Reg. No. : .....

Name : .....HSSLIVE

Code No. 7018-A

SECOND YEAR
SAY/IMPROVEMENT
JUNE 2017

Time : 2½ Hours
Cool-off time : 15 Minutes

Part – III

MATHEMATICS (SCIENCE)

Maximum : 80 Scores

General Instructions to Candidates :

- There is a 'cool-off time' of 15 minutes in addition to the writing time of 2½ hrs.
- You are not allowed to write your answers nor to discuss anything with others during the 'cool-off time'.
- Use the 'cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

നിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും. ഈ സമയത്ത് ചോദ്യങ്ങൾക്ക് ഉത്തരം എഴുതാനോ, മറ്റുള്ളവരുമായി ആശയ വിനിമയം നടത്താനോ പാടില്ല.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- എല്ലാ ചോദ്യങ്ങൾക്കും ഉത്തരം എഴുതണം.
- ഒരു ചോദ്യനമ്പർ ഉത്തരമെഴുതാൻ തെരഞ്ഞെടുത്തു കഴിഞ്ഞാൽ ഉപചോദ്യങ്ങളും അതേ ചോദ്യ നമ്പരിൽ നിന്ന് തന്നെ തെരഞ്ഞെടുക്കേണ്ടതാണ്.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

1. (a) If $R = \{(x, y) : x, y \in \mathbb{Z}, x - y \in \mathbb{Z}\}$, then the relation R is

(a) Reflexive but not transitive

(b) Reflexive but not symmetric

(c) Symmetric but not transitive

(d) an Equivalence relation **(Score : 1)**

(b) Let $*$ be a binary operation on the set Q of rational numbers by $a * b = 2a + b$.

Find $2 * (3 * 4)$ and $(2 * 3) * 4$. **(Scores : 2)**

(c) Let $f : R \to R$, $g : R \to R$ be two one-one functions. Check whether gof is one-one or not. **(Scores : 2)**

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2. (a) $\sin(\tan^{-1} 1)$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) 1

(c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$ **(Score : 1)**

(b) If $x \in (0, \pi/2)$, show that

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}$$ **(Scores : 3)**

3. (a) The number of all possible 2×2 matrices with entries 0 or 1 is

(a) 8 (b) 9

(c) 16 (d) 25 **(Score : 1)**

(b) If the area of a triangle whose vertices are (k, 0), (5, 0), (0, 1) is 10 square units, then find k. **(Scores : 2)**

(c) Using elementary transformations find the inverse of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ **(Scores : 3)**

4. (a) If A is a 2×2 matrix with $|A| = 5$, then $|\text{adj } A|$ is

(a) 5 (b) 25

(c) $\frac{1}{5}$ (d) $\frac{1}{25}$ **(Score : 1)**

(b) Solve the system of equations using matrix method

$x + y + z = 1$

$2x + 3y - z = 6$

$x - y + z = -1$ **(Scores : 4)**

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5. (a) Examine whether the function defined by $f(x) = \begin{cases} x + 5 & \text{if } x \leq 1 \\ x - 5 & \text{if } x > 1 \end{cases}$ is continuous or not. **(Scores : 2)**

(b) If $x = a^{\sin^{-1}t}$, $y = a^{\cos^{-1}t}$, $a > 0$, show that $\frac{dy}{dx} = \frac{-y}{x}$. **(Scores : 2)**

(c) If $y = \sin^{-1}x$, show that $(1 - x^2)\frac{d^2y}{dx^2} = x\frac{dy}{dx}$. **(Scores : 2)**

6. (a) Slope of the tangent to the curve $y = 5 - 10x^2$ at the point $(-1, -5)$ is

(a) 10 (b) -10

(c) 20 (d) -20 **(Score : 1)**

(b) Show that of all rectangles inscribed in a fixed circle, the square has the maximum area. **(Scores : 4)**

OR

(a) Maximum value of $f(x) = \log x$ in $[1, e]$ is

(a) 1 (b) e

(c) 1/e (d) 0 **(Score : 1)**

(b) Using differentials, find the approximate value of $(255)^{1/4}$. **(Scores : 4)**

7. Find the following :

(a) $\int \frac{4x - 10}{\sqrt{(x-2)(x-3)}}\,dx$ **(Scores : 3)**

(b) $\int \frac{1}{(x^2+1)(x^2+4)}\,dx$ **(Scores : 3)**

8. Evaluate $\int_0^{\pi/4} \log(1 + \tan x)\,dx$ **(Scores : 4)**

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OR

Evaluate $\int_0^1 e^x dx$ as the limit of a sum. **(Scores : 4)**

9. (a) Area below the curve $y = -2x + 3$ in the first quadrant.

(a) 1/4 (b) 9/4

(c) 2 (d) 8 **(Score : 1)**

(b) Draw a rough sketch of the curves $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$. Also find the area between these two curves. **(Scores : 5)**

10. (a) The degree of the differential equation $2x^2 \frac{d^2y}{dx^2} + x^4 \left(\frac{dy}{dx}\right)^3 + 7 = 0$ is

(a) 4 (b) 3

(c) 2 (d) 1 **(Score : 1)**

(b) Solve the differential equation $(1 + y^2)\frac{dx}{dy} + x = e^{-\tan^{-1}y}$. **(Scores : 5)**

11. (a) The value of $|\vec{x}|$, if $\vec{b}$ is a unit vector and $(2\vec{x} - 2\vec{b}) \cdot (\vec{x} + \vec{b}) = 30$.

(a) $\sqrt{6}$ (b) 6

(c) 4 (d) 2 **(Score : 1)**

(b) If $\vec{a} = \hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{j} + \hat{k}$, then find a unit vector which is perpendicular to both $\vec{a}$ and $\vec{b}$. **(Scores : 2)**

12. (a) Cosine of the angle between the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is

(a) 1/3 (b) 2/3

(c) 1/2 (d) 1 **(Score : 1)**

(b) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 4$ and each one of them is perpendicular to the sum of other two, then find $|\vec{a} + \vec{b} + \vec{c}|$. **(Scores : 4)**

13. (a) Which of the following is a plane perpendicular to $x + 3y + 4z = 7$?

(a) $4x + 3y + z = 7$ (b) $4x - z = 7$

(c) $3x + 4y + z = 0$ (d) $x + y + z = 0$ **(Score : 1)**

(b) Find the shortest distance between the lines

$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t(-\hat{i} - 2\hat{j} - 2\hat{k})$ and

$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s(\hat{i} - \hat{j} - \hat{k})$ **(Scores : 3)**

14. (a) Distance of the point (1, 0, 0) from the plane $x + 2y + 2z = 0$.

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 1 **(Score : 1)**

(b) Find the Cartesian equation of a line passing through (1, 2, – 4) and perpendicular to the lines $\frac{x-2}{2} = \frac{y-1}{-1} = \frac{z-1}{1}$ and $\frac{x-5}{1} = \frac{y}{1} = \frac{z-2}{1}$. **(Scores : 3)**

15. Consider the linear programming problem.

Maximize $z = x + y$

Subject to the constraints

$x - y \leq -1$

$-2x + y \geq 0,$

$x, y \geq 0$

(a) Find the feasible region. **(Scores : 3)**

(b) Find the corner points of the feasible region. **(Scores : 2)**

(c) Find the maximum point. **(Score : 1)**

16. A man is known to be speak truth 3 out of 4 times.

(a) Write the probability that he speaks truth. **(Score : 1)**

(b) He throws a die.

(i) Find the probability that he reports that "it is a six". **(Scores : 2)**

(ii) If he reports "it is six". Find the probability that it is actually not a six.

(Scores : 2)

OR

Let x denote the number of study hours of a person during a randomly selected day. The probability distribution is given below :

x :	0	1	2	3	4
P(x)	0.1	k	2k	2k	k

(a) Find k. **(Score : 1)**

(b) Find the probability that he studies at least two hours. **(Scores : 4)**