QUESTION PAPER CODE 65/1/1

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Q. No. Marks

1-10. 1. $x = 25$ 2. $x = \frac{1}{5}$ 3. 10 4. $x = 2$ 5. $x = \pm 6$

6. $2x^{3/2} + 2\sqrt{x} + c$ 7. $\frac{\pi}{12}$ 8. 5 9. $\frac{2\pi}{3}$

10. $\left\{\vec{r} - \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\right\} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

or

$\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = a + b + c$ 1×10 =10 m

SECTION - B

11. $\forall\ (a, b) \in A \times A$

$a + b = b + a \quad \therefore \quad (a, b)\ R\ (a, b) \quad \therefore$ R is reflexive 1 m

For $(a, b), (c, d) \in A \times A$

If $(a, b)\ R\ (c, d)$ i.e. $a + d = b + c \Rightarrow c + b = d + a$

then $(c, d)\ R\ (a, b) \quad \therefore$ R is symmetric 1 m

For $(a, b), (c, d), (e, f) \in A \times A$

If $(a, b)\ R\ (c, d)$ & $(c, d)\ R\ (e, f)$ i.e. $a + d = b + c$ & $c + f = d + e$

Adding, $a + d + c + f = b + c + d + e \Rightarrow a + f = b + e$

then $(a, b)\ R\ (e, f) \quad \therefore$ R is transitive 1 m

$\therefore$ R is reflexive, symmetric and transitive

hence R is an equivalance relation ½ m

$[(2, 5)] = \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$ ½ m

12. $\cot^{-1}\left\{\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$

$$= \cot^{-1}\left\{\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right\}$$ 2½ m

$$= \cot^{-1}\left\{\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right\} = \cot^{-1}\left(\cot\frac{x}{2}\right)=\frac{x}{2}$$ 1½ m

OR

$$\text{LHS} = 2\left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right) + \tan^{-1}\frac{1}{7}$$ 1½+½ m

$$= 2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{2\cdot\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right) + \tan^{-1}\frac{1}{7}$$ 1 m

$$= \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7} = \tan^{-1}\frac{25}{25} = \tan^{-1}(1) = \frac{\pi}{4} = \text{RHS}$$ 1 m

13. $\text{LHS} = \begin{vmatrix} 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix}$

$$= \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ 2z & 2z & z-x-y \\ x-y-z & 2x & 2x \end{vmatrix} \quad R_1 \to R_1 + R_2 + R_3$$ 1 m

$$= \begin{vmatrix} x+y+z & 0 & 0 \\ 2z & 0 & -(x+y+z) \\ x-y-z & x+y+z & x+y+z \end{vmatrix}; \quad \begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix}$$ 2 m

$$= (x+y+z) \cdot \{0 \cdot (x+y+z) + (x+y+z)^2\} = (x+y+z)^3$$ 1 m

14. let $u = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$, $v = \cos^{-1}\left(2x\sqrt{1-x^2}\right)$, $x = \cos\theta \ \therefore\ \theta = \cos^{-1}x$

$\therefore \quad u = \tan^{-1}\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right) = \tan^{-1}(\tan\theta) = \theta = \cos^{-1}x$ 1 m

and $v = \cos^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right) = \cos^{-1}(\sin 2\theta) = \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

$= \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\cos^{-1}x$ 1 m

$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}}, \quad \frac{dv}{dx} = \frac{2}{\sqrt{1-x^2}}$ 1 m

$\therefore \quad \frac{du}{dv} = \frac{-1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{2} = \frac{-1}{2}$ 1 m

(In case, If $x = \sin\theta$ then answer is $\frac{1}{2}$)

15. $y = x^x \quad \therefore \ \log y = x\log x,$ Taking log of both sides ½ m

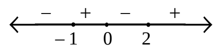
$\Rightarrow \ \frac{1}{y}\frac{dy}{dx} = \log x + 1,$ Diff. w r t "x" 1½ m

$\Rightarrow \ \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x},$ Diff. w r t "x" 1½ m

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$ ½ m

16. $f'(x) = 12x^3 - 12x^2 - 24x = 12x(x+1)(x-2)$ 1+½ m

$f'(x) > 0, \ \forall x \in (-1, 0) \cup (2, \infty)$ 1 m



$f'(x) < 0, \ \forall x \in (-\infty, -1) \cup (0, 2)$ 1 m

$\therefore$ f(x) is strictly increasing in $(-1, 0) \cup (2, \infty)$

and strictly decreasing in $(-\infty, -1) \cup (0, 2)$ ½ m

OR

Point at $\theta = \pi/4$ is $\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$ ½ m

$$\frac{dy}{d\theta} = -3a\cos^2\theta\sin\theta; \quad \frac{dx}{d\theta} = 3a\sin^2\theta\cos\theta$$ 1 m

$$\therefore \text{ slope of tangent at } \theta = \frac{\pi}{4} \text{ is } \left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}} = \left.\frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}\right]_{\theta=\frac{\pi}{4}}$$

$$= -\cot\frac{\pi}{4} = -1$$ 1 m

Equation of tangent at the point :

$$y - \frac{a}{2\sqrt{2}} = -1\left(x - \frac{a}{2\sqrt{2}}\right) \Rightarrow x + y - \frac{a}{\sqrt{2}} = 0$$ 1 m

Equation of normal at the point :

$$y - \frac{a}{2\sqrt{2}} = 1\left(x - \frac{a}{2\sqrt{2}}\right) \Rightarrow x - y = 0$$ ½ m

17. $$\int \frac{\sin^6x + \cos^6x}{\sin^2x \cdot \cos^2x}\,dx = \int \frac{(\sin^2x + \cos^2x)\left[(\sin^2x + \cos^2x)^2 - 3\sin^2x\cos^2x\right]}{\sin^2x \cdot \cos^2x}\,dx$$ 1½ m

$$= \int \left[\frac{1}{\sin^2x \cdot \cos^2x} - 3\right] dx$$

$$= \int \left[\frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} - 3 \right] dx$$ ½ m

$$= \int \left(\sec^2 x + \operatorname{cosec}^2 x - 3 \right) dx$$ ½ m

$$= \tan x - \cot x - 3x + c$$ 1½ m

(Accept $-2 \cot 2x - 3x + c$ also)

OR

$$\int (x-3)\sqrt{x^2+3x-18}\ dx$$

$$= \frac{1}{2}\int (2x+3)\sqrt{x^2+3x-18}\ dx \ - \frac{9}{2}\int \sqrt{x^2+3x-18}\ dx$$ 1 m

$$= \frac{1}{2}\cdot\frac{2}{3}\ (x^2+3x-18)^{3/2} - \ \frac{9}{2}\int \sqrt{\left(x+3/2\right)^2 - \left(\frac{9}{2}\right)^2}\ dx$$ 1½ m

$$= \frac{1}{3}(x^2+3x-18)^{3/2} - \ \frac{9}{2}$$

$$\left\{ \frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x-18} - \frac{81}{8}\log\left| x+\frac{3}{2}+\sqrt{x^2+3x-18} \right| + c \right.$$ 1½ m

or $$= \frac{1}{3}(x^2+3x-18)^{3/2} - \ \frac{9}{8}$$

$$\left\{ (2x+3)\sqrt{x^2+3x-18} - \frac{81}{2}\log\left| x+\frac{3}{2}+\sqrt{x^2+3x-18} \right| + c \right.$$

18. $$e^x\sqrt{1-y^2}\ dx = \frac{-y}{x}\ dy \ \Rightarrow\ xe^x\ dx = \frac{-y}{\sqrt{1-y^2}}\ dy$$ 1 m

Integrating both sides

$$\int xe^x\ dx = \frac{1}{2}\int \frac{-2y}{\sqrt{1-y^2}}\ dy$$

$$\Rightarrow\ xe^x - e^x = \sqrt{1-y^2} + c$$ 1+1 m

For $x = 0,\ y = 1,\ c = -1$ $\therefore$ solution is : $e^x\ (x-1) = \sqrt{1-y^2} - 1$ ½+½ m

19. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{2x}{x^2-1}y = \frac{2}{(x^2-1)^2}$$ 1 m

Integrating factor $= e^{\int \frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2-1$ 1 m

$\therefore$ Solution is $y\cdot(x^2-1) = \int \frac{2}{(x^2-1)^2}\cdot(x^2-1)\ dx + c$ 1 m

$\Rightarrow \quad y(x^2-1) = 2\int \frac{1}{x^2-1}\ dx + c$

$\Rightarrow \quad y(x^2-1) = \log\left|\frac{x-1}{x+1}\right| + c$ 1 m

20. $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = \left(\vec{a}+\vec{b}\right)\cdot\left\{\left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\right\}$ ½ m

$= \left(\vec{a}+\vec{b}\right)\left\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right\}$ 1 m

$= \vec{a}\cdot\left(\vec{b}\times\vec{c}\right)+\vec{a}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{a}\cdot\left(\vec{c}\times\vec{a}\right)+\vec{b}\cdot\left(\vec{b}\times\vec{c}\right)$ 1½ m

$+\ \vec{b}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{b}\cdot\left(\vec{c}\times\vec{a}\right)$

$\left\{\vec{a}\cdot\left(\vec{b}\times\vec{a}\right),\ \left\{\vec{a}\cdot\left(\vec{c}\times\vec{a}\right)=\vec{b}\cdot\left(\vec{b}\times\vec{c}\right)=\vec{b}\cdot\left(\vec{b}\times\vec{a}\right)=0\right\}\right.$

$= 2\left\{\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)\right\}=2\left[\vec{a},\vec{b},\vec{c}\right]$ 1 m

OR

$\vec{a}+\vec{b}+\vec{c}=\vec{0} \quad \therefore \quad \vec{a}+\vec{b} = -\vec{c}$ ½ m

$\Rightarrow \quad \left(\vec{a}+\vec{b}\right)^2 = \left(-\vec{c}\right)^2 = \left(\vec{c}\right)^2$ ½ m

$$\Rightarrow \quad \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\,\vec{a}\cdot\vec{b} = \left|\vec{c}\right|^2$$ 1 m

$$\Rightarrow \quad 9 + 25 + 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = 49, \quad \theta \text{ being angle between } \vec{a}\ \&\ \vec{b}$$ 1 m

$$\therefore \quad \cos\theta = \frac{15}{2\cdot 3\cdot 5} = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$$ 1 m

21. let $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = u;\ \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = v$

General points on the lines are

$(3u - 1, 5u - 3, 7u - 5)$ & $(v + 2, 3v + 4, 5v + 6)$ 1 m

lines intersect if

$3u - 1 = v + 2,\ 5u - 3 = 3v + 4,\ 7u - 5 = 5v + 6$ for some u & v 1 m

or $3u - v = 3$ .... (1), $5u - 3v = 7$ .... (2), $7u - 5v = 11$ .... (3)

Solving equations (1) and (2), weget $u = \frac{1}{2},\ v = -\frac{3}{2}$ ½ m

Putting u & v in equation (3), $7\cdot\frac{1}{2} - 5\left(-\frac{3}{2}\right) = 11$ $\therefore$ lines intersect ½ m

Point of intersection of lines is : $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$ 1 m

22. let b_2, g_2 be younger boy and girl

and b_1, g_1 be elder, then, sample space of two children is

$S = \{(b_1, b_2), (g_1, g_2), (b_1, g_2), (g_1, b_2)\}$ 1 m

A = Event that younger is a girl = $\{(g_1, g_2), (b_1, g_2)\}$

B = Event that at least one is a girl = $\{(g_1, g_2), (b_1, g_2), (g_1, b_2)\}$

E = Event that both are girls = $\{(g_1, g_2)\}$

(i) $P(E/A) = \dfrac{P(E \cap A)}{P(A)} = \dfrac{1}{2}$ 1½ m

(ii) $P(E/B) = \dfrac{P(E \cap B)}{P(B)} = \dfrac{1}{3}$ 1½ m

SECTION - C

23. Here
$$\begin{aligned} 3x + 2y + z &= 1000 \\ 4x + y + 3z &= 1500 \\ x + y + z &= 600 \end{aligned}$$ 1½

$$\therefore \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1000 \\ 1500 \\ 600 \end{pmatrix} \text{ or } A \cdot X = B$$

$|A| = 3(-2) - 2(1) + 1(3) = -5 \neq 0 \;\therefore\; X = A^{-1}B$ ½ m

Co-factors are

$$\begin{aligned} &A_{11} = -2, \quad A_{12} = -1, \quad A_{13} = 3 \\ &A_{21} = -1, \quad A_{22} = 2, \quad A_{23} = -1 \\ &A_{31} = 5, \quad A_{32} = -5, \quad A_{33} = -5 \end{aligned}$$ 1½ m

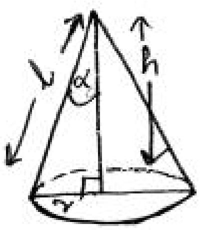
$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5} \begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix} \begin{pmatrix} 1000 \\ 1500 \\ 600 \end{pmatrix}$$

$\therefore$ x = 100, y = 200, z = 300 1½ m

i.e. Rs. 100 for discipline, Rs 200 for politeness & Rs. 300 for punctuality

One more value like sincerity, truthfulness etc. 1 m

24.



For correct figure ½ m

Let radius, height and slant height of cone be r, h & l

$\therefore \quad r^2 + h^2 = \mathbf{l}^2, \ \mathbf{l}$ (constant) ½ m

Volume of cone (V) = $\frac{1}{3}\pi r^2 h$ ½ m

$\therefore \quad V = \frac{\pi}{3} h \left(\mathbf{l}^2 - h^2\right) = \frac{\pi}{3}\left(\mathbf{l}^2 h - h^3\right)$ 1 m

$\frac{dv}{dh} = \frac{\pi}{3}\left(\mathbf{l}^2 - 3h^2\right)$ 1 m

$\therefore \quad \frac{dv}{dh} = 0 \ \Rightarrow \ h = \frac{\mathbf{l}}{\sqrt{3}}$ ½ m

$\frac{d^2v}{dh^2} = -2\pi h = -2\pi \cdot \frac{\mathbf{l}}{\sqrt{3}} = -\frac{2\pi \mathbf{l}}{\sqrt{3}} < 0$ 1 m

$\therefore$ at $h = \frac{\mathbf{l}}{\sqrt{3}}$, volume is maximum

$\cos\alpha = \frac{h}{\mathbf{l}} = \frac{1}{\sqrt{3}} \quad \therefore \quad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 1 m

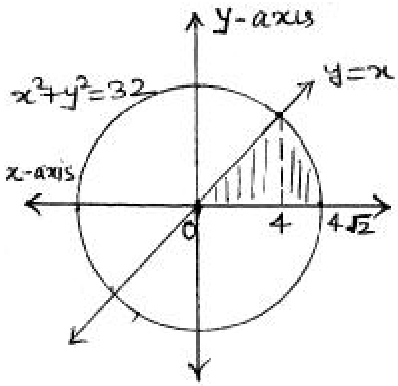
25. $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$ 1 m

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)} + \sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}\, dx$ 1 m

$\therefore \quad I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\, dx$ 1 m

Adding we get, $2I = \int_{\pi/6}^{\pi/3} dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ 2 m

$\therefore \quad I = \frac{\pi}{12}$ 1 m

26. 

Correct Figure 1 m

The line and circle intersect each other at $x = \pm 4$ 1 m

Area of shaded region

$$= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{\left(4\sqrt{2}\right)^2 - x^2}\,dx$$ 1½ m

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\left\{\frac{x\sqrt{32-x^2}}{2} + 16\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right\}\right]_4^{4\sqrt{2}}$$ 1½ m

$$= 8 + 4\pi - 8 = 4\pi \text{ sq.units}$$ 1 m

27. Equation of plane through points A, B and C is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0 \Rightarrow 16x + 24y + 32z - 56 = 0$$
$$\text{i.e. } 2x + 3y + 4z - 7 = 0$$ 3+1 m

Distance of plane from (7, 2, 4) = $\left|\dfrac{2(7)+3(2)+4(4)-7}{\sqrt{9+16+4}}\right|$ 1 m

$$= \sqrt{29}$$ 1 m

OR

General point on the line is $(2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)\hat{k}$ 1 m

Putting in the equation of plane; we get

$$1\cdot(2+3\lambda) - 1\cdot(-1+4\lambda) + 1\cdot(2+2\lambda) = 5$$ 1½ m

$$\therefore \quad \lambda = 0$$ 1 m

Point of intersection is $2\hat{i} - \hat{j} + 2\hat{k}$ or $(2, -1, 2)$ 1½ m

Distance = $\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = \sqrt{169} = 13$ 1 m

28. Let x and y be electronic and manually operated sewing machines purchased respectively

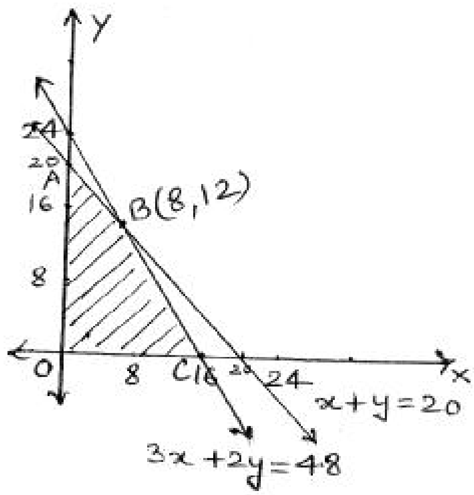
$\therefore$ L.P.P. is Maximize $P = 22x + 18y$ ½ m

subject to $360x + 240y \le 5760$

or $3x + 2y \le 48$

$x + y \le 20$

$x \ge 0, y \ge 0$ 2 m



For correct graph 2 m

Vertices of feasible region are

A (0, 20), B(8, 12), C(16, 0) & O(0, 0)

P(A) = 360, P(B) = 392, P(C) = 352 ½ m

$\therefore$ For Maximum P, Electronic machines = 8

Manual machines = 12 1 m

29. Let E_1 : Event that lost card is a spade

E_2 : Event that lost card is a non spade ½ m

A : Event that three spades are drawn without replacement from 51 cards

$$P(E_1) = \frac{13}{52} = \frac{1}{4}, \quad P(E_2) = 1 - \frac{1}{4} = \frac{3}{4}$$ 1 m

$$P(A/E_1) = \frac{{}^{12}C_3}{{}^{51}C_3}, \quad P(A/E_2) = \frac{{}^{13}C_3}{{}^{51}C_3}$$ 1½ m

$$P(E_1/A) = \frac{\frac{1}{4} \cdot \frac{{}^{12}C_3}{{}^{51}C_3}}{\frac{1}{4} \cdot \frac{{}^{12}C_3}{{}^{51}C_3} + \frac{3}{4} \cdot \frac{{}^{13}C_3}{{}^{51}C_3}}$$ 1+1 m

$$= \frac{10}{49}$$ 1 m

OR

X = No. of defective bulbs out of 4 drawn = 0, 1, 2, 3, 4 1 m

Probability of defective bulb = $\frac{5}{15} = \frac{1}{3}$ ½ m

Probability of a non defective bulb = $1 - \frac{1}{3} = \frac{2}{3}$ ½ m

Probability distribution is :

x :	0	1	2	3	4
P(x) :	$\frac{16}{81}$	$\frac{32}{81}$	$\frac{24}{81}$	$\frac{8}{81}$	$\frac{1}{81}$
x P(x) :	0	$\frac{32}{81}$	$\frac{48}{81}$	$\frac{24}{81}$	$\frac{4}{81}$

2½ m

½ m

Mean = $\sum x\ P(x) = \frac{108}{81}$ or $\frac{4}{3}$ 1 m