

SECOND YEAR HIGHER SECONDARY EXAMINATION, JUNE 2017
(Finalised Scheme of Valuation)

Subject: Mathematics (Commerce)

Code No: 7053

Qn.No	Scoring Indicators	Split Score	Total Score
1(i)	(a) Range of $f = \text{Codomain of } f'$	1	1
(ii)	$f(x) = x+1$, $g(x) = x^2$ $f \circ g(x) = f[g(x)] = f[x^2] = x^2+1$	1	2
	$g \circ f(x) = g[f(x)] = g[x+1] = (x+1)^2$	1	
(iii)	$f(x) = 3x+2$ Domain of $f = \text{Range of } f = \mathbb{R}$ $f(x_1) = f(x_2) \Rightarrow 3x_1+2 = 3x_2+2$ $\Rightarrow x_1 = x_2$ $\therefore f$ is 1-1	1	5
	Let $y = 3x+2$ $\Rightarrow x = \frac{y-2}{3}$ $\Rightarrow f(x) = f\left[\frac{y-2}{3}\right] = 3\left[\frac{y-2}{3}\right]+2$ $\Rightarrow f(x) = y$ $\therefore f$ is onto	1	2
	Remarks: - (ii) $f \circ g(x) = f[g(x)]$, $g \circ f(x) = g[f(x)]$ give 1 mark (iii) If f is 1-1 & onto give 1 score.		
2	(i) (c) $\pi/6$	1	1
	(ii) $\tan^{-1} \left[\frac{2x+3x}{1-2x \cdot 3x} \right] = \pi/4$	1	

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Qn. No	Sub Qns	Answer Key / Value points	Score	Total
		$\frac{5x}{1-6x^2} = \tan \frac{\pi}{4} = 1$ $\Rightarrow 6x^2 + 5x - 1 = 0$ $(6x-1)(x+1) = 0 \Rightarrow x = \frac{1}{6} \text{ or } x = -1$ $x = -1 \text{ is not acceptable.}$ $\therefore x = \frac{1}{6} \text{ is the solution}$	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	<p>3</p> <p>④</p>
3	(i)	(b) 5×7	1	1
	(ii)	$(A+2B)' = A' + 2B'$ $= \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + 2 \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$ $= \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$ <p>Remark: $A+2B = \begin{bmatrix} -4 & 3 \\ 3 & 6 \end{bmatrix}$ give full score.</p>	<p>1</p> <p>1</p> <p>1</p>	<p>2</p> <p>⑤</p>
	(iii)	$A = \begin{bmatrix} -2 & 1 \\ 2 & 3 \end{bmatrix}, A = -6 - 2 = -8$ $A^{-1} = \frac{1}{ A } \cdot (\text{adj } A)$ $A^{-1} = \frac{1}{-8} \begin{bmatrix} 3 & -1 \\ -2 & -2 \end{bmatrix}$ <p>Remarks: - Any other method give full score</p>	<p>1/2</p> <p>1/2</p> <p>1</p>	<p>2</p>
4	(i)	(b) singular	1	1

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Qn. No	Sub Qns	Answer Key / Value points	Score	Total
	(ii)	$\begin{vmatrix} 2x+2y+2z & x & y \\ 2x+2y+2z & y+z+2x & y \\ 2x+2y+2z & x & z+x+2y \end{vmatrix} \quad C_1 \rightarrow C_1 + C_2 + C_3$ $= (2x+2y+2z) \begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$ $= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 0 & y+z+x & 0 \\ 0 & 0 & z+x+y \end{vmatrix}$ $(R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1)$ $= 2(x+y+z) \begin{vmatrix} y+z+x & 0 \\ 0 & z+x+y \end{vmatrix}$ $= 2(x+y+z)^3$	<p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1</p>	<p>3</p> <p>(4)</p>
5	(i) (ii)	<p>(b) 1</p> $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$ $ A = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 40 \neq 0$ $\text{adj } A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } \text{adj } A = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$	<p>1</p> <p>1/2</p> <p>1</p> <p>1</p>	<p>1</p>

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Qn. No	Sub Qns	Answer Key / Value points	Score	Total
6		$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ $x=1, y=2, z=-1$	1/2	4 (5)
	(i)	Not continuous	1	1
	(ii)	When $x^2 - 5x + 6 = 0 \Rightarrow x = \{2, 3\}$ $f(x)$ is discontinuous at $x=2$ and $x=3$	1 1	2 (3)
7	(i)	$y = e^{2x + \log x} = e^{2x} \cdot e^{\log x} = e^{2x} \cdot x$ $\frac{dy}{dx} = e^{2x} \cdot 1 + x \cdot e^{2x} \cdot 2$ $= e^{2x} [1 + 2x]$ <p>Remarks:- using chain rule</p> $\frac{dy}{dx} = e^{2x + \log x} \cdot (2 + \frac{1}{x})$ <p>OR use logarithmic differentiation give full score</p>	1 1 1	2
	(ii)	$x = at^2, y = 2at$ $\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a$ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{t}$ Diff w.r.t 'x'	1 1	3

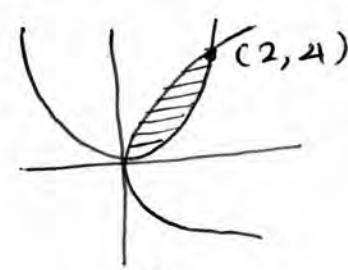
$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{1}{t} \right) \cdot \frac{dt}{dx} = \frac{-1}{t^2} \times \frac{1}{2at}$$

$$= -\frac{1}{2at^3} \quad 1$$

Remark: For formula give 1/2 score

Qn no:	Sub Qns	Answer Key / Value points		
	iii	$f(x) = x(x-2) = x^2 - 2x$ $f'(x) = 2x - 1 = 2(x-1)$ <p>Since $f'(x)$ exist, f is differentiable on $(1, 3)$, the conditions of MVT are satisfied</p> $f(a) = -1, f(b) = 3$ $f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{3 - (-1)}{3 - 1} = \frac{4}{2} \Rightarrow 2(c-1) = \frac{4}{2}$ $\Rightarrow c - 1 = 1 \Rightarrow c = 2 \in (1, 3)$ <p>Mean value theorem is satisfied</p>	1 1 $\frac{1}{2}$ $\frac{1}{2}$	3 ⑧
8	(i)	<p>Cost function = $1500 + 30 \times 5 + 5^2$ $= 1500 + 150 + 25 = 1675$</p>	1	
	(ii)	<p>Marginal cost function</p> $\frac{dc}{dx} = 30 + 2x$	1	④
	(iii)	<p>Marginal cost when $x = 20$ (MC) at $x = 20 = 30 + 2(20)$ $= 70$</p>	2	
		(OR)		
	(i)	(b) decreasing	1	1
	(ii)	<p>$y = x^3$ diff w.r.t x, $\frac{dy}{dx} = 3x^2$ At $(1, 1)$ $\frac{dy}{dx} = 3 \times 1^2 = 3$</p>	1	
		<p>At $(1, 1)$, slope of the tangent = 3 Equation of the tangent at $(1, 1)$ is $y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$</p>	1	

Qn no:	Sub Qns	Answer key / value points	Score	Total
	(iii)	At (1,1) Slope of the normal = $-\frac{1}{3}$ Equation of the normal at (1,1) $y-1 = -\frac{1}{3}(x-1)$ $\Rightarrow x+3y-4=0$ Remark:- For formulae give $\frac{1}{2}$ mark each	1	3 (4)
9	(i)	(b) $2\sqrt{x} + c$	1	1
	(ii)	$\int \sin^3 x = \int \sin^2 x \cdot \sin x dx$ $= \int (1 - \cos^2 x) \sin x dx$ Put $\cos x = t \Rightarrow -dt = \sin x dx$ $= \int (1-t^2)(-dt) = \int (t^2-1) dt$ $= \left(\frac{t^3}{3} - t\right) + c$ $= \frac{\cos^3 x}{3} - \cos x + c$	1	2 (5)
	(iii)	Remark: for ³ alternative method give full score $\frac{1}{2}$ $\int x \log x dx = \int \log x \cdot x dx$ $= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2}$ $= \frac{x^2}{2} \log x - \frac{1}{2} \left(\frac{x^2}{2}\right) + c$ $= \frac{x^2}{2} \log x - \frac{x^2}{4} + c$ OR	$\frac{1}{2}$	2
	(i)	(c) e^{-1}	1	1
	(ii)	Let $I = \int_0^{\pi/2} \cos^2 x dx$ Also $I = \int_0^{\pi/2} \cos^2(\pi/2 - x) dx = \int_0^{\pi/2} \sin^2 x dx$	1	

Qn No:	Sub Qns	Answer key / Value points	Score	Total
	(iii)	$2I = \int_0^{\pi/2} 1 \, dx = [x]_0^{\pi/2} = \pi/2$ $I = \pi/4$ $\int_0^1 x \cdot e^x \, dx$ $\Rightarrow x \cdot e^x - \int 1 \cdot e^x \cdot dx$ $\Rightarrow x \cdot e^x - e^x + c \Rightarrow e^x [x-1]_0^1$ $\Rightarrow [e(1-1) - e^0(0-1)] = 0+1=1$	<p>1</p> <p>1</p> <p>1</p>	<p>2</p> <p>(5)</p> <p>2</p>
10	<p>(i)</p> <p>(ii)</p>	<p>Solving $y=x^2$ and $y^2=8x$,</p> $x(x^3-8)=0 \Rightarrow x=0, x=2$ <p>When $x=0, y=0$ $x=2, y=4$</p> <p>Points of intersection are $(0,0), (2,4)$</p>  $\text{Req. area} = \int_0^2 \sqrt{8x} \, dx - \int_0^2 x^2 \, dx$ $= \left[2\sqrt{2} \frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right]_0^2$ $= \left[\frac{4\sqrt{2}}{3} \cdot 2^{3/2} - \frac{8}{3} \right] = \frac{16}{3} - \frac{8}{3}$ $= \frac{8}{3} \text{ sq. units}$ <p>Remarks: For fig only give 1/2 score</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p>	<p>1</p> <p>(4)</p> <p>3</p>

Qn no:	Sub Qns	Answer key/Value points	Score	Total
11	(i)	$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = \log x$	1	1
	(ii)	Integrating factor, $e^{\int p dx}$ $P = 1/x, Q = \log x$ $I.F = e^{\log x} = x.$	1	1
	(iii)	Solution is $y(IF) = \int Q(IF) + C$ $\Rightarrow xy = \int x \log x dx$ $\Rightarrow xy = \frac{1}{2} \left[x^2 \log x - \frac{x^2}{2} \right] + C$	1	2
		Remark:- For formulae give $\frac{1}{2}$ score each	1	1
12	(i)	$\vec{AB} = \vec{OB} - \vec{OA}$ $= (2\hat{i} - \hat{j} + 3\hat{k}) - (\hat{i} + 2\hat{j} + 4\hat{k})$ $= (\hat{i} - 3\hat{j} - \hat{k})$	1	1
	(ii)	Unit vector along $\vec{AB} = \frac{\vec{AB}}{ \vec{AB} }$ $= \frac{\hat{i} - 3\hat{j} - \hat{k}}{\sqrt{1^2 + (-3)^2 + (-1)^2}} = \frac{1}{\sqrt{11}} (\hat{i} - 3\hat{j} - \hat{k})$	1	2
	(iii)	$\vec{a} \cdot (\vec{b} + \vec{c}) \times (\vec{a} + \vec{b} + \vec{c})$ $= \vec{a} \cdot \{ \vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b} + \vec{c} \times \vec{c} \}$ $= \vec{a} \cdot \{ \vec{b} \times \vec{a} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b} \}$ $= \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{c} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{b})$ $= [\vec{a} \vec{b} \vec{a}] + [\vec{a} \vec{b} \vec{c}] + [\vec{a} \vec{c} \vec{a}] + [\vec{a} \vec{c} \vec{b}]$ $= [\vec{a} \vec{b} \vec{c}] - [\vec{a} \vec{b} \vec{c}] = 0$	1	3

(4)

(6)

Qn no.s	Sub Qns	Answer key / Value points	Score	Total
13	(i)	<p>Let θ be the angle between any two diagonals of the cube</p> $\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$ $= \frac{-a \cdot a + a \cdot (-a) + a \cdot a}{\sqrt{3a^2} \sqrt{3a^2}} = \frac{-a^2}{3a^2} = -\frac{1}{3}$ <p>$\therefore \theta = \cos^{-1}(1/3)$</p>	1 1 1	3 5
	(ii)	<p>Equation of the plane in intercept form is</p> $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1 \Rightarrow 6x + 4y + 3z = 12$ <p>OR</p> $\vec{a}_1 = c\hat{i} + 2\hat{j} + \hat{k} \quad \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$ $\vec{a}_2 = 2\hat{i} - \hat{j} + \hat{k} \quad \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$ $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j}, \quad \vec{b}_1 \times \vec{b}_2 = -3\hat{i} + 3\hat{k}$ $ \vec{b}_1 \times \vec{b}_2 = \sqrt{18}$ $\text{s.d} = \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 } = \frac{ -3/\sqrt{18} }{\sqrt{18}} = \frac{3}{\sqrt{18}}$	1 1 1	2 3

Qn no's	Sub Qns	Answer key / value points	Score	Total																					
	(ii)	<p>Given plane $2x - 3y + 4z = 6$</p> <p>The \perpr distance from origin is $\frac{ d }{\sqrt{a^2 + b^2 + c^2}} = \frac{ 6 }{\sqrt{2^2 + (-3)^2 + 4^2}}$</p> <p style="text-align: center;">$= \frac{6}{\sqrt{29}}$</p>	1 1	2 (5)																					
14	(i)	<p>Let the number of shirts be 'x' & pants be 'y'</p> <table border="1" style="margin: 10px auto;"> <thead> <tr> <th rowspan="2">Cloth</th> <th rowspan="2">No. of clo. pieces</th> <th colspan="3">Item req.</th> <th rowspan="2">Profit</th> </tr> <tr> <th>A</th> <th>B</th> <th>C</th> </tr> </thead> <tbody> <tr> <td>shirt</td> <td>x</td> <td>3</td> <td>2</td> <td>1</td> <td>150</td> </tr> <tr> <td>Pant</td> <td>y</td> <td>4</td> <td>3</td> <td>2</td> <td>200</td> </tr> </tbody> </table> <p>(i) The objective function is $Z = 150x + 200y$</p> <p>(ii) Constraints are $3x + 4y \leq 12$ $2x + 3y \geq 5$ $x + 2y \leq 12$ $x, y \geq 0$</p>	Cloth	No. of clo. pieces	Item req.			Profit	A	B	C	shirt	x	3	2	1	150	Pant	y	4	3	2	200	1 3	(4)
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15	(i)	<p>$x + 2y = 10$</p> <table border="1" style="margin: 10px auto;"> <tr><td>x</td><td>0</td><td>10</td></tr> <tr><td>y</td><td>5</td><td>0</td></tr> </table> <p>$3x + y = 15$</p> <table border="1" style="margin: 10px auto;"> <tr><td>x</td><td>0</td><td>5</td></tr> <tr><td>y</td><td>15</td><td>0</td></tr> </table>	x	0	10	y	5	0	x	0	5	y	15	0											
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Qn no.s	Sub Qns	Answer key / value points	Score	Total								
			2	2								
	(ii)	<p>The feasible region is the shaded region in the figure</p> <table border="1"> <thead> <tr> <th>Corner points</th> <th>$z = 3x + 2y$</th> </tr> </thead> <tbody> <tr> <td>A (5, 0)</td> <td>$z = 15$</td> </tr> <tr> <td>B (4, 3)</td> <td>$z = 18$</td> </tr> <tr> <td>C (0, 5)</td> <td>$z = 10$</td> </tr> </tbody> </table> <p>Maximum value of z is 18 at (4, 3)</p>	Corner points	$z = 3x + 2y$	A (5, 0)	$z = 15$	B (4, 3)	$z = 18$	C (0, 5)	$z = 10$	1	2
Corner points	$z = 3x + 2y$											
A (5, 0)	$z = 15$											
B (4, 3)	$z = 18$											
C (0, 5)	$z = 10$											
16	(i)	<p>Remark:- For axis $\frac{1}{2}$ marks each. Any two correct two corner pts give</p> <p>$P(A) = \frac{4}{7}$ $P(B) = \frac{2}{7}$ 1 mark</p>	1	1								
	(ii)	<p>P (only one of them passing)</p> $= P(A \cup B)$ $= P(A) \cdot P(B) + P(A') \cdot P(B)$ $= \frac{3}{7} \times \frac{2}{7} + \frac{4}{7} \times \frac{5}{7} = \frac{26}{49}$	1	2								
	(iii)	<p>P (both of them passing) = $P(A \cap B)$</p> $= P(A) \cdot P(B) = \frac{3}{7} \times \frac{2}{7} = \frac{6}{49}$	1	2								

Qn nos	Sub Ans	Answer key / Value Points	Score	Total
17	(i)	$0 + k + 2k + 3k + 4k = 1$ $\Rightarrow 10k = 1 \Rightarrow k = \frac{1}{10}$	1	1
	(ii)	$P(X < 3) = P(X=0) + P(X=1) + P(X=2)$ $= 0 + k + 2k = \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$	1 1	2
	(iii)	Mean of the random variable $E(X) = 0 + 1 \times \frac{1}{10} + 2 \times \frac{2}{10} + 3 \times \frac{3}{10} + 4 \times \frac{4}{10}$ $= \frac{1}{10} [1 + 4 + 9 + 16] = \frac{30}{10} = 3$	1 1	2 (5)
	1.	Beena Mathew Mae Basil HSS Kothamangalam Ernakulam (dist)	Beena	
	2.	Santosh Kumar. S DUMNIM HSS. Mavankkay Trivandrum Dist (Mob) 9447000141	Santosh	
	3	Jessymol Varghese Union HSS Mambra Thiruvananthapuram (D)	Jessymol Varghese	