obtained contains the line $\vec{r} = -\hat{i} + 3\hat{j} + 4\hat{k} + \lambda(3\hat{i} - 2\hat{j} - 5\hat{k})$.

Q28. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs ` 10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B that costs ` 4. How many packets of mixes from S and T should the company purchase to honour the contract requirement and yet minimize cost ? Make a LPP and solve graphically.

Q29. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.