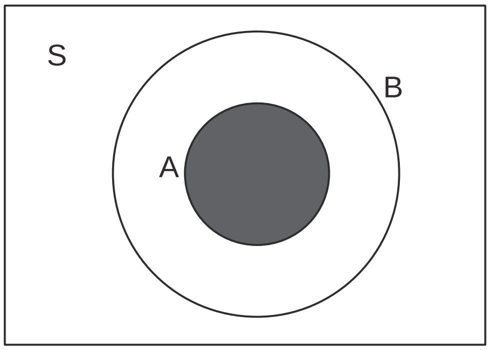
MATHEMATICS MARCH 2016
SOLUTIONS

1. a) A 1

b)



(2)

c) Let M = set of teachers, who teach Mathematics
P = set of teachers, who teach Physics

$n(M)=12;\ n(P)=12;\ n(M\cup P)=20$

$\therefore n(M\cap P)=n(M)+n(P)-n(M\cup P)$

$=12+12-20=4$ (2)

2. (a) $x+1=3\Rightarrow x=3-1=2$

$y-2=1\Rightarrow y=1+2=3$ 1

(b) $\mathrm{A}=\{1,2,3,4,5\};\ \mathrm{B}=\{4,6,9\}$

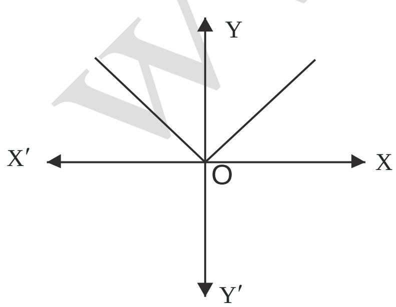
$\mathrm{R}=\mathrm{x}-\mathrm{y}$ is a positive integer.

$\therefore\ \mathrm{R}=\{(5,4)\}$ 2

(c) A real function R is said to be a modulus function, if

$f(x)=|x|, x\in R,$ is known as modulus function. 1

Domain of modulus function is R 1

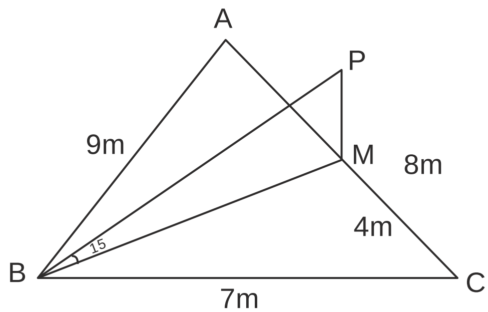


1

3. a) 210^0 1

b) $\text{LHS} = \dfrac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = \dfrac{2\cos 6x \cos x}{2\cos 6x \sin x} = \cot x = \text{RHS}$ 2

c)



$$\cos C = \frac{BC^2 + AC^2 - AB^2}{2(BC)(AC)} = \frac{49+64-81}{2(7)(8)} = \frac{32}{112} = \frac{2}{7}$$

In ΔBMC, $BM^2 = BC^2 + MC^2 - 2(BC)(MC)\cos C$

$$= 49 + 16 - 2\times 7\times 4\times \frac{2}{7} = 65 - 16 = 49$$

$$= BM = 7m$$

In ΔBMP, $\tan 15 = \dfrac{PM}{BM} \Rightarrow PM = BM \tan 15 = 7\left(2-\sqrt{3}\right)m$

$\therefore$ Height of the lamp post $= 7\left(2-\sqrt{3}\right)m$ 4

4. $P(n): a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a\left(r^n - 1\right)}{r-1}$

a) $P(1): a = \dfrac{a\left(r^1 - 1\right)}{r-1} \Rightarrow a = a$

Hence, P(1) is true. 1

b) Assume that P(k) be true.

$\therefore$ $P(k): a + ar + ar^2 + \ldots + ar^{k-1} = \dfrac{a\left(r^k - 1\right)}{r-1}$ ……………. (1)

To prove that P(k+1) is true.

$P(k+1): a + ar + ar^2 + \ldots + ar^{k-1} + ar^k = \dfrac{a\left(r^{k+1} - 1\right)}{r-1}$

Using (1),

$$\frac{a\left(r^k - 1\right)}{r-1} + ar^k = \frac{a\left(r^{k+1} - 1\right)}{r-1}$$

$$\text{LHS} = \frac{a\left(r^k - 1\right)}{r-1} + ar^k = \frac{ar^k - a + ar^k\left(r-1\right)}{r-1}$$

$$= \frac{ar^k - a + ar^k r - ar^k}{r-1} = \frac{ar^{k+1} - a}{r-1} = \text{RHS}$$

Hence P(k+1) is true. Hence, P(n) is true for all $n \in N$ 3

5. a) ii) 0 and 2 1

(b) *Let* $z = i = 0 + 1i$

$$r = \sqrt{0^2 + 1^2} = 1$$

Let $\tan\theta = \frac{y}{x} = \frac{1}{0} = \infty$

$$\therefore \theta = \frac{\pi}{2}$$

$\therefore$ $z = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$. 2

c) $\Delta = b^2 - 4ac = 1^2 - 4\times\sqrt{5}\times\sqrt{5} = 1 - 20 = -19$

$\therefore$ $x = \frac{-1 \pm \sqrt{-19}}{2\sqrt{5}} = \frac{-1 \pm i\sqrt{19}}{2\sqrt{5}}$ 2

6. a) iii) $(-2,3]$ 1

b) $2x + y = 4$

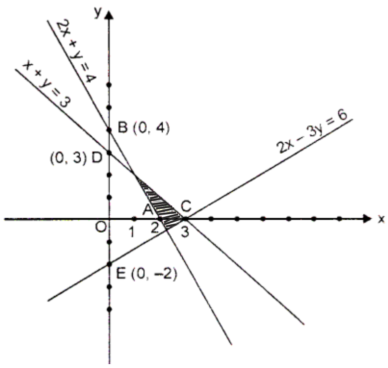
x	0	2
y	4	0

$x + y = 3$

x	0	3
y	3	0

$2x - 3y = 6$

x	0	3
y	–2	0



Solution region:

$2x+y\geq 4$

$\Rightarrow 0\geq 4,$ which is false. $\quad\|$ by putting $x=0, y=0$

Hence shade the half plane, which does not contain the origin.

$x+y\leq 3$

$\Rightarrow 0\leq 3,$ which is true.

Hence shade the half plane, which contains the origin.

$2x-3y\leq 6$

$\Rightarrow 0\leq 6,$ which is true.

Hence shade the half plane, which contains the origin.

The common region shown in the figure is the solution region. 4

7. a) $^{7}C_5={}^{7}C_2=\dfrac{7\times 6}{1\times 2}=21$ 1

b) $3\times{}^{n}P_4=5\times{}^{n-1}P_4$

$3\times n(n-1)(n-2)(n-3)=5\times(n-1)(n-2)(n-3)(n-4)$ 2

$3n=5n-20\Rightarrow 2n=20\Rightarrow n=10$

c) Since 4 cards belong to four different suits,

No. of ways = $^{13}C_1\times{}^{13}C_1\times{}^{13}C_1\times{}^{13}C_1=13^4$ 3

OR

a) $^{29}C_{29}=1$ 1

b) $12\times{}^{n-1}P_3=5\times{}^{n+1}P_3$

$12\times(n-1)(n-2)(n-3)=5\times(n+1)n(n-1)$

$12\times(n-2)(n-3)=5\times(n+1)n$

$12\left(n^2-5n+6\right)=5n^2+5n$

$12n^2-60n+72-5n^2-5n=0$

$7n^2-65n+72=0$

$7n^2 - 56n - 9n + 72 = 0$

$7n(n-8) - 9(n-8) = 0$

$(n-8)(7n-9) = 0$

$n = 8 \text{ or } n = \frac{9}{7}$

But $n = \frac{9}{7}$ is inadmissible.

$\therefore n = 8$ 2

c) Required number of selections $= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7\times1 + 21\times4 + 35\times6 + 35\times4 = 7 + 84 + 210 + 140 = 441$ 3

8. a) $a = \sqrt{2}$; $b = \sqrt{3}$ $n = 7$

$8^{\text{th}} \text{ term} = t_{7+1} = {}^7C_7 \times \left(\sqrt{2}\right)^{7-7} . \left(\sqrt{3}\right)^7$ 1

$= 1 \times 1 \times \left(\sqrt{3}\right)^7 = 27\sqrt{3}$

b) $a = x, b = \frac{1}{2x}; n = 18$

$t_{r+1} = {}^{18}C_r \times (x)^{18-r} . \left(\frac{1}{2x}\right)^r = {}^{18}C_r \times (x)^{18-r} . \frac{1}{2^r x^r}$

$= {}^{18}C_r \times \frac{1}{2^r} \times (x)^{18-2r}$

To find the term independent of x, put $r = 9$, we have

$t_{9+1} = {}^{18}C_9 \times \frac{1}{2^9} \times (x)^{18-2(9)} = \frac{1}{512} \times {}^{18}C_9$ 3

9. a) $a = 5; r = 5$

$a_n = ar^{n-1} = 5 \times 5^{n-1} = 5^n$ 1

b) $a = 210; a_n = 990$; $d = 10$

$n = \frac{990 - 210}{10} + 1 = 79$

$S_n = \frac{79}{2} \times \left[2 \times 210 + (79-1)10\right]$

$= \frac{79}{2} \times [420 + 780]$

$= \frac{79}{2} \times 1200 = 79 \times 600 = 68400$ 2

c) $a_n = n(n+3)$

$$S_n = \sum_{n=1}^{n}\left(n^2+3n\right) = \sum n^2 + 3\sum n$$
$$= \frac{n(n+1)(2n+1)}{6} + 3\frac{n(n+1)}{2}$$
$$= \frac{n(n+1)(2n+1)}{6} + \frac{9n(n+1)}{6}$$
$$= \frac{n(n+1)}{6}\left[2n+1+9\right]$$
$$= \frac{n(n+1)}{6}\left[2n+10\right]$$
$$= \frac{n(n+1)}{6}\times 2(n+5)$$
$$= \frac{n(n+1)(n+5)}{3}$$ 2

10. a) ii) $x-2y-4=0;\ x-2y-5=0$ 1

b) $3x-4y+10=0$
$3x-4y=-10$
$$\frac{3x}{-10}-\frac{4y}{-10}=1$$
$$\frac{x}{-\frac{10}{3}}+\frac{y}{\frac{10}{4}}=1$$
x- intercept $=-\frac{10}{3}$
y-intercept $=\frac{10}{4}=\frac{5}{2}$ 2

c) Slope of the given line is $-\frac{A}{B}=-\frac{1}{-7}=\frac{1}{7}$
$\therefore$ slope of the required line is -7
Given x intercept of the required line is 3.
$\therefore$ the point is $(3,0)$
Hence equation of the required line is $y-0=-7(x-3)$
$y+7x=21$ or $7x+y-21=0$` 2

12. a) i) $(-4,2,-5)$ 1

b) Let YZ plane divides the line joining the points $A(-2,4,7)$ and $B(3,-5,8)$ at $R(x,y,z)$ in the ratio k:1.
Then x coordinates of R = 0.

$$\Rightarrow \frac{k(3)+1(-2)}{k+1}=0$$

$3k-2=0 \Rightarrow 3k=2$ 3

$$\therefore k=\frac{2}{3}$$

13. a) $\frac{d}{dx}\left(\frac{x^n}{n}\right)=\frac{1}{n}.nx^{n-1}=x^{n-1}$ 1

b) $y=\frac{\sin x}{x+1}$

$$\frac{dy}{dx}=\frac{(x+1)\cos x-\sin x(1+0)}{(x+1)^2}$$

$$=\frac{(x+1)\cos x-\sin x}{(x+1)^2}$$

c) Let $f(x)=\cos x$

$$f(x+h)=\cos(x+h)$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\left(\frac{\cos(x+h)-\cos x}{h}\right)$$

$$=\lim_{h\to 0}\left[\frac{-2\sin\left(\frac{x+h+x}{2}\right).\sin\left(\frac{x+h-x}{2}\right)}{h}\right]$$

$$=\lim_{h\to 0}\frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$=\lim_{h\to 0}-\sin\left(\frac{2x+h}{2}\right)\times\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

$$=-\sin\left(\frac{2x+0}{2}\right)\times 1 \qquad \| \lim_{h\to 0}\frac{\sin x}{x}=1$$

$=-\sin x$ 3

OR

a) $\frac{d}{dx}(-\sin x) = -\cos x$ 1

b) $y = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$

$$\frac{dy}{dx} = a \times \frac{-4}{x^5} - b \times \frac{-2}{x^3} - \sin x$$

$$= \frac{-4a}{x^5} + \frac{2b}{x^3} - \sin x$$ 2

c) Let $f(x) = sin\, x$

$$f(x+h) = sin(x+h)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \left(\frac{sin(x+h) - sin\, x}{h} \right)$$

$$= \lim_{h \to 0} \left[\frac{2\, cos\left(\frac{x+h+x}{2}\right).sin\left(\frac{x+h-x}{2}\right)}{h} \right]$$

$$= \lim_{h \to 0} \frac{2\, cos\left(\frac{2x+h}{2}\right) sin\left(\frac{h}{2}\right)}{h}$$

$$= \lim_{h \to 0} cos\left(\frac{2x+h}{2}\right) \times \frac{sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}$$

$$= cos\left(\frac{2x+0}{2}\right) \times 1 \qquad || \lim_{h \to 0} \frac{sin\, x}{x} = 1$$

$$= cos\, x$$ 3

14. a) Let p: "Every natural number is greater than zero".

~p: "Every natural number is not greater than zero". 1

b) Let us assume that $\sqrt{13}$ is a rational number.

$\therefore \sqrt{13} = \frac{a}{b}$, where a and b are co-prime. i.e., a and b have no common factors, which

implies that

$13b^2 = a^2 \Rightarrow 13$ divides a.

$\therefore$ there exists an integer 'k' such that $a = 13k$

$\therefore\ a^2 = 169k^2 \Rightarrow 13b^2 = 169k^2 \Rightarrow b^2 = 13k^2 \Rightarrow 13$ divides b.

i.e., 13 divides both a and b, which is contradiction to our assumption that a and b have no common factor. $\therefore$ our supposition is wrong.

$\therefore\ \sqrt{13}$ is an irrational number. 3

15. a) $\frac{\sum x}{n} = 50 \Rightarrow \frac{450}{n} = 50$

$50n = 450 \Rightarrow n = \frac{450}{50} = 9$ 1

b)

x_i	f_i	$f_i x_i$	$\lvert x_i - \overline{x}\rvert$	$f_i \lvert x_i - \overline{x}\rvert$
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
	40	300		92

$\overline{x} = \frac{\sum f_i x_i}{N} = \frac{300}{40} = 7.5$

$M.D(\overline{x}) = \frac{1}{N} \sum f_i \lvert x_i - \overline{x}\rvert$

$= \frac{1}{40} \times 92 = 2.3$ 4

16. a) 2^6 1

b) $P(A) = 0.5;\ P(B) = 0.6; P(A \cap B) = 0.3$

$$P(A') = 1 - P(A) = 1 - 0.5 = 0.5$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.5 + 0.6 - 0.3 = 0.8$$

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.8 = 0.2$$

$$P(A' \cup B') = 1 - P(A \cap B) = 1 - 0.3 = 0.7$$ 4

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