Arithmetic Fundamentals

Table of Squares

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	20	25
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	400	625

Table of Cubes

Number	2	3	4	5	10`	20	100
Cube	8	27	64	125	1000	8000	1000000

Commonly used Decimal, Percent and Fractions(Less than 1)

Percent	10%	20%	25%	30%	33%	40%	50%	60%	66%	75%	80%	90%	100%
Fractions	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{9}{10}$	*1*
Decimals	*0.1*	*0.2*	*0.25*	*0.3*	*0.33*	*0.4*	*0.5*	*0.6*	*0.66*	*0.75*	*0.8*	*0.9*	*1*

Commonly used Decimal, Percent and Fractions (Greater than 1)

Percent	100%	125%	133.33%	150%	200%
Fractions	*1*	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	*2*
Decimals	*1*	*1.25*	*1.33*	*1.5*	*2.0*

Divisibility Rule

Number	Rule	Example
2	If last digit is 0,2,4,6, or 8	22, 30, 50, 68, 1024
3	If sum of digits is divisible by 3	123 is divisible by 3 since 1 + 2 + 3 = 6 (and 6 is divisible by 3)
4	If number created by the last 2 digits is divisible by 4	864 is divisible by 4 since 64 is divisible by 4
5	If last digit is 0 or 5	5, 10, 15, 20, 25, 30, 35, 2335
6	If divisible by 2 & 3	522 is divisible by 6 since it is divisible by 2 & 3
9	If sum of digits is divisible by 9	621 is divisible by 9 since 6 + 2 + 1 = 9 (and 9 is divisible by 9)
10	If last digit is 0	10, 20, 30, 40, 50, 5550

Operations Involving Exponents

Multiplication	$a^m \times a^n = a^{(m+n)}$
Division	$a^m \div a^n = a^{(m-n)}$
Power	$(a^m)^n = a^{m\times n}$
Roots	$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Special Exponents

Reciprocal	$\frac{1}{a^m} = a^{-m}$
Power 0	$a^0 = 1$
Power 1	$a^1 = a$

Logarithms

Definition: $y = log_b x \rightarrow x = b^y$

Example: $log_2 32 = 5 \rightarrow 32 = 2^5$

Properties

$log_x x = 1 \quad log_x 1 = 0$

$log_x x^n = n \quad x^{log_x y} = y$

$log_x(y^n) = n log_x y$

$log_x(a \times b) = log_x a + log_x b$

$log_x\left(\frac{a}{b}\right) = log_x a - log_x b$

Progressions

Arithmetic Progression	Geometric Progression
Arithmetic Progression: n^{th} term of an Arithmetic Progression $a_n = a_1 + (n-1)d$ Sum of ***n*** terms of an arithmetic expression $s_n = \frac{n(a_1 + a_n)}{2} = \frac{n[2a_1 + (n-1)d]}{2}$ The first term is a_1, the common difference is d, and the number of terms is n.	**Geometric Progression:** n^{th} term of a Geometric Progression $a_n = a_1 r^{(n-1)}$ Sum of ***n*** terms of a geometric progression: $s_n = \frac{a(r^{n+1} - 1)}{r - 1}$ The first term is a_1, the common ratio is r, and the number of terms is n.

Infinite Geometric Progression

Sum of all terms in an infinite geometric series = $\frac{a_1}{(1-r)}$ $where -1 < r < 1$

Roots of a Quadratic Equation

A quadratic equation of type $\mathbf{ax^2 + bx + c}$ has two solutions, called roots. These two solutions may or may not be distinct. The roots are given by the quadratic formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where the $\pm$ sign indicates that both $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ are solutions

Common Factoring Formulas

1. $x^2 - y^2 = (x - y) \times (x + y)$
2. $x^2 + 2xy + y^2 = (x + y)^2$
3. $x^2 - 2xy + y^2 = (x - y)^2$
4. $x^3 + 3yx^2 + 3y^2x + y^3 = (x + y)^3$
5. $x^3 - 3yx^2 + 3y^2x - y^3 = (x - y)^3$

Binomial Theorem

The coefficient of $x^{(n-k)}y^k$ in $(x + y)^n$ is:

$$ {}^{n}_{k}C = \frac{n!}{k!\,(n-k)!} $$

Applies for any real or complex numbers x and y, and any non-negative integer n.

Summary of counting methods

	Order matters	Order doesn't matter
With Replacement	If r objects are taken from a set of n objects, in a specific order with replacement, how many different samples are possible? n^r	N/A
Without Replacement	**Permutation Rule**: If r objects are taken from a set of n objects without replacement, in a specific order, how many different samples are possible? $^n_rP = \frac{n!}{(n-r)!}$	**Combination Rule**: If r objects are taken from a set of n objects without replacement and disregarding order, how many different samples are possible? $^n_rC = \frac{n!}{r!(n-r)!}$

Probability

The probability of an event *A*, $P(A)$ is defined as

$$P(A) = \frac{Number\ of\ outcomes\ that\ occur\ in\ event\ A}{Total\ number\ of\ likely\ outcomes}$$

Independent Events: If *A* and *B* are independent events, then the probability of *A* happening and the probability of *B* happening is:

$$P(A\ and\ B) = P(A) \times P(B)$$

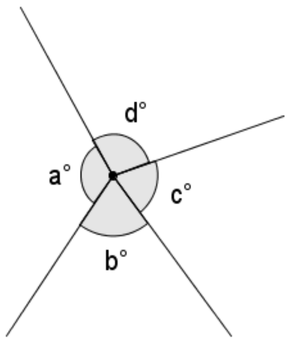
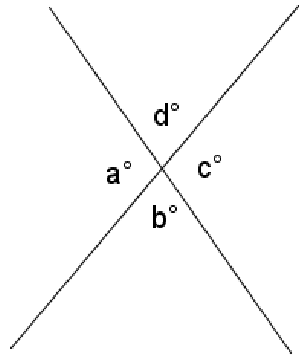
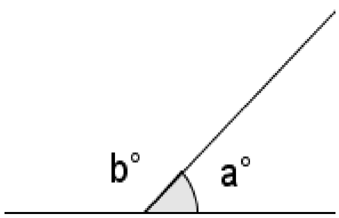
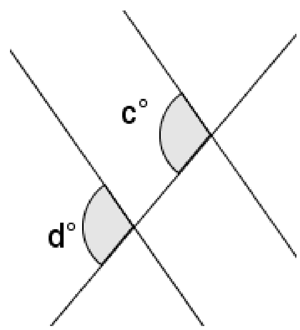
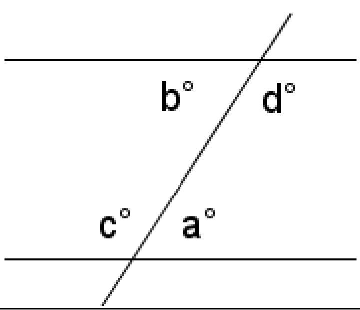
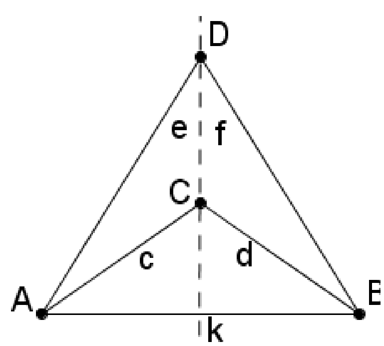
Dependent Events: If *A* and *B* are dependent events, then the probability of *A* happening and the probability of *B* happening, given *A,* is:

$$P(A\ and\ B) = P(A) \times P(B \mid A)$$

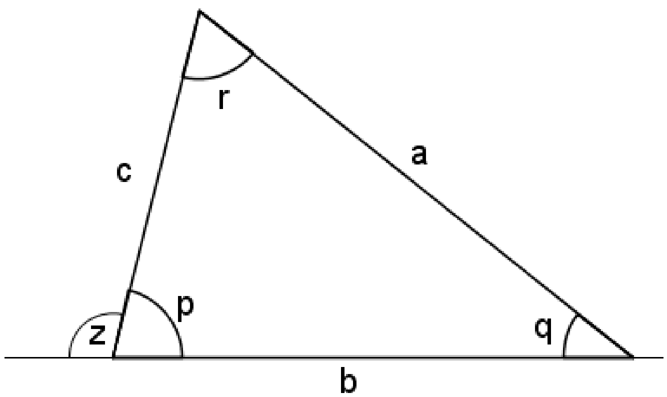
Conditional Probability: The probability of an event occurring given that another event has already occurred e.g. what is the probability that B will occur after A

$$P(B|A) = \frac{P(A\ and\ B)}{P(A)}$$

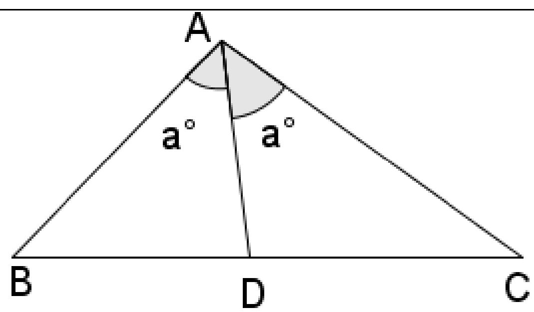
Geometry Fundamentals

The sum of angles around a point will always be **360** degrees.  In the adjacent figure $a + b + c + d = 360°$	Vertical angles are equal to each other.  In the adjacent figure $a = c$ and $b = d$
The sum of angles on a straight line is 180˚.  In the adjacent figure sum of angles and b is 180 i.e. $a + b = 180°$	When a line intersects a pair of parallel lines, the corresponding angles are formed are equal to each other  In the figure above $c = d$
When a line intersects a pair of parallel lines, the alternate interior and exterior angles formed are equal to each other  In the adjacent figure alternate interior angles a=b and alternate exterior angles c=d	Any point on the perpendicular bisector of a line is equidistant from both ends of the line.  In the figure, k is the perpendicular bisector of segment AB and c=d, e=f

Triangle Properties

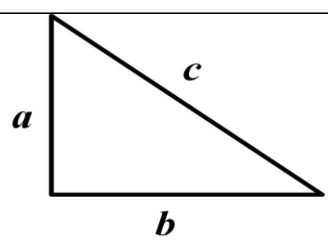


1. Sum of all internal angles of a triangle is 180 degrees i.e. $p + q + r = 180°$
2. Sum of any two sides of a triangle is greater than the third i.e. $\mathbf{a} + \mathbf{b} > \mathbf{c}$ or $\mathbf{c} + \mathbf{b} > \mathbf{a}$ or $\mathbf{a} + \mathbf{c} > \mathbf{b}$
3. The largest interior angle is opposite the largest side; the smallest interior angle is opposite the smallest side i.e. if $\mathbf{p} > \mathbf{q} \rightarrow \mathbf{a} > \mathbf{c}$
4. The exterior angle is supplemental to the adjoining interior angle i.e. $p + z = 180°$**.** Since $p + q + r = 180°$ it follows that $z = q + r$



5. The internal bisector of an angle bisects the opposite side in the ratio of the other two sides. In the adjoining figure $\frac{BD}{DC} = \frac{AB}{BC}$

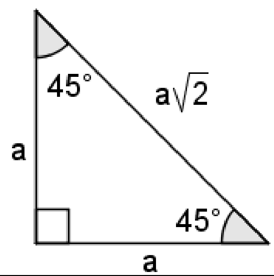
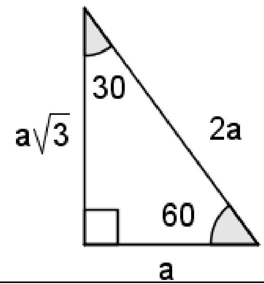
Pythagoras Theorem



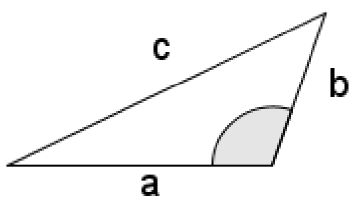
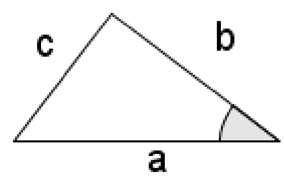
$c^2 = a^2 + b^2$

Commonly Used Pythagorean Triples	
Height, Base	**Hypotenuse**
3,4 or 4,3	5
6,8 or 8,6	10
5, 12 or 12,5	13
7, 24 or 24,7	25

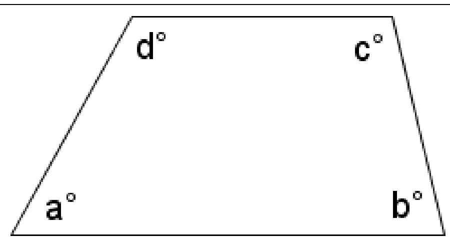
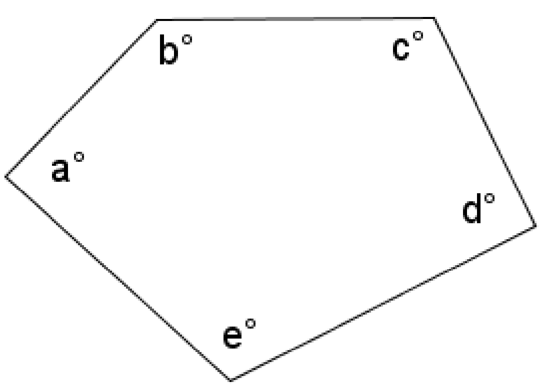
Special Right Triangles

	The lengths of the sides of a 45°- 45°- 90° triangle are in the ratio of $\mathbf{1:1:\sqrt{2}}$		The lengths of the sides of a 30°- 60°- 90° triangle are in the ratio of $\mathbf{1:\sqrt{3}:2}$

Test of Acute and Obtuse Triangles

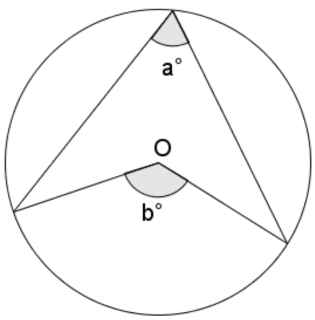
If $c^2 < a^2 + b^2$ then it is an acute-angled triangle, i.e. the angle facing side *c* is an acute angle. 	If $c^2 > a^2 + b^2$ then it is an obtuse-angled triangle, i.e. the angle facing side *c* is an obtuse angle. 

Polygons

	Sum of interior angles of a quadrilateral is 360° $a + b + c + d = 360°$
	Sum of interior angles of a pentagon is 540° $a + b + c + d + e = 540°$
If *n* is the number of sides of the polygon then, sum of interior angles = (*n* - 2)180°	

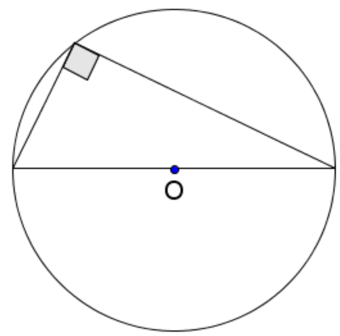
Properties of a Circle

The angle at the centre of a circle is twice any angle at the circumference subtended by the same arc.

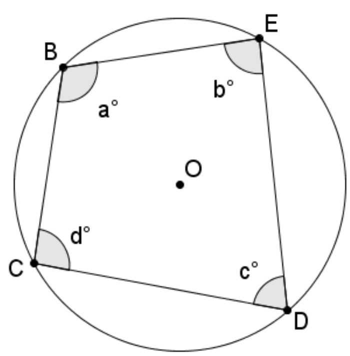


In the adjacent figure angle $b = 2a$

Every angle subtended at the circumference by the diameter of a circle is a right angle (90˚).

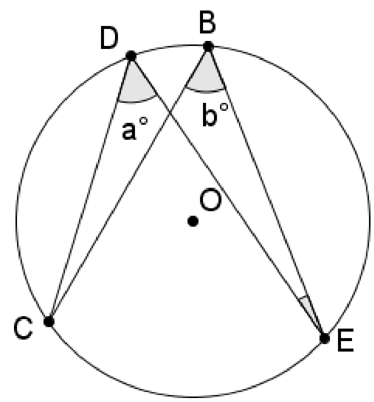


In a cyclic quadrilateral, the opposite angles are supplementary i.e. they add up to 180˚.



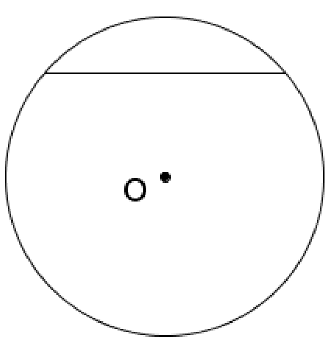
In the figure above angle $a + c = 180°$ and $b + d = 180°$

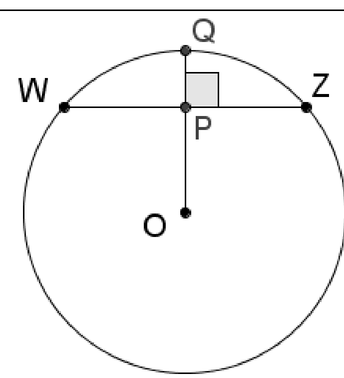
The angles at the circumference subtended by the same arc are equal.



In the adjacent figure angle $b = a$

A **chord** is a straight line joining 2 points on the circumference of a circle.



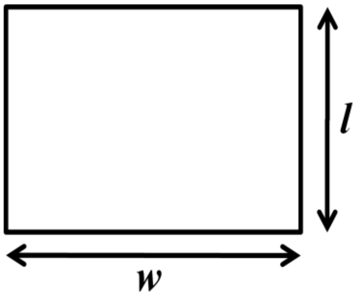


A radius that is perpendicular to a chord bisects the chord into two equal parts and vice versa.

In the adjacent figure PW=PZ

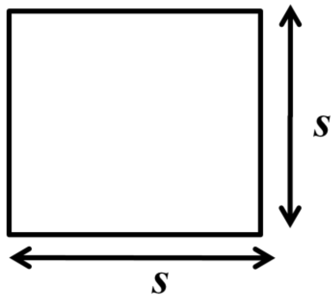
Area and Perimeter of Common Geometrical Figures

Rectangle



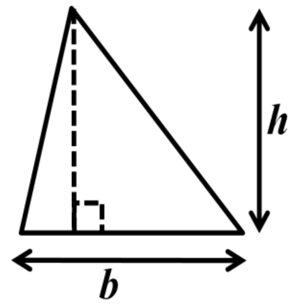
Area $A = l \times w$
Perimeter $P = 2 \times (l + w)$

Square



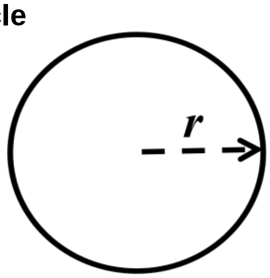
Area $A = s^2$
Perimeter $P = 4s$

Triangle



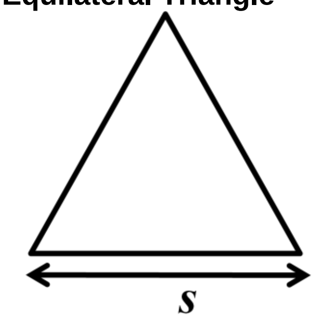
Area $A = \frac{1}{2} \times b \times h$

Circle



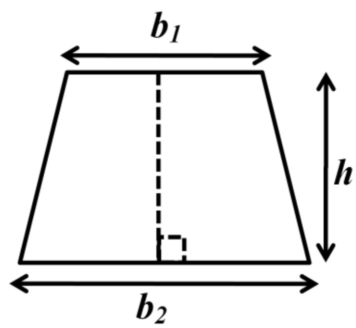
Area $A = \pi r^2$
Perimeter $P = 2\pi r$

Equilateral Triangle



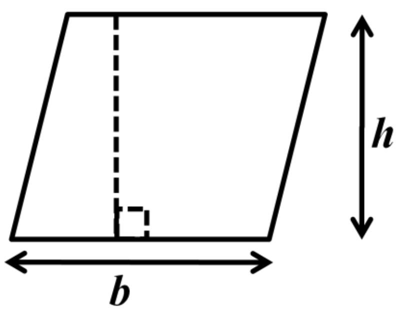
Area $A = \frac{\sqrt{3}}{4} \times s^2$
Perimeter $P = 3s$
Altitude $h = a\frac{\sqrt{3}}{2}$

Trapezoid



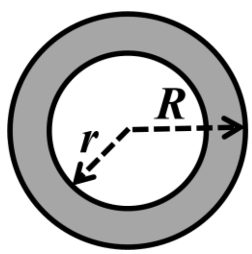
Area $A = \frac{1}{2} h\,(b_1 + b_2)$

Parallelogram



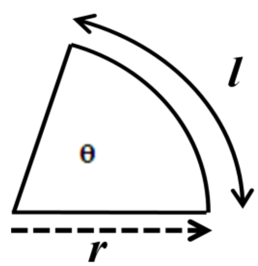
Area $A = h \times b$
Perimeter $P = 2 \times (h + b)$

Ring



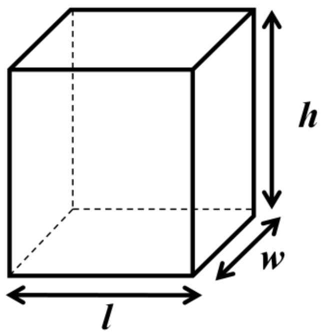
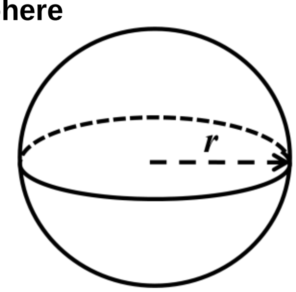
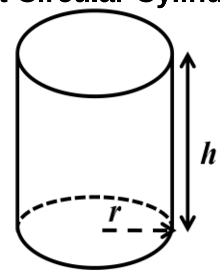
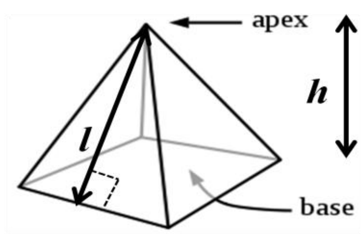
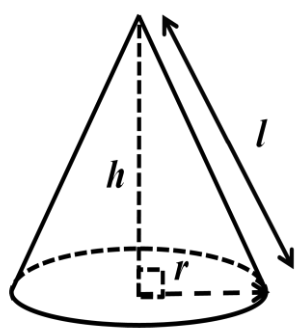
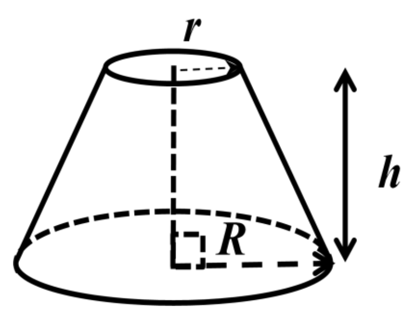
Area $A = \pi(R^2 - r^2)$

Sector

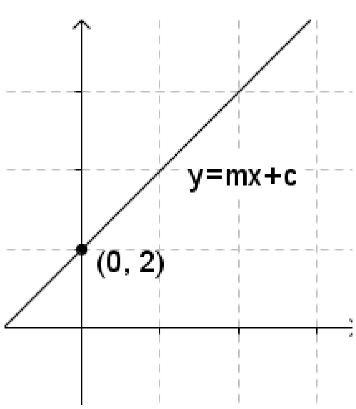
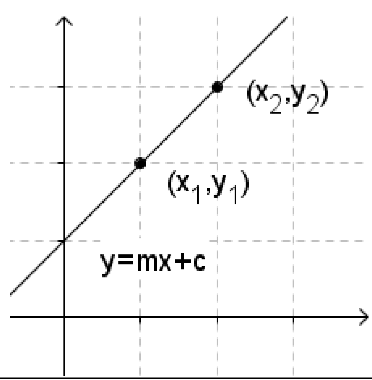


θ in degrees
Area $A = \pi r^2 \times \frac{\theta}{360}$
Length of Arc $L = \pi r \times \frac{\theta}{180}$
Perimeter $P = L + 2r$

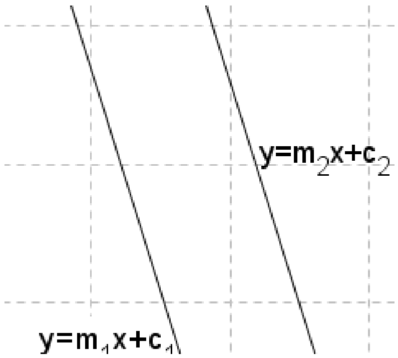
Volume and Surface Area of 3 Dimensional Figures

Cube	**Sphere**	**Right Circular Cylinder**
 Volume $v = l \times w \times h$ Surface Area $A = 2(lw + wh + hl)$	 Volume $v = \frac{4}{3}\pi r^3$ Surface Area $A = 4\pi r^2$	 Volume $v = \pi r^2 \mathrm{h}$ Surface Area $A = 2\pi \mathrm{rh}$
Pyramid	**Right Circular Cone**	**Frustum**
 Volume $v = \frac{1}{3}Bh$ Surface Area = $A = B + \frac{pl}{2}$ **B** is the area of the base l is the slant height	 Volume $v = \frac{1}{3}\pi r^2 \mathrm{h}$ Surface Area $A = \pi r(l + r)$	 Volume $v = \frac{1}{3}\pi \mathrm{h}(\mathrm{r}^2 + \mathrm{rR} + \mathrm{R}^2)$

Coordinate Geometry

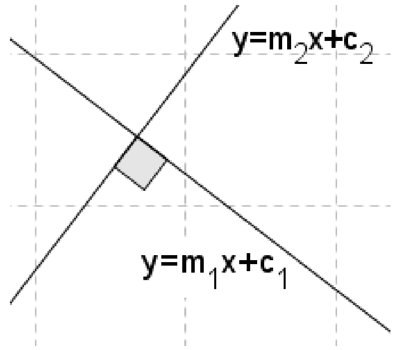
Line: Equation of a line $y = mx + c$  In the adjoining figure, c is the intercept of the line on Y-axis i.e. *c=2, m* is the slope	Slope of a line $m = \frac{y_2 - y_1}{x_2 - x_1}$ 

The slopes of parallel lines are equal



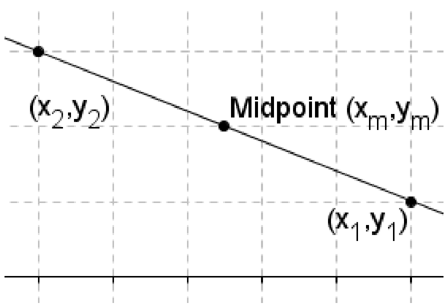
In the adjoining figure, the two lines are parallel to each other i.e. $m_1 = m_2$

The slopes of perpendicular lines are opposite reciprocals of one another.



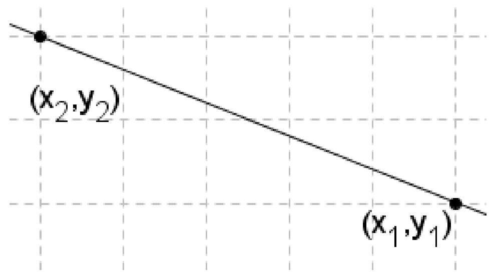
In the adjoining the two lines are perpendicular to each other i.e. $m_1 = \frac{-1}{m_2}$

Midpoint between two points (x_1, y_1) and (x_2, y_2) in a x-y plane:



$$(x_m, y_m) = \left(\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2}\right)$$

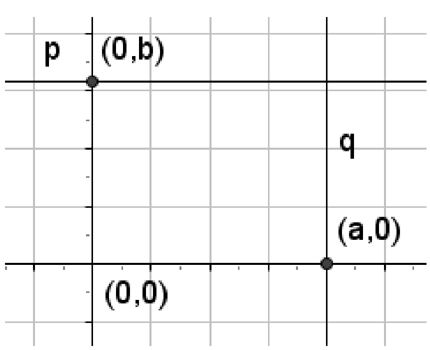
Distance between two points (x_1, y_1) and (x_2, y_2) in a x-y plane:



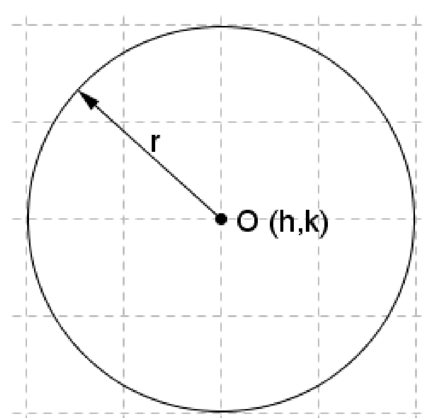
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Horizontal and Vertical line

Equation of line parallel to x-axis (line p) with intercept on y-axis at (0, b) is $y = b$. Equation of line parallel to y-axis (line q) with intercept on x-axis at (a,0) is $x = a$



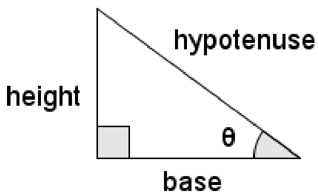
Equation of a circle with center (*h,k*) and radius *r*



$$(x - h)^2 + (y - k)^2 = r^2$$

Trigonometry Basics

Definition: Right Triangle definition for angle θ such that $0 < \theta < 90°$



$$sin\theta = \frac{height}{hypotenuse}, \quad csc\theta = \frac{1}{sin\theta}$$

$$cos\theta = \frac{base}{hypotenuse}, \quad sec\theta = \frac{1}{cos\theta}$$

$$tan\theta = \frac{height}{base}, \quad cot\theta = \frac{1}{tan\theta}$$

Pythagorean Relationships

$$sin^2\theta + cos^2\theta = 1$$

$$tan^2\theta + 1 = sec^2\theta$$

$$cot^2\theta + 1 = csc^2\theta$$

Tan and Cot

$$tan\theta = \frac{sin\theta}{cos\theta},$$

$$cot\theta = \frac{cos\theta}{sin\theta}$$

Half Angle Formulas

$$sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$tan^2\theta = \frac{(1 - \cos(2\theta))}{(1 + \cos(2\theta))}$$

Even/Odd

$$\sin(-\theta) = -sin\theta$$

$$\cos(-\theta) = cos\theta$$

$$\tan(-\theta) = -tan\theta$$

$$\sin(2\theta) = 2sin\theta cos\theta, \cos(2\theta) = cos^2\theta - sin^2\theta$$

Periodic Formula: Where n is an integer

$$\sin(\theta + 2\pi n) = \sin\theta \rightarrow \sin(\theta + 180°) = sin\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \rightarrow \cos(\theta + 180°) = cos\theta$$

$$\tan(\theta + \pi n) = \tan\theta \rightarrow \tan(\theta + 90°) = \tan\theta$$

	$0°$	$30°$	$45°$	$60°$	$90°$
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞