

Reg. No. :

Code No. 8018

Name :

For Scheme-I Candidates only

**Second Year – 2015
SAY / IMPROVEMENT**

Time : 2½ Hours
Cool-off time : 15 Minutes

Part – III

MATHEMATICS (SCIENCE)

Maximum : 80 Scores

General Instructions to Candidates :

- There is a 'cool-off time' of 15 minutes in addition to the writing time of 2½ hrs.
- You are not allowed to discuss anything with others during the 'cool-off time'.
- Use the 'cool-off time' to plan your answers.
- Read question carefully.
- All questions are compulsory and only internal choice is allowed.
- When you select a question, all the sub-questions must be answered from the same question itself.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

നിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും. ഈ സമയത്ത് ചോദ്യങ്ങൾക്ക് ഉത്തരം എഴുതാനോ, മറ്റുള്ളവരുമായി ആശയവിനിമയം നടത്താനോ പാടില്ല.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- എല്ലാ ചോദ്യങ്ങൾക്കും ഉത്തരം എഴുതണം.
- ഒരു ചോദ്യനമ്പർ ഉത്തരമെഴുതാൻ തെരഞ്ഞെടുത്തു കഴിഞ്ഞാൽ ഉപചോദ്യങ്ങളും അതേ ചോദ്യനമ്പറിൽ നിന്ന് തന്നെ തെരഞ്ഞെടുക്കേണ്ടതാണ്.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നൽകിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

1. (a) When a relation R on a set A is said to be reflexive ? (Score : 1)
- (b) Find gof and fog, if $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$. (Scores : 2)
- (c) Show that $f : [-1, 1] \longrightarrow \mathbb{R}$ given by $f(x) = \frac{x}{x+2}$ is one-one. (Scores : 2)
2. (a) Write the principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$. (Score : 1)
- (b) Express $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$ in the simplest form. (Scores : 3)
3. Consider a 2×2 matrix $A = [a_{ij}]$ with $a_{ij} = 2^i + j$. (HS SLIVE.IN)
- (a) Construct A (Score : 1)
- (b) Find $A + A'$ and $A - A'$ (Score : 1)
- (c) Express A as sum of a symmetric and skew-symmetric matrix. (Score : 1)
4. (a) Find the relationship between 'a' and 'b' if the function f defined by
- $$f(x) = ax + 1, \quad x \leq 3$$
- $$= bx + 3, \quad x > 3$$
- is continuous (Scores : 2)
- (b) Find $\frac{dy}{dx}$, if $y^x = x^y$. (Scores : 2)
- (c) If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$. (Scores : 2)

5. Consider the matrix $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$.

(a) Find A^2 (Scores : 2)

(b) Find k so that $A^2 = kA - 2I$ (Score : 1)

6. (a) If $\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} = 15$, then find the values of x .

(b) Solve the following system of equations :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

7. (a) Find the equation of the tangent to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at $(1, 1)$. (Scores : 2)

(b) Find two positive numbers whose sum is 15 and the sum of whose squares is minimum. (Scores : 3)

8. (a) $\int \frac{2x}{1+x^2} dx =$ _____ (Score : 1)

(b) Find $\int \frac{1}{x^2 - 6x + 13} dx$ (Scores : 2)

(c) Find $\int x^2 e^{x^3} dx$ (Scores : 2)

9. Consider the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = -\hat{i} + 7\hat{k}$

(a) Find $\vec{a} + \vec{b}$ (Score : 1)

(b) Find a unit vector in the direction of $\vec{a} + \vec{b}$. (Scores : 2)

10. (a) Evaluate $\int_1^2 \frac{x}{(x+1)(x+2)} dx$. (Scores : 2)

(b) Evaluate $\int_0^\pi \frac{x}{1+\sin x} dx$. (Scores : 3)

OR

Evaluate $\int_0^2 e^x dx$ as the limit of a s. (Scores : 5)

11. Consider the triangle ABC with vertices A(1, 2, 3), B(-1, 0, 4) and C(0, 1, 2)

(a) Find \vec{AB} and \vec{AC} . (Score : 1)

(b) Find $\angle A$. (Scores : 2)

(c) Find the area of triangle ABC. (Scores : 2)

12. (a) Find the area of the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Scores : 3)

(b) Find the area of the region bounded by the parabolas $y = x^2$ and $y^2 = x$. (Scores : 3)

13. (a) Write the order and degree of the differential equation

$$xy \left(\frac{d^2y}{dx^2} \right)^2 + x \left(\frac{dy}{dx} \right)^3 - y \frac{dy}{dx} = 0$$
 (Score : 1)

(b) Find the general solution of the differential equation $y \log y dx - x dy = 0$. (Scores : 3)

(c) Find the integrating factor of the differential equation $x \frac{dy}{dx} - y = 2x^2$. (Scores : 2)

14. (a) Find the value of 'p' if the lines

$$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1} \text{ and } \frac{x}{1} = \frac{y}{p} = \frac{z}{3} \text{ are perpendicular.}$$

(Score : 1)

- (b) Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}) \text{ and}$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

(Scores : 3)

15. Consider the linear programming problem :

Maximize $z = 4x + y$

Subject to the constraints :

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x \geq 0, y \geq 0$$

- (a) Draw its feasible region
- (b) Find the corner points of the feasible region.
- (c) Find the corner at which z attains its maximum

(Scores : 3)

(Scores : 2)

(Score : 1)

16. Consider a plane which passes through the point $(5, 2, -4)$ and perpendicular to the line

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$$

- (a) Write its equation in Cartesian form.
- (b) Find its distance from the point $(1, 2, -1)$
- (c) Find the angle made by it with the line

(Score : 1)

(Score : 1)

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$$

(Scores : 2)

17. (a) A die is thrown thrice. Find the probability of getting an odd number atleast once.

(Scores : 2)

(b) Two cards are drawn successively with replacement from a pack of 52 cards. Find the probability distribution of the number of aces.

(Scores : 3)

OR

(a) If $P(A) = 0.8$, $P(B) = 0.5$ and $P\left(\frac{B}{A}\right) = 0.4$, then find $P(A \cup B)$.

(Scores : 2)

(b) If a fair coin is tossed 10 times, then find the probability of getting exactly 6 heads.

(Scores : 3)

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