SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2016.
(Finalised Scheme of Valuation)

Subject: Mathematics (Science) **Code No: 1018**

Qn.No		Scoring Indicators	Split Score	Total Score
1.	(a)	(ii) one-one but not onto	1	1
	(b)	$gof(x) = g(f(x))$	1	2
		$= g(8x^3)$	1/2	
		$= (8x^3)^{1/3} = 2x$	1/2	
	(c)	No	1	2
		Any counter example for justification	1	
		Remark: (b) $gof(x) = 2x$ (2)		
		(c) Checking any property of binary operation (1)		
2	(a)	$\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	1
	(b)	$2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left[\frac{2\times\frac{1}{2}}{1-(\frac{1}{2})^2}\right] + \tan^{-1}\frac{1}{7}$	1	3
		$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$	1	
		$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right)$	1/2	
		$= \tan^{-1}\left(\frac{31\times 21}{21\times 17}\right) = \tan^{-1}\left(\frac{31}{17}\right)$	1/2	
		OR		
		$2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$		
		$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{2}}{1-(\frac{1}{2})^2}\right) + \tan^{-1}\frac{1}{7}$	1	
		$= \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7}$	1	
		$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right)$	1/2	
		$= \tan^{-1}(31/17)$	1/2	

Remark: $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$ (1/2).

: Applying the formula for $\tan^{-1}x + \tan^{-1}y$ (1)

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3	(a)	(iii) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	1	1
	(b)	$A^T = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$	1	
		$A + A^T = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$ symmetric	½	
		$A - A^T = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$ skew-symmetric	½	3
		$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$	1	
		$= \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$		
	(c)	$\text{Adj } A = \begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}$	1	
		$\lvert A \rvert = 2$	½	2
		$A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix}$	½	
		Remark. (i) $A^{-1} = \frac{\text{Adj } A}{\lvert A \rvert}$ (½)		
		(ii) Consider other method also to find A^{-1}		
4.	(a)	(i) 1	1	1
	(b)	$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$	1	
		$R_1 \to R_1 + R_2 + R_3$		
		$= (1+x+x^2)\begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$	½	
		$= (1+x+x^2)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$	1	
		$C_2 \to C_2 - C_1 \quad C_3 \to C_3 - C_1$		

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		$=(1+x+x^2)\begin{vmatrix}1 & 0 & 0\\ x^2 & (1-x)(1+x) & x(1-x)\\ x & x(x-1) & 1-x\end{vmatrix}$	½	4
		$=(1+x+x^2)(1-x)(1-x)\begin{vmatrix}1 & 0 & 0\\ x^2 & 1+x & x\\ x & -x & 1\end{vmatrix}$	½	
		$=(1+x+x^2)(1-x)(1-x)(1+x+x^2)=\underline{(1-x^3)^2}$	½	
		Remark: Direct expansion (1) score.		
5	(a)	LHL $\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 2x+3 = 7$	½	2
		RHL $\Rightarrow \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 2x-3 = 1$	½	
		$\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$ (LHL $\neq$ RHL)	½	
		$\therefore$ 2 is a point of discontinuity	½	
		Remark $f(x)$ is discontinuous at $x=2$ (1)		
	(b)	$(x-y)\log e = y\log x$	1	4
		$x-y = y\log x \Rightarrow x = y\log x + y$	½	
		$x = y(\log x + 1)$	½	
		$\therefore y = \dfrac{x}{1+\log x}$	½	
		$\dfrac{dy}{dx} = \dfrac{1+\log x - 1}{(1+\log x)^2} = \dfrac{\log x}{(1+\log x)^2}$	1	
		$= \dfrac{\log x}{(\log ex)^2}$	½	
6	(a)	(iii) 1	1	1
	(b)	$f'(x) = 2x-4$	½	2
		$f'(x)=0 \Rightarrow 2x-4=0 \Rightarrow x=2$	½	
		Intervals $(-\infty, 2)$ $(2,\infty)$	½	
		$f(x)$ is decreasing at $(-\infty, 2)$	½	

Remark : for decreasing $f'(x) < 0$ (½)

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6	(c)	$f(x) = (2x-1)^2+3$ $\quad f'(x) = 4(2x-1)$	½	
		$f''(x) = 8$	½	
		$f'(x)=0 \Rightarrow 2x-1=0 \Rightarrow x=1/2$	½	2
		$f(x)$ has minimum value at $x=1/2$; minimum value is $f(1/2)=3$	½	
		Remark: (i) $f(x)=(2x-1)^2+3$, where $(2x-1)^2 \geq 0$ ∴ It has only minimum value which is zero ∴ The minimum value of $f(x)=(2x-1)^2+3$ for all x = 3 (2)		
		(ii) If $f'(x)=0$ and $f''(x)>0$, then $f(x)$ has minimum value (1)		
		OR		OR
6	a)	(ii) $f(x) = \log x$.	1	1
	(b)	$y = 3x^2$ ∴ $\frac{dy}{dx} = 6x$	½	
		Slope $= \left.\frac{dy}{dx}\right\rvert_{(1,1)} = 6$	½	2
		∴ eqⁿ of tangent $y-y_1 = \left.\frac{dy}{dx}\right\rvert_{(1,1)}(x-x_1)$	½	
		$y-1 = 6(x-1)$	½	
		$y = 6x-5$		
		Remark: slope $= \frac{dy}{dx} = 6$ (1)		
	c)	$y = \sqrt{x}$	½	
		$x = 36 \quad \Delta x = 0.6$	½	
		$dy = \left(\frac{dy}{dx}\right)\Delta x = \frac{1}{2\times\sqrt{x}} \times 0.6 = 0.05$	½	2
		$\sqrt{36.6} = 6+0.05 \approx 6.05$	½	
		Remark: $\sqrt{36.6} = \sqrt{x} + \frac{1}{2\sqrt{x}}\Delta x$. and finding the value (2)		

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Qn.No		Scoring Indicators	Split Score	Total Score
7	(a)	(ii) $\pi/3$	1	1
	(b)	unit vector along $\bar{a}-\bar{b} = \frac{\bar{a}-\bar{b}}{\lvert\bar{a}-\bar{b}\rvert}$	½	
		$\bar{a}-\bar{b} = -2\bar{i}+\bar{j}-2\bar{k}$	½	2
		$\lvert\bar{a}-\bar{b}\rvert = \sqrt{9} = 3$	½	
		unit vector $= \frac{1}{3}(-2\bar{i}+\bar{j}-2\bar{k})$	½	
8	(a)	(ii) $\hat{i}-\hat{j}$	1	1
	(b)	$\begin{vmatrix} 2 & -4 & 5 \\ 1 & -\lambda & 1 \\ 3 & 2 & -5 \end{vmatrix} = 0$	1	
		$2(5\lambda-2)+4(-5-3)+5(2+3\lambda)=0$	½	2
		$\lambda = \frac{26}{25}$	½	
		<u>Remark:</u> For coplanar $[\bar{a}\ \bar{b}\ \bar{c}]=0$ Or $\bar{a}\cdot(\bar{b}\times\bar{c})=0$ (½)		
	(c)	$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{\lvert\bar{a}\rvert\lvert\bar{b}\rvert}$	½	
		$\bar{a}\cdot\bar{b} = -1$	½	2
		$\lvert\bar{a}\rvert = \sqrt{3}$ $\lvert\bar{b}\rvert = \sqrt{3}$	½	
		$\cos\theta = \frac{-1}{3}$ or $\theta = \cos^{-1}(-1/3)$	½	
		<u>Remark:</u> $\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$ (½)		
		$\theta = \cos^{-1}(-1/3)$ (1½)		
9	(a)	$\cos^2 x = \frac{1+\cos 2x}{2}$	½	
		$\int \cos^2 x\,dx = \frac{1}{2}\int (1+\cos 2x)\,dx$	½	2
		$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + C$	1	
	(b)	$2x - x^2 = -(x^2-2x)$	½	
		$= -[x^2-2x+1-1]$		
		$= -[(x-1)^2-1^2] = 1-(x-1)^2$	½	

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Qn.No		Scoring Indicators	Split Score	Total Score
		$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{1-(x-1)^2}}$	½	2
		$= \underline{\sin^{-1}(x-1)+C}$	½	
		Remark: $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+C$ (½)		
	c)	$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$	1	2
		$= x\sin x + \cos x + C$	1	
		Remark: $\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\cdot\int v\,dx\right)dx$ (½)		
		$\int \sin x\,dx = -\cos x$ (½)		
		$\int \cos x\,dx = \sin x$ (½)		
10.	(a)	$I = \int_0^{\pi} \log(1+\cos x)\,dx$ — (1)		
		$\Rightarrow \int_0^{\pi} \log(1+\cos(\pi-x))\,dx$	1	
		$= \int_0^{\pi} \log(1-\cos x)\,dx$.		
		$= \int_0^{\pi} \log 2\sin^2 x/2\,dx$	1	
		$= \int_0^{\pi} (\log 2 + 2\log\sin x/2)\,dx$	1	
		$= \int_0^{\pi} \log 2\,dx + 2\int_0^{\pi} \log\sin x/2\,dx$		4.
		put $\frac{x}{2} = t \Rightarrow dx = 2dt$		
		$x=0$ $t=0$ and $x=\pi$ $t=\pi/2$	½	
		$I = \log 2\,(x)_0^{\pi} + 2\int_0^{\pi/2} \log\sin t\cdot 2dt$		
		$= \log 2(\pi-0) + 4\left(-\frac{\pi}{2}\log 2\right)$		
		$= \underline{\underline{-\pi\log 2}}$	½	

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Qn.No	Scoring Indicators	Split Score	Total Score
10	Alternative method		
	$I = \int_0^{\pi} \log(1+\cos x)\,dx \quad (1)$		
	$= \int_0^{\pi} \log(1+\cos(\pi - x))\,dx$	1	
	$I = \int_0^{\pi} \log(1-\cos x)\,dx \quad (2)$		
	$2I = \int_0^{\pi} [\log(1+\cos x) + \log(1-\cos x)]\,dx$	1	
	$= \int_0^{\pi} \log(1-\cos^2 x)\,dx$		4
	$= \int_0^{\pi} \log \sin^2 x\,dx = 2\int_0^{\pi} \log \sin x\,dx$	1	
	$\therefore I = \int_0^{\pi} \log \sin x\,dx = -\pi \log 2$	½+½	
	OR	OR	
	$\int_a^b f(x)\,dx = \lim_{h\to 0} h[f(a)+f(a+h)+ \dots + f(a+(n-1)h)]$	1	
	$\therefore \int_0^5 (x+1)\,dx = \lim_{h\to 0} h[1+(1+h)+(1+2h)+ \dots + 1+(n-1)h]$	½	
	$= \lim_{h\to 0} h[n + h(1+2+3 \dots + n-1)]$		
	$= \lim_{h\to 0} h\left[n + h\frac{n(n-1)}{2}\right]$	½	
	$= \lim_{h\to 0} \left[nh + \frac{n^2h^2 - h^2 n}{2}\right]$	½	
	$= \lim_{h\to 0} \left[nh + nh\frac{(nh-h)}{2}\right]$	½	

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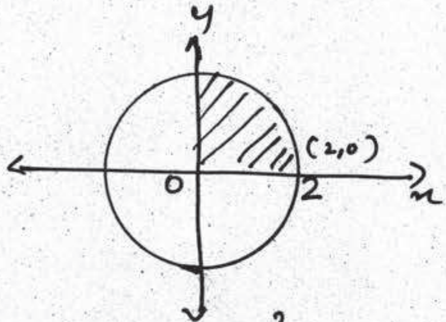
Qn.No		Scoring Indicators	Split Score	Total Score
		$= \lim_{h \to 0}\left(5 + \frac{(5-h)5}{2}\right) = \frac{35}{2}$	1	4
		Remark: For $f(x) = x+1$ $a=0$ $b=5$ (1)		
		For directer method $\int_0^5 (x+1)dx = \frac{35}{2}$ (1)		
11	(a)	(iv) $\int_a^b y\,dx$	1	1
	(b)		1	
		Area of the shaded region $= \int_0^2 y\,dx$	1	
		$= \int_0^2 \sqrt{4-x^2}\,dx$	1	5
		$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$	1	
		$= \frac{4}{2}\sin^{-1} 1$		
		$= 2 \cdot \frac{\pi}{2}$	½	
		$\therefore$ Required area $= 4\pi$	½	
		Remark: (1) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ (1)		
		(2) Without figure and correct answer (5)		
12	(a)	(i) $\frac{d^2y}{dx^2} + y = 0$	1	1
	(b)	$\frac{dy}{dx} + \frac{2y}{x} = x$	1	
		$P = \frac{2}{x}$ $Q = x$	½	
		$I.F = e^{\int P dx}$	½	
		$= e^{(2\log x)} = x^2$	½	

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Qn.No		Scoring Indicators	Split Score	Total Score
		Solution $y \cdot x^2 = \int x \cdot x^2 dx$.	1	
		$= \int x^3 dx$		
		$yx^2 = \frac{x^4}{4} + c$	½	5
		when $x = 1$, $y = 0$ $0 = \frac{1}{4} + c \quad \therefore c = \frac{-1}{4}$	½	
		∴ particular solution $yx^2 = \frac{x^4}{4} - \frac{1}{4}$	½	
		Remark: Solution $y \cdot IF = \int Q \cdot IF\, dx$ (1)		
13		$SD = \left\| \frac{(a_2 - a_1) \cdot (b_1 \times b_2)}{\|b_1 \times b_2\|} \right\|$	1	
		$a_1 = \hat{i} + \hat{j} \quad a_2 = 2\hat{i} + \hat{j} - \hat{k}$	½	
		$b_1 = 2\hat{i} - \hat{j} + \hat{k} \quad b_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$	½	
		$a_2 - a_1 = \hat{i} - \hat{k}$	½	4
		$b_1 \times b_2 = 3\hat{i} - \hat{j} + 7\hat{k}$	½	
		$\|b_1 \times b_2\| = \sqrt{59}$	½	
		$SD = \frac{10}{\sqrt{59}}$	½	
14	(a)	(iv) $6x + 4y + 3z = 12$	1	1
	(b)	eqⁿ is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$	1	
		$\Rightarrow \begin{vmatrix} x - 2 & y - 5 & z + 3 \\ -2 - 2 & -3 - 5 & 5 + 3 \\ 5 - 2 & 3 - 5 & -3 + 3 \end{vmatrix} = 0$	1	

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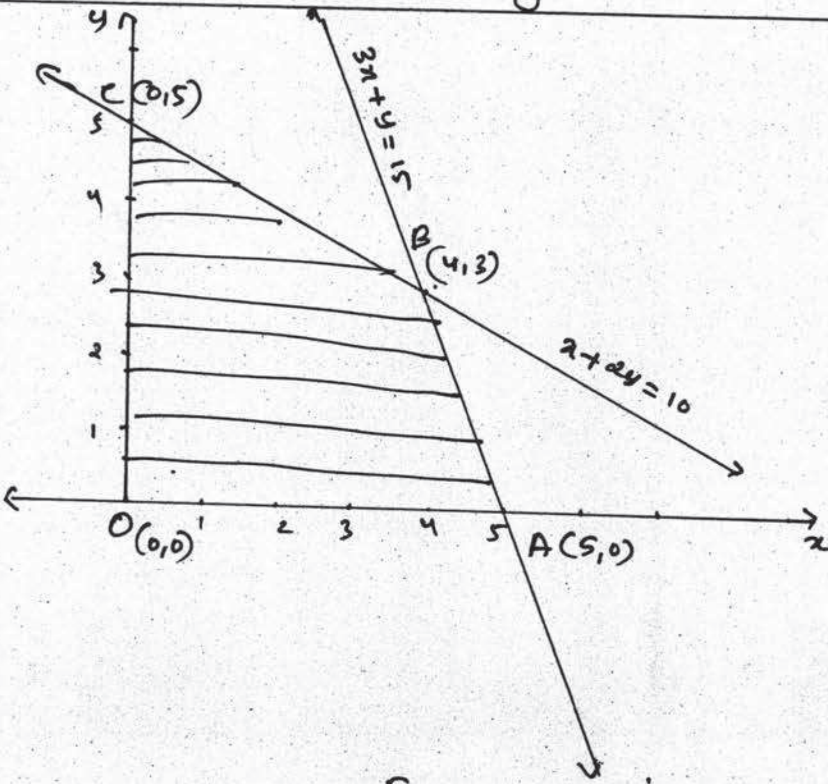
		$\Rightarrow (x-2)16 + (y-5)24 + (z+3)32 = 0$	1/2	3
		$2x + 3y + 4z = 7$	1/2	
		Remark: Using vector method and correct answer give (3)		
15.	(a)			
		For x, y axis	1/2	3
		For the line $x+2y=10$	1	
		For the line $3x+y=15$	1	
		fessible region	1/2	
	(b)	Corner points O(0,0) A(5,0) B(4,3) C(0,5) (1/2 mark each)	2	2
	(c)	Z is maximum at B(4,3) z = 18	1	1
		Remark: For any two correct corner points 2 score		

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16	(a)	(iv) 0.58	1	1
	(b)	E_1 : lost card is a diamond E_2 : lost card is not a diamond A : Selecting '2' diamonds from the remaining cards.	1	
		$P(E_1) = \frac{13}{52} = \frac{1}{4}$ $P(E_2) = \frac{39}{52} = \frac{3}{4}$	1	
		$P(A/E_1) = \frac{12C_2}{51C_2} = \frac{12\times11}{51\times50}$ $P(A/E_2) = \frac{13C_2}{51C_2} = \frac{13\times12}{51\times50}$		4.
		$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)}$	1	
		$= \frac{\frac{1}{4}\times\frac{12\times11}{51\times50}}{\frac{1}{4}\times\frac{12\times11}{51\times50}+\frac{3}{4}\times\frac{13\times12}{51\times50}}$	½	
		$= \frac{132}{600} = \frac{11}{50}$	½.	
		OR.		
16	1)	$\frac{1}{6}$	1	1
	2)	$n = 4$	½	
		$q = 1-p = 1-1/6 = 5/6$	½	
		$P(x=r) = nC_r\, q^{n-r}\, p^r$	1	

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		$P(x=2) = 4C_2\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2$	1	
		$= \frac{4\times 3}{1\cdot 2} \times \frac{25}{36} \times \frac{1}{36}$		4.
		$= \frac{25}{216}$	½.	
		Remark:		
	(a)	Writing $\{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$ (½)		
	b)	$P(x=2) = 4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$ (3½)		