Reg. No. : ....

Name : ....

Code No. 2018

Second Year – JUNE 2016
SAY / IMPROVEMENT

Time : 2½ Hours
Cool-off time : 15 Minutes

Part – III

MATHEMATICS (SCIENCE)

Maximum : 80 Scores

General Instructions to Candidates :

- There is a 'cool-off time' of 15 minutes in addition to the writing time of 2½ hrs.
- You are not allowed to write your answers nor to discuss anything with others during the 'cool-off time'.
- Use the 'cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

നിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും. ഈ സമയത്ത് ചോദ്യങ്ങൾക്ക് ഉത്തരം എഴുതാനോ, മറ്റുള്ളവരുമായി ആശയവിനിമയം നടത്താനോ പാടില്ല.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- എല്ലാ ചോദ്യങ്ങൾക്കും ഉത്തരം എഴുതണം.
- ഒരു ചോദ്യനമ്പർ ഉത്തരമെഴുതാൻ തെരഞ്ഞെടുത്തു കഴിഞ്ഞാൽ ഉപചോദ്യങ്ങളും അതേ ചോദ്യനമ്പരിൽ നിന്ന് തന്നെ തെരഞ്ഞെടുക്കേണ്ടതാണ്.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നൽകിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

1. (a) If the matrix A is both symmetric and skew-symmetric, then A is a

(i) diagonal matrix (ii) zero matrix

(iii) square matrix (iv) scalar matrix **(Score : 1)**

(b) If $A = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$, then show that $A^2 - 5A + 10I = 0$ **(Scores : 3)**

(c) Hence find A^{-1}. **(Scores : 2)**

2. (a) The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix}$ is

(i) -4 (ii) 0

(iii) 1 (iv) 4 **(Score : 1)**

(b) Using matrix method, solve the system of linear equations,

$x + y + 2z = 4$

$2x - y + 3z = 9$

$3x - y - z = 2$ **(Scores : 4)**

3. (a) If $f : R \to R$ and $g : R \to R$ defined by $f(x) = x^2$ and $g(x) = x + 1$, then $gof(x)$ is

(i) $(x + 1)^2$ (ii) $x^3 + 1$

(iii) $x^2 + 1$ (iv) $x + 1$ **(Score : 1)**

(b) Consider the function $f : N \to N$, given by $f(x) = x^3$. Show that the function f is injective but not surjective. **(Scores : 2)**

(c) The given table shows an operation * on A = {p, q}

*	p	q
p	p	q
q	p	q

(i) Is * a binary operation on A ?

(ii) Is * commutative ? Give reason. **(Scores : 2)**

4. (a) The principal value of $\tan^{-1}(-\sqrt{3})$ is

(i) $\frac{\pi}{3}$ (ii) $\frac{-\pi}{3}$

(iii) $\frac{\pi}{4}$ (iv) $\frac{-\pi}{6}$ **(Score : 1)**

(b) Show that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}$ **(Scores : 3)**

5. (a) Find $\frac{dy}{dx}$, if $x = a\cos^2\theta$, $y = b\sin^2\theta$. **(Scores : 3)**

(b) Find the second derivative of the function

$y = e^x \sin x$ **(Scores : 3)**

6. (a) The slope of the normal to the curve, $y = x^3 - x^2$ at $(1, -1)$ is

(i) 1 (ii) –1

(iii) 2 (iv) 0 **(Score : 1)**

(b) Find the intervals in which the function $f(x) = 2x^3 - 24x + 25$ is increasing or decreasing. **(Scores : 4)**

OR

(a) The rate of change of the area of a circle with respect to radius r, when r = 5 cm

(i) 25π cm²/cm (ii) 25 cm²/cm

(iii) 10π cm²/cm (iv) 10 cm²/cm **(Score : 1)**

(b) Show that of all rectangles with a given area, the square has the least perimeter.

(Scores : 4)

7. Find the following :

(a) $\int \cot x \log \sin x \, dx$ **(Scores : 2)**

(b) $\int \frac{1}{x^2 + 2x + 2} dx$ **(Scores : 2)**

(c) $\int x e^{9x} dx$ **(Scores : 2)**

8. (a) The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ is

(i) 2 (ii) 1

(iii) 0 (iv) Not defined **(Score : 1)**

(b) Solve $\frac{dy}{dx} + 2y \tan x = \sin x$, $y = 0$, when $x = \frac{\pi}{3}$. **(Scores : 5)**

9. (a) The projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ is

(i) 1 (ii) 0

(iii) 2 (iv) –1 **(Score : 1)**

(b) Find the area of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ **(Scores : 2)**

10. Find $\int_0^{\pi} \frac{x}{1 + \sin x} dx$. **(Scores : 4)**

OR

Find $\int_0^2 e^x dx$ as the limit of a sum. **(Scores : 4)**

11. (a) The area bounded by the curve $y = 2 \cos x$, the x-axis from $x = 0$ to $x = \frac{\pi}{2}$ is

(i) 0 (ii) 1

(iii) 2 (iv) –1 **(Score : 1)**

(b) Find the area of the region bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$.

(Scores : 5)

12. (a) $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$ is equal to

(i) $\vec{0}$ (ii) $|\vec{a}|^2 - |\vec{b}|^2$

(iii) $\vec{a} \times \vec{b}$ (iv) $2(\vec{a} \times \vec{b})$ **(Score : 1)**

(b) If $\vec{a}$ and $\vec{b}$ are any two vectors, then prove that $(\vec{a} \times \vec{b})^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} \end{vmatrix}$

(Scores : 2)

(c) Using vectors, show that the points

A(1, 2, 7), B(2, 6, 3) and C(3, 10, –1) are collinear. **(Scores : 2)**

13. (a) The equation of the line which passes through the point (1, 2, 3) and parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$ is

(i) $3\hat{i} + 2\hat{j} - 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

(ii) $2\hat{i} - 5\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$

(iii) $\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-2\hat{i} + 4\hat{j} - 2\hat{k})$

(iv) $\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$ **(Score : 1)**

(b) Find the angle between the pair of lines

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ **(Scores : 3)**

14. (a) The distance of the plane $x + y + z + 1 = 0$ from the point (1, 1, 1) is

(i) 4 units (ii) $\frac{1}{\sqrt{3}}$ units

(iii) $\frac{4}{\sqrt{3}}$ units (iv) $\frac{1}{4\sqrt{3}}$ units **(Score : 1)**

(b) Find the equation of the plane passing through (1, 0, –2) and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$. **(Scores : 3)**

15. Consider the following L.P.P.

Maximise, $Z = 3x + 9y$

Subject to the constraints $x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$

(a) Draw its feasible region. **(Scores : 3)**

(b) Find the corner points of the feasible region. **(Scores : 3)**

16. (a) If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$ then P(A/B) is

(i) $\frac{9}{4}$ (ii) $\frac{16}{13}$

(iii) $\frac{4}{9}$ (iv) $\frac{11}{13}$ **(Score : 1)**

(b) Probability of solving a specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, then

(i) Find the probability that the problem is solved. **(Scores : 2)**

(ii) Find the probability that exactly one of them solves the problem. **(Scores : 2)**

OR

A die is thrown 6 times. If getting an odd number is a success

(i) Find probability of success and failure **(Score : 1)**

(ii) Find the probability of 5 success. **(Scores : 2)**

(iii) Find the probability of atleast 5 successes. **(Scores : 2)**