SECOND YEAR HIGHER SECONDARY SAY/IMP. EXAMINATION, JUNE 2016.
(Finalised Scheme of Valuation)

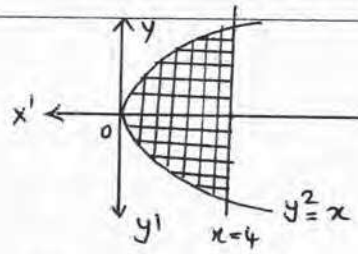
Subject: Mathematics (Commerce) Code No: 2053

Qn.No		Scoring Indicators	Split Score	Total Score
1	(i)	$A' = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 5 & 0 \\ 3 & 7 & -2 \end{bmatrix}$	1	1
	(ii)	$A + A' = \begin{bmatrix} 2 & 6 & 6 \\ 6 & 10 & 7 \\ 6 & 7 & -4 \end{bmatrix}$	1	5
		$A - A' = \begin{bmatrix} 0 & -2 & 0 \\ 2 & 0 & 7 \\ 0 & -7 & 0 \end{bmatrix}$	1	2
	(iii)	$\frac{1}{2}(A+A') = \begin{bmatrix} 1 & 3 & 3 \\ 3 & 5 & 7/2 \\ 3 & 7/2 & -2 \end{bmatrix}$, $\frac{1}{2}(A-A') = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 7/2 \\ 0 & -7/2 & 0 \end{bmatrix}$	1	
		$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A') = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 3 & 0 & -2 \end{bmatrix}$	1	2
2	(i)	(b) $K^n \lvert A \rvert$	1	1
	(ii)	$C_1 \to C_1 + C_2 + C_3$ $\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix}$	1	
		$\Rightarrow (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$	$\frac{1}{2}$	4
		$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$		
		$\Rightarrow (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$	1	
		$\Rightarrow x^2(x+a+b+c)$	$\frac{1}{2}$	3

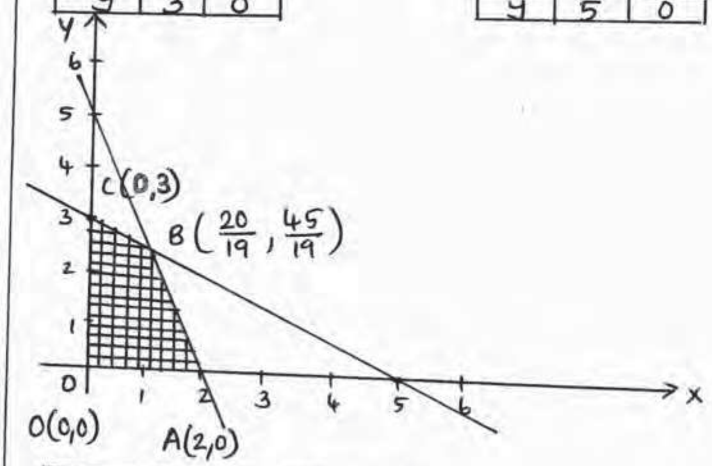
Qn.No		Scoring Indicators	Split Score	Total Score
3	(i)	$\lvert A\rvert = \begin{vmatrix}1 & 2 & 2\\ 2 & -1 & 3\\ 3 & -1 & -1\end{vmatrix} = 1(1+3)-2(-2-9)+2(-2+3)$ $= 4+22+2 = \underline{\underline{28}}$	1	1
	(ii)	Co-factor $A = \begin{bmatrix}4 & 11 & 1\\ 0 & -7 & 7\\ 8 & 1 & -5\end{bmatrix}$	2	
		Adj $A = \begin{bmatrix}4 & 0 & 8\\ 11 & -7 & 1\\ 1 & 7 & -5\end{bmatrix}$	1	
		$A^{-1} = \dfrac{\text{Adj } A}{\lvert A\rvert} = \dfrac{\begin{bmatrix}4 & 0 & 8\\ 11 & -7 & 1\\ 1 & 7 & -5\end{bmatrix}}{28}$		5
		$X = A^{-1}B = \frac{1}{28}\begin{bmatrix}4 & 0 & 8\\ 11 & -7 & 1\\ 1 & 7 & -5\end{bmatrix}\begin{bmatrix}4\\ 9\\ 2\end{bmatrix} = \frac{1}{28}\begin{bmatrix}32\\ -17\\ 57\end{bmatrix}$	$\frac{1}{2}$	
		$x = \dfrac{32}{28}, \quad y = \dfrac{-17}{28}, \quad z = \dfrac{57}{28}$	$\frac{1}{2}$	4
		<u>Remarks</u>: Give $\frac{1}{2}$ score for $A^{-1} = \frac{\text{Adj}A}{\lvert A\rvert}$, $X = A^{-1}B$		
4	(i)	(c) $27x^4$	1	1
	(ii)	(a) $a*b = ab+1$ $b*a = ba+1$, Hence $*$ is Commutative	1 1	2
		(b) $(a*b)*c = (ab+1)*c = abc+c+1$ $a*(b*c) = a*(bc+1) = abc+a+1$ $(a*b)*c \neq a*(b*c)$, Hence $*$ is not associative	1 1	5 2
5	(i)	$-\dfrac{\pi}{3}$	1	1
	(ii)	$2\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \tan^{-1}x$ $\Rightarrow \tan^{-1}\left[\dfrac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2}\right] = \tan^{-1}x$	1	
		$\Rightarrow \dfrac{2(1-x)(1+x)}{(1+x)^2-(1-x)^2} = x \Rightarrow \dfrac{2-2x^2}{4x} = x$	1	4

Qn.No		Scoring Indicators	Split Score	Total Score
		$\Rightarrow 6x^2=2 \Rightarrow x^2=\frac{1}{3} \Rightarrow x=\pm\frac{1}{\sqrt{3}}$ Since $x>0$, $x=\frac{1}{\sqrt{3}}$ Remark Give full score for alternate method	1	3
6	(i)	$f(-2)=-5$	1	1
	(ii)	$\lim_{x\to-2} f(x)=\lim_{x\to-2}\frac{x^2-x-6}{x+2}=\lim_{x\to-2}\frac{(x-3)(x+2)}{(x+2)}$ $=\lim_{x\to-2}(x-3)=-2-3=-5$ $\therefore \lim_{x\to-2} f(x)=f(-2)=-5$ So f is continuous at $x=-2$	1 1	3 2
7	(i)	$y=\log\left(\frac{1}{x}\right)$, $\frac{dy}{dx}=-\frac{1}{x}$ $\frac{dy}{dx}+\frac{1}{x}=-\frac{1}{x}+\frac{1}{x}=0$ Remarks: Give full score for alternate method	1 1	2
	(ii)	$\frac{dy}{dx}=-a\sin(\log x)\frac{1}{x}+b\cos(\log x)\frac{1}{x}$ $\Rightarrow x\frac{dy}{dx}=-a\sin(\log x)+b\cos(\log x)$ $\Rightarrow x\frac{d^2y}{dx^2}+\frac{dy}{dx}=-a\cos(\log x)\frac{1}{x}-b\sin(\log x)\frac{1}{x}$ $\Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$	1 $\frac{1}{2}$ 1 $\frac{1}{2}$	8 3
	(iii)	$\frac{dy}{dx}=xe^x+e^x-\frac{1}{x^2}=e^x(x+1)-\frac{1}{x^2}$	3	3
8	(i)	Revenue when $x=20$ units $=280$	1	1
	(ii)	$MR=\frac{d}{dx}(R(x))=\frac{2x}{5}$	1	1
	(iii)	Marginal Revenue at $x=10$ units $=4$ Marginal Revenue at $x=25$ units $=10$	1 1	4 2

Qn.No		Scoring Indicators	Split Score	Total Score
		OR		
	(i)	Since $f(x)$ is an exponential function, f is continuous and differentiable in R		
		$f'(x) = 2e^{2x} > 0 \quad \forall x \in R$	1	1
		Hence $f(x)$ is an increasing on R		
	(ii)	$f'(x) = 4 - x$, $f'(x) = 0 \Rightarrow x = 4$	1	
		$f(-2) = -10$, $f(4) = 8$, $f\left(\frac{9}{2}\right) = \frac{63}{8}$	1	4
		Hence maximum value of the function $= \underline{8}$	1	3
9	(i)	$e^{\log x} = x$	$\frac{1}{2}$	
		$\int e^{\log x} dx = \int x\, dx = \underline{\frac{x^2}{2} + C}$	$\frac{1}{2}$	1
	(ii)	$\int \frac{1}{x(1+\log x)} dx$ $\quad t = 1 + \log x$, $dx = x\, dt$	1	5
		$\int \frac{1}{t} dt = \log\lvert t\rvert + C = \underline{\log\lvert 1 + \log x\rvert + C}$	1	2
	(iii)	$\int x \sin x\, dx = x\, {}^{-}\cos x - \int 1\, {}^{-}\cos x\, dx$	1	
		$= -x\cos x + \int \cos x\, dx$		
		$= \underline{-x\cos x + \sin x + C}$	1	2
		OR		
	(i)	(a) 2	2	2
	(ii)	$I = \int_0^{\frac{\pi}{2}} \log(\tan x)\, dx$		
		$= \int_0^{\frac{\pi}{2}} \log(\cot x)\, dx$	1	5
		$2I = \int_0^{\frac{\pi}{2}} \left(\log(\tan x) + \log(\cot x)\right) dx$		
		$= \int_0^{\frac{\pi}{2}} \log 1\, dx = 0$	1	2
	(iii)	$f(-x) = (-\sin x)^7 = -\sin^7 x = -f(x)$, $f(x)$ is an odd function		
		$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx = 0$	1	1

Qn.No		Scoring Indicators	Split Score	Total Score
10	(i)		1	1
	(ii)	Required area = 2 x shaded area		(4)
		$= 2\int_0^4 y\,dx$	1	
		$= 2\int_0^4 \sqrt{x}\,dx = 2\left[\frac{x^{3/2}}{3/2}\right]_0^4$	1	
		$= \frac{4}{3} \times 8 = \frac{32}{3}$ sq. units	1	3
11	(i)	$xy = c^2 \Rightarrow x\frac{dy}{dx} + y = 0$	1	1
	(ii)	(a) $\frac{dy}{1+y^2} = (1+x^2)\,dx$, which is the variable separable form	1	1
		(b) $\int \frac{dy}{1+y^2} = \int (1+x^2)\,dx$	1	(4)
		$\Rightarrow \tan^{-1}y = x + \frac{x^3}{3} + C$, which is the required general solution	1	2
12	(i)	$\overrightarrow{AB} = -\hat{i} + 3\hat{j} + 5\hat{k}$, $\overrightarrow{BC} = -\hat{i} - 2\hat{j} - 6\hat{k}$	$\frac{1}{2}+\frac{1}{2}$	1
	(ii)	$\overrightarrow{AB} \times \overrightarrow{BC} = -8\hat{i} - 11\hat{j} + 5\hat{k}$	1	
		$\lvert\overrightarrow{AB} \times \overrightarrow{BC}\rvert = \sqrt{210}$	1	
		Unit vector perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{BC} = \frac{\overrightarrow{AB} \times \overrightarrow{BC}}{\lvert\overrightarrow{AB} \times \overrightarrow{BC}\rvert}$	$\frac{1}{2}$	(6)
		$= \frac{-8\hat{i} - 11\hat{j} + 5\hat{k}}{\sqrt{210}}$		3
	(iii)	$\overrightarrow{CA} = 2\hat{i} - \hat{j} + \hat{k}$	$\frac{1}{2}$	
		$\overrightarrow{AB} \cdot \overrightarrow{CA} = -2 - 3 + 5 = 0$, Hence $\overrightarrow{AB} \perp r\ \overrightarrow{CA}$,	1	
		ΔABC is a right angled triangle	1	2
		Remark Give full score for alternate method		

Qn.No		Scoring Indicators	Split Score	Total Score
13	(i)	$\vec{b}_1 = 3\hat{i}+2\hat{j}+6\hat{k}$, $\vec{b}_2 = \hat{i}+2\hat{j}+2\hat{k}$	1	
		$\cos\theta = \left\lvert \dfrac{\vec{b}_1 \cdot \vec{b}_2}{\lvert\vec{b}_1\rvert\,\lvert\vec{b}_2\rvert} \right\rvert$	1	
		$= \left\lvert \dfrac{(3\hat{i}+2\hat{j}+6\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{49}\ \sqrt{9}} \right\rvert = \dfrac{19}{21}$	1	3
		$\therefore \theta = \cos^{-1}\left(\dfrac{19}{21}\right)$		5
	(ii)	Required equation is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$	1	
		$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$	1	2
		$-2x-3y+3z+5=0$ or $2x+3y-3z-5=0$		
		OR		
	(i)	$SD = \left\lvert \dfrac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{\lvert\vec{b}_1\times\vec{b}_2\rvert} \right\rvert$	$\frac{1}{2}$	
		$\vec{a}_1 = -\hat{i}-\hat{j}-\hat{k}$, $\vec{a}_2 = 3\hat{i}+5\hat{j}+7\hat{k}$		
		$\vec{b}_1 = 7\hat{i}-6\hat{j}+\hat{k}$, $\vec{b}_2 = \hat{i}-2\hat{j}+\hat{k}$	1	
		$\vec{a}_2-\vec{a}_1 = 4\hat{i}+6\hat{j}+8\hat{k}$		
		$\vec{b}_1\times\vec{b}_2 = -4\hat{i}-6\hat{j}-8\hat{k}$	$\frac{1}{2}$	
		$\lvert\vec{b}_1\times\vec{b}_2\rvert = \sqrt{116}$	$\frac{1}{2}$	3
		$d = \dfrac{116}{\sqrt{116}} = \sqrt{116}$	$\frac{1}{2}$	5
	(ii)	$\cos\theta = \left\lvert \dfrac{\vec{n}_1\cdot\vec{n}_2}{\lvert\vec{n}_1\rvert\,\lvert\vec{n}_2\rvert} \right\rvert = \left\lvert \dfrac{(2\hat{i}+2\hat{j}-3\hat{k})\cdot(3\hat{i}-3\hat{j}+5\hat{k})}{\sqrt{17}\ \sqrt{43}} \right\rvert$	1	
		$= \dfrac{15}{\sqrt{731}}$	1	2
		$\therefore \theta = \cos^{-1}\left(\dfrac{15}{\sqrt{731}}\right)$		

<table>
<tr><th>Qn.No</th><th></th><th>Scoring Indicators</th><th>Split Score</th><th>Total Score</th></tr>
<tr><td>14</td><td>(i)</td><td>Objective function is Maximise $Z = 75x + 25y$
$400x + 100y \leq 12,000$ or $4x + y \leq 120$
$x + y \leq 90$
$x, y \geq 0$</td><td>1
1
1
1</td><td>1
4
3</td></tr>
<tr><td>15</td><td></td><td>$3x + 5y = 15$
<table><tr><td>x</td><td>0</td><td>5</td></tr><tr><td>y</td><td>3</td><td>0</td></tr></table>$5x + 2y = 10$
<table><tr><td>x</td><td>0</td><td>2</td></tr><tr><td>y</td><td>5</td><td>0</td></tr></table>

<table><tr><th>Corner points</th><th>Z</th></tr><tr><td>O (0,0)</td><td>0</td></tr><tr><td>A (2,0)</td><td>10</td></tr><tr><td>B$\left(\frac{20}{19}, \frac{45}{19}\right)$</td><td>$\frac{235}{19}$</td></tr><tr><td>C (0, 3)</td><td>9</td></tr></table>Maximum value of Z is $\frac{235}{19}$ at $B\left(\frac{20}{19}, \frac{45}{19}\right)$</td><td>1
2
1</td><td>4</td></tr>
<tr><td>16</td><td>(i)

(ii)</td><td>P (both balls are red) = P(first ball is red and Second ball is red)
$= \frac{8}{18} \times \frac{8}{18} = \frac{16}{81}$
P(first ball is black and second ball is red)
$= \frac{10}{18} \times \frac{8}{18} = \frac{20}{81}$</td><td>1

1+1</td><td>1

2</td></tr>
</table>

Qn.No		Scoring Indicators	Split Score	Total Score
	(iii)	P(one of them black and other is red)		5
		$= P(B)\,P(R) + P(R)\,P(B)$	1	
		$= 2\,P(B)\,P(R)$		
		$= 2 \times \frac{10}{18} \times \frac{8}{18} = \frac{40}{81}$	1	2
17	(i)	$n = 6,\ P = \frac{1}{2},\ q = \frac{1}{2}$		
		$P(X=x) = {}^nC_x\, p^x q^{n-x}$	$\frac{1}{2}$	
		$P(X=x) = 6C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{n-x}$	$\frac{1}{2}$	1
	(ii)	P(at least 5 Successes) $= P(X=5) + P(X=6)$	1	
		$= 6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right) + 6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0$	1	2
		$= 7\left(\frac{1}{2}\right)^6 = \frac{7}{64}$		5
	(iii)	P(at most 5 Successes) $= 1 - P(6 \text{ Successes})$	1	
		$= 1 - P(X=6)$		
		$= 1 - \left(\frac{1}{2}\right)^6$		
		$= 1 - \frac{1}{64} = \frac{63}{64}$	1	2