EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\vec{a} + \vec{b} = 6\hat{i} + \hat{k}$ ½ m

    $\therefore$ Reqd. unit vector $= \dfrac{6}{\sqrt{37}}\hat{i} + \dfrac{1}{\sqrt{37}}\hat{k}$ ½ m

2. Reqd. area $= \left| \vec{a} \times \vec{b} \right|$ ½ m

    $\therefore \left| 12\hat{i} - 4\hat{j} + 8\hat{k} \right| = \sqrt{144 + 16 + 64} = \sqrt{224}$ or $4\sqrt{14}$ sq. units   ½ m

3. Getting $x-$ intercept $= \dfrac{5}{2}$, $y-$ intercept $= 5$, $z-$ intercept $= -5$ ½ m

    $\therefore$ Their sum $= \dfrac{5}{2}$ ½ m

4. $\text{co} - \text{factor of } a_{21} = 3$ 1 m

5. Degree = Order = 2   any one correct   ½ m

    $\therefore$ Degree + order = 4   ½ m

6. $2^y \, dy = dx \Rightarrow \dfrac{2^y}{\log 2} = x + c$ ½+½ m

SECTION - B

7.   Getting $A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$ 1 m

$4A - 3I = \begin{bmatrix} 8-3 & -4 \\ -4 & 8-3 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} = A^2$ 1 m

$A^2 = 4A - 3I$ .... (i)

Multiply both sides by A^{-1} ½ m

$A = 4I - 3A^{-1}$ or $A^{-1} = \dfrac{1}{3}(4I - A) = \dfrac{1}{3}\begin{pmatrix} 4-2 & 0+1 \\ 0+1 & 4-2 \end{pmatrix} = \dfrac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ 1½ m

OR

$A^2 = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B^2 = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} a^2+b & a-1 \\ b(a-1) & b+1 \end{bmatrix}$ 1½ m

$(A+B)^2 = \begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} 1+a & 0 \\ 2+b & -2 \end{pmatrix}\begin{pmatrix} (1+a)^2 & 0 \\ (2+b)(1+a)-2(2+b) & 4 \end{pmatrix}$ .... (i)   1½ m

$A^2 + B^2 = \begin{pmatrix} a^2+b-1 & a-1 \\ b(a-1) & b \end{pmatrix}$ ....(ii)

Equating (i) and (ii), we get $b = 4, a = 1$ 1 m

8.   Using $C_1 \rightarrow C_1 + C_2 + C_3$ and taking $a^2 + a + 1$ common from C_1

$\Delta = (a^2+a+1)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}$, using $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$ 1½ m

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$$\Delta = \left(a^2 + a + 1\right) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & a(a-1) & (1-a)(1+a) \end{vmatrix} = \left(a^2 + a + 1\right)(1-a)^2 \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & 1+a \end{vmatrix}$$ 1½ m

$$= \left(a^2 + a + 1\right)(1-a)^2 \left(1 + a + a^2\right)$$

$$= \left[(1-a)\left(1 + a + a^2\right)\right]^2 = \left(1 - a^3\right)^2$$ 1 m

9.   Let $x + a = t \implies dx = dt$ and $x = t - a \implies x - a = t - 2a$ 1 m

$$\therefore \; I = \int \frac{\sin(t - 2a)\, dt}{\sin t} = \int \frac{(\sin t \cos 2a - \cos t \cdot \sin 2a)\, dt}{\sin t}$$ 1 m

$$= \cos 2a \int dt - \sin 2a \int \cot t \, dt = \cos 2a \, t - \sin 2a \cdot \log |\sin t| + c$$ 1 m

$$= \cos 2a \,(x + a) - \sin 2a \, \log |\sin(x + a)| + c$$ 1

<p align="center">OR</p>

consider $\dfrac{x^2}{\left(x^2 + 4\right)\left(x^2 + 9\right)}$ · Let $x^2 = t$ ½ m

$$\therefore \; \frac{t}{(t+4)(t+9)} = -\frac{4}{5}\frac{1}{t+4} + \frac{9}{5}\frac{1}{t+9}$$ 1 m

$$I = \int \frac{t \; dt}{(t+4)(t+9)} = -\frac{4}{5}\int \frac{dt}{t+4} + \frac{9}{5}\int \frac{dt}{t+9}$$ ½ m

$$= \frac{-4}{5} \log |t+4| + \frac{9}{5} \log |t+9| + c$$ 1½ m

$$\therefore \; I = -\frac{4}{5} \log \left|x^2 + 4\right| + \frac{9}{5} \log \left|x^2 + 9\right| + c$$ ½ m

10.   Writing given integral as   1 m

$$I = \int_{-\pi/2}^{0} \frac{\cos x}{1 + e^x}\, dx + \int_{0}^{\pi/2} \frac{\cos x}{1 + e^x}\, dx \qquad \text{Let } x = -t, \, dx = -dt$$ 1½ m

$$\text{when } x = -\frac{\pi}{2}, \, t = \pi/2$$

$$x = 0, \, t = 0$$

<p align="center">4</p>

$$\therefore \ I \ = \ \int_0^{\frac{\pi}{2}} \frac{e^t \cos t}{1+e^t} \, dt + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} \, dx = \int_0^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x} \, dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} \, dx \qquad 1\tfrac{1}{2} \text{ m}$$

$$I \ = \ \int_0^{\frac{\pi}{2}} \frac{\left(1+e^x\right)\cos x \, dx}{\left(1+e^x\right)} = \int_0^{\frac{\pi}{2}} \cos x \, dx = \left(\sin x\right)_0^{\frac{\pi}{2}} = 1 \qquad 1 \text{ m}$$

11.   Let B_1, B_2, B_3 be the events that the bolts produced by machines   ½ m

E_1, E_2, E_3 and A be the event that the selected bulb is defective

$$\therefore \ P\left(B_1\right) = \frac{1}{2}, \ P\left(B_2\right) = P\left(B_3\right) = \frac{1}{4}$$

1½ m

$$P\left(\frac{A}{B_1}\right) = \frac{1}{25}, \ P\left(\frac{A}{B_2}\right) = \frac{1}{25}, \ P\left(\frac{A}{B_3}\right) = \frac{1}{20}$$

$$P\left(A\right) = \sum_{c=1}^{3} P\left(B_c\right) P\left(\frac{A}{B_c}\right) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20} + \frac{1}{4} \times \frac{1}{25} \qquad 1+\tfrac{1}{2} \text{ m}$$

$$= \ \frac{17}{400} \qquad \text{½ m}$$

OR

$$\therefore \ P\left(x=3\right) = \frac{1}{6} \times \frac{1}{5} \times 2 = \frac{1}{15}, \ P\left(x=4\right) = \frac{2}{6} \times \frac{1}{5} \times 2 = \frac{2}{15}$$

$$\text{Simlarly} \ P\left(x=5\right) = \frac{3}{15}, \ P\left(x=6\right) = \frac{4}{15}, \ P\left(x=7\right) = \frac{5}{15}$$

Prob. distribution is

x :	3	4	5	6	7
P(x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

2 m

x . P (x) :	$\dfrac{3}{15}$	$\dfrac{8}{15}$	$\dfrac{15}{15}$	$\dfrac{24}{15}$	$\dfrac{35}{15}$
$x^2 P(x)$:	$\dfrac{9}{15}$	$\dfrac{32}{15}$	$\dfrac{75}{15}$	$\dfrac{144}{15}$	$\dfrac{245}{15}$

$\text{Mean} = \sum x_i \cdot P(x_i) = \dfrac{85}{15} = \dfrac{17}{3}$ 1 m

$\text{Variance} = \sum x_i^2 \, P(x_i) - (\text{Mean})^2 = \dfrac{101}{3} - \dfrac{289}{9} = \dfrac{14}{9}$ 1 m

12.

$\vec{BC} = \left(3\hat{i} - \hat{j} + 4\hat{k} \right) - \left(\hat{j} + \hat{k} \right) = 3\hat{i} - 2\hat{j} + 3\hat{k}$ 1½ m
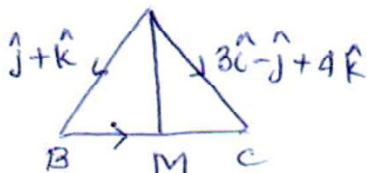
$\therefore \ \vec{BM} = \dfrac{3}{2}\hat{i} - \hat{j} + \dfrac{3}{2}\hat{k}$ 1 m

$\text{AM} = \left| \hat{j} + \hat{k} + \dfrac{3\hat{i} - 2\hat{j} + 3\hat{k}}{2} \right| = \left| \dfrac{3\hat{i} + 5\hat{k}}{2} \right| = \dfrac{\sqrt{34}}{2}$ 1½ m

13.   Any plane through given point is $a(x-3) + b(y-6) + c(z-4) = 0 \ldots\ldots\ldots$(i)   1 m

with $a + 5b + 4c = 0 \ldots\ldots$(A)   ½ m

(i) passes through $(3, 2, 0) \implies -4b - 4c = 0$ or $b + c = 0 \ldots\ldots$(B)   ½ m

From (A) and (B) $a + b + (4b + 4c) = 0 \implies a = -b$ 1 m

$\therefore \ a = -b = c$ 1 m

$\therefore \ $ Required eqn. of plane is $x - y + z - 1 = 0$

14. $\text{LHS} = \tan^{-1}\left(\dfrac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta}$ 2 m

$\therefore \tan^{-1}\dfrac{2\cos\theta}{\sin^2\theta} = \tan^{-1}\left(\dfrac{2}{\sin\theta}\right)$ 1 m

$\Rightarrow \cot\theta = 1$ or $\theta = \dfrac{\pi}{4}$ 1 m

OR

The given equation can be written

$\left(\tan^{-1}2 - \tan^{-1}1\right) + \left(\tan^{-1}3 - \tan^{-1}2\right) + \left(\tan^{-1}4 - \tan^{-1}3\right) + \ldots + \tan^{-1}(n+1) - \tan^{-1}n = \tan^{-1}\theta$ 2 m

$\Rightarrow \tan^{-1}(n+1) - \tan^{-1}1 = \tan^{-1}\theta$ 1 m

$\Rightarrow \tan^{-1}\dfrac{n+1-1}{1+(n+1)} = \tan^{-1}\theta \Rightarrow \theta = \dfrac{n}{n+2}$ 1 m

15. $9y^2 = x^3 \Rightarrow 18y\dfrac{dy}{dx} = 3x^2 \Rightarrow \dfrac{dy}{dx} = \text{slope of tangent} = \dfrac{x^2}{6y}$ ⎫

$\therefore$ Slope of normal $= -\dfrac{6y}{x^2}$ 1+½ m

As the intercepts by normal on both axes are equal

$\therefore$ Slope of normal $= \pm 1 \Rightarrow \dfrac{-6y}{x^2} = \pm 1 \Rightarrow y = \pm\dfrac{x^2}{6}$ 1 m

$\therefore 9\left(\dfrac{x^4}{36}\right) = x^3 \Rightarrow x = 4$ and $y^2 = \dfrac{64}{9} \Rightarrow y = \pm\dfrac{8}{3}$ 1 m

$\therefore$ The points are $\left(4, \dfrac{8}{3}\right), \left(4, -\dfrac{8}{3}\right)$ ½ m

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16. $\dfrac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left[1+\dfrac{x}{\sqrt{1+x^2}}\right] = \dfrac{n}{\sqrt{1+x^2}}\left[x+\sqrt{1+x^2}\right]^n = \dfrac{ny}{\sqrt{1+x^2}}$ 1½ m

$\therefore\ \sqrt{1+x^2}\,\dfrac{dy}{dx} = ny\dots\dots\dots(i)$ ½ m

$\therefore\ \sqrt{1+x^2}\,\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx}\cdot\dfrac{x}{\sqrt{1+x^2}} = n\dfrac{dy}{dx}$ 1 m

$\Rightarrow\ \left(1+x^2\right)\dfrac{d^2y}{dx^2} + x\,\dfrac{dy}{dx} = n\sqrt{1+x^2}\,\dfrac{dy}{dx} = n\cdot ny\ \ (\text{from (i)})$

$= n^2\,y$ 1 m

17. L H D at $x=1: \lim\limits_{x\to1^-}\left(\dfrac{x-1}{x-1}\right) = 1$

   R H D at $x=1$, $\lim\limits_{x\to1^+}\dfrac{2-x-1}{x-1} = -1$ 2 m

   $\therefore$ f is not differentiable at $x=1$

   L H D at $x=2$, $\lim\limits_{x\to2^-}\dfrac{2-x-0}{x-2} = -1$

   R H D at $x=2$, $\lim\limits_{x\to2^+}\dfrac{-2+3x-x^2}{(x-2)} = \lim\limits_{x\to2^+} -\dfrac{(x-1)(x-2)}{(x-2)} = -1$ 2 m

   $\therefore$ f is diff. at $x=2$

18. Communication Matrix $A = \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix}$ Telephone
House calls
Letters

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$$\text{Cost Matrix } B = \begin{pmatrix} \overset{\text{Tele}}{1000} & \overset{\text{House calls}}{500} & \overset{\text{Letters}}{5000} \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{matrix} \text{City x} \\ \text{City y} \end{matrix}$$

$$\therefore \text{ Total cost Matrix} = \begin{pmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{pmatrix} \begin{pmatrix} 140 \\ 200 \\ 150 \end{pmatrix} = \begin{pmatrix} 990000 \\ 2120000 \end{pmatrix}$$ 3 m

any relevant value   1 m

19. $$I = \int e^{2x} \sin(3x+1)\,dx = \left[\frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \int e^{2x} \cos(3x+1)\,dx \right]$$ 1½ m

$$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{2} \cdot \frac{e^{2x}}{2} \cdot \cos(3x+1) - \frac{9}{4} \int e^{2x} \sin(3x+1)\,dx \, \Big]$$ 1 m

$$= \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1) - \frac{9}{4} I$$ 1 m

$$\frac{13}{4} I = \frac{e^{2x}}{2} \cdot \sin(3x+1) - \frac{3}{4} e^{2x} \cdot \cos(3x+1)$$

$$I = \frac{4}{13} \left[\frac{e^{2x}}{2} \left(-\frac{3}{2} \cos(3x+1) + \sin(3x+1) \right) \right] + c$$ ½ m

SECTION - C

20. Let $x_1, x_2, \in R$ such that $f(x_1) = f(x_2) \Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$ 1½+½ m

$$\Rightarrow 4(x_1 - x_2)[x_1 + x_2 + 3] = 0 \Rightarrow x_1 = x_2$$

$$\Rightarrow \text{ f is one - one}$$

f is clearly onto and hence intertible   1 m

Let y be an arbitrary element of S

$$f(x) = y = 4x^2 + 12x + 15 = (2x+3)^2 + 6$$ 1 m

$$\therefore f^{-1} : R \to S \text{ is given by } f^{-1}(y) = \left(\frac{\sqrt{y-6} - 3}{2} \right)$$ 2 m

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21.
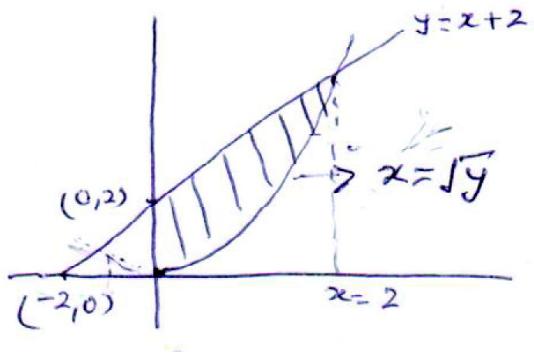
Correct Figure   1m

Points of intersection

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$ 1½ m

$x = 2, -1$ (−1 is rejected)

$\therefore$ Reqd. area $= \int\limits_{0}^{2} \{(x+2) - x^2\} dx$ 1½ m

$= \left[\dfrac{x^2}{2} + 2x - \dfrac{x^3}{3} \right]_{0}^{2}$

$= \left(2 + 4 - \dfrac{8}{3} \right) = \dfrac{10}{3}$ sq.units   2 m

22.   Let $z = ax + by$, also $xy = c^2 \Rightarrow y = \dfrac{c^2}{x}$

$\therefore$ $z = ax + \dfrac{bc^2}{x}$ ½ + ½ m

$\therefore \dfrac{dz}{dx} = a + bc^2 \left(\dfrac{-1}{x^2} \right)$, $\dfrac{dz}{dx} = 0 \Rightarrow bc^2 = ax^2$

or $x = \sqrt{\dfrac{b}{a}} \, c$ 1½ m

showing $\dfrac{d^2 z}{ax^2}$ at $x = \sqrt{\dfrac{b}{a}} \, c > 0 \Rightarrow$ minima   1½ m

$y = \dfrac{c^2}{x} = \dfrac{c^2}{c} \sqrt{\dfrac{a}{b}} = c \sqrt{\dfrac{a}{b}}$ 1 m

$\therefore$ minimum $z = a \sqrt{\dfrac{b}{a}} \, c + bc \sqrt{\dfrac{a}{b}} \, c = 2 \sqrt{ab} \, c$ 1 m

OR

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$y = x^2 + 7x + 2$, $3x - y - 3 = 0$ ....(i)

$\therefore 3x - (x^2 + 7x + 2) - 3 = 0$ 1 m

Distance of (x, y) from (i)

$$D = \left| \frac{3x - (x^2 + 7x + 2) - 3}{\sqrt{10}} \right| \text{ or } D = \left| \frac{(-x^2 - 4x - 5)}{\sqrt{10}} \right| = \left| \frac{(x+2)^2 + 1}{\sqrt{10}} \right|$$ 2 m

$$\frac{dD}{dx} = \frac{2}{\sqrt{10}}(x+2), \frac{dD}{dx} = 0 \text{ at } x = -2$$ 1 m

$$\frac{d^2D}{dx^2} > 0 \Rightarrow \text{minima}$$ 1 m

$\therefore$ D is minimum at $x = -2$

at $x = -2$, $y = -8$ 1 m

$\therefore$ The required pt. on the parabola is $(-2, -8)$
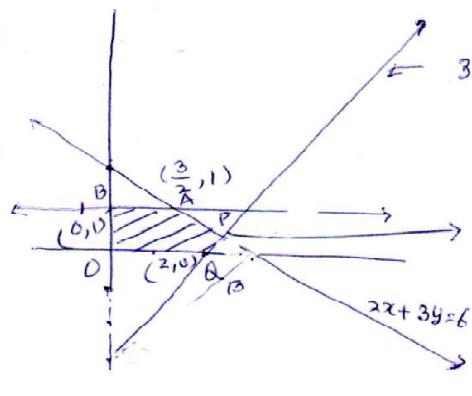
23.   Figure   3 m

Feasible region is B A PQO   1 m

$$z_B = 9, z_P = \frac{240}{13} + \frac{54}{13} = \frac{294}{13}$$

$$= 22\frac{8}{13}$$ 1 m

$Z_Q = 16$

$\therefore$ Z is maximum at $\left(\frac{30}{13}, \frac{6}{13} \right)$

and maximum value $= 22\frac{8}{13}$ 1 m

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24. Any line through $(1, -2, 3)$ with d. r's as $2, 3 - 6$ is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda \qquad \text{1 ½ m}$$

$$\therefore \ x = 2\lambda + 1, \ y = 3\lambda - 2, \ z = -6\lambda + 3 \qquad \text{1½ m}$$

It lies on the plane $x - y + z = 5$

$$\therefore \ 2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$$

$$\Rightarrow \ \lambda = \frac{1}{7} \qquad \text{1 m}$$

Reqd. point is $\left(\dfrac{9}{7}, -\dfrac{11}{7}, \dfrac{15}{7} \right)$ \qquad 1 m

$$\therefore \ \text{Reqd distance} = \sqrt{\left(\frac{9}{7} - 1\right)^2 + \left(-\frac{11}{7} + 2\right)^2 + \left(\frac{15}{7} - 3\right)^2} = \frac{1}{7}\sqrt{4 + 9 + 36} = \frac{7}{7} = 1 \qquad \text{1 m}$$

25. $$\frac{dy}{dx} = \frac{2x \sin\left(\dfrac{y}{x}\right) - y \cos\left(\dfrac{y}{x}\right)}{y - x \cos\left(\dfrac{y}{x}\right)} = \frac{2 \sin\left(\dfrac{y}{x}\right) - \dfrac{y}{x} \cos\left(\dfrac{y}{x}\right)}{\dfrac{y}{x} - \cos\left(\dfrac{y}{x}\right)} \qquad \text{1 m}$$

Let $y = vx \ \Rightarrow \ \dfrac{dy}{dx} = v + x \dfrac{dv}{dx}$ \qquad 1 m

$$\therefore \ v + x \frac{dv}{dx} = \frac{2 \sin v - v \cos v}{v - \cos v} \ \Rightarrow \ x \frac{dv}{dx} = \frac{2 \sin v - v^2}{v - \cos v} \qquad \text{1 m}$$

$$\Rightarrow \ \frac{v - \cos v}{-2 \sin v + v^2} = \frac{-dx}{x} \ \text{or} \ \frac{1}{2}\left[\frac{2v - 2\cos v}{-2\sin v + v^2}\right] dv = -\frac{dx}{x} \qquad \text{1 m}$$

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$$\Rightarrow \quad \frac{1}{2} \log \left| v^2 - 2\sin v \right| = -\log x + \log c$$

or $\log \left| \sqrt{v^2 - 2\sin v} \right| = \log c - \log x$ 1 m

$$\sqrt{v^2 - 2\sin v} = \frac{c}{x}$$

or $x\sqrt{\dfrac{y^2}{x^2} - 2\sin \dfrac{y}{x}} = c$ ½ m

$$y^2 - 2x^2 \sin\left(\frac{y}{x}\right) = c'$$ ½ m

<div align="center">OR</div>

$$\left(\sqrt{(1+x^2)(1+y^2)}\right) dx + xy\, dy = 0$$

$$\Rightarrow \quad \frac{y}{\sqrt{1+y^2}}\, dy + \frac{\sqrt{1+x^2}}{x}\, dx = 0$$ 1 m

$$\frac{1}{2} \int \frac{2y}{\sqrt{1+y^2}}\, dy + \int \frac{\sqrt{1+x^2}}{x}\, dx = 0$$

$$\sqrt{1+y^2} + \int \frac{(1+x^2)}{x\sqrt{1+x^2}}\, dx = c$$ 1½ m

$$I_1 = \int \frac{1}{x\sqrt{1+x^2}}\, dx + \int \frac{x}{\sqrt{1+x^2}}\, dx = I_2 + \sqrt{1+x^2}$$ 1 m

For I_2, Let $x = \dfrac{1}{t}$, $dx = \dfrac{-1}{t^2}\, dt$ 1 m

$$I_2 = \int \frac{-1}{t^2 \cdot \frac{1}{t}\sqrt{1+\frac{1}{t^2}}}\, dt = -\int \frac{dt}{\sqrt{t^2+1}} = -\log\left[t + \sqrt{1+t^2}\right]$$ 1 m

$$= -\log\left[\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right] = -\log\left[\frac{1 + \sqrt{1 + x^2}}{x}\right]$$

½ m

$$\therefore \text{ The solution is } \sqrt{1 + x^2} + \sqrt{1 + y^2} - \log\left(\frac{1 + \sqrt{1 + x^2}}{2}\right) = c$$

26. $P\text{(Doublet)} = \frac{1}{6}$, $P\text{(not a doublet)} = \frac{5}{6}$

1 m

The random variate x can take values $0, 1, 2, 3, 4$

x	0	1	2	3	4
P(x)	$\left(\frac{5}{6}\right)^4$	$4\frac{1}{6}\left(\frac{5}{6}\right)^3$	$6\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$	$4\left(\frac{1}{6}\right)^3\frac{5}{6}$	$\left(\frac{1}{6}\right)^4$
	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

2½ m

$$\text{Mean} = \sum x\, P(x) = \frac{500 + 300 + 60 + 4}{1296} = \frac{864}{1296} = \frac{2}{3}$$

1 m

$$\sum x^2\, P(x) = \frac{500 + 600 + 180 + 16}{1296} = \frac{1296}{1296} = 1$$

1 m

$$\therefore \text{ Variance} = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$$

½ m

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