MATHEMATICS

CLASS X

TIME: - 3 HOURS   M.M.100

> **GENERAL INSTRUCTIONS:**
- **All questions are compulsory.**
- **The question paper consists of 25 questions divided in to three sections-A, B and C. section A contains 10 questions of 3 marks each. Section B is of 10 questions of 4 marks each and section C is of 5 questions of 6 marks each.**
- **There is no overall choice. However internal choice has been provided in two questions of three marks each, two questions of four marks each and two questions of six marks each.**
- **In question on theorems, the drawing should be neat and exactly as per the given measurements.**

Use of calculator is not permitted.

Section - A

(1) Solve the following system of linear equations:

$$\frac{15}{u} + \frac{2}{v} = 17$$

$$\frac{1}{u} + \frac{1}{v} = \frac{36}{5}$$

(2) Express in the lowest terms: $\dfrac{2x^4 - 162}{(x^2 + 9)(2x - 6)}$

(3) A burner is available for Rs.1600 cash or Rs.360 as cash down payment followed by three monthly instalments of Rs.420 each. Find the rate of interest under the instalment plan.

(4) A loan of Rs.8400 is to be paid in two equal instalments, the interest being charged at 10% per annum compounded annually. Find each instalment.

(5) If $(x - k)$ is the HCF of $x^2 - x - 6$ and $x^3 + 3x - 18$, find the value of k.

(6) In $\triangle$ABC, AD bisects $\angle$A. AB = 6. 3cm, AC = 4. 5cm and BD = 3. 8cm, find DC.
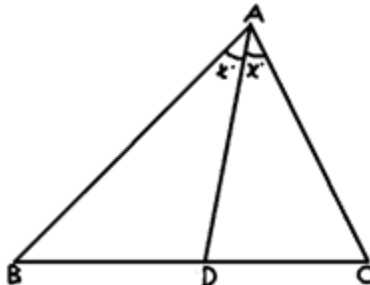
OR

In Fig.2 XY is parallel to AC and XY divides triangular region ABC into two parts equal in area. Determine AX
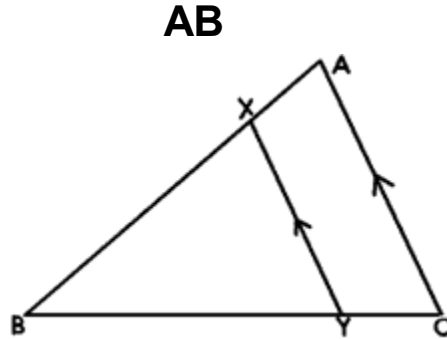
AB

Fig.2

(7) Solve for x: $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$

(8) If 7 times the 7th term of an A.P. is equal to 11 times its 11th term, show that the 18th term of the A.P. is zero.

(9)Find the sum of the first 51 terms of the AP whose second and third terms are respectively 14 and 18.

(10) In the given Fig.3 ABCD is a cyclic quadrilateral whose side AB is a diameter of the circle through A,B,C,D. If $\angle ADC = 130^{0}$, find $\angle BAC$.
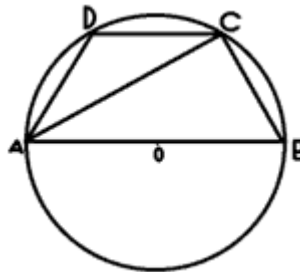
Fig . 3

OR

In Fig.4 AT is a tangent to the circle. If $\angle ABC = 50^{0}$, AC =BC, find $\angle BAT$.
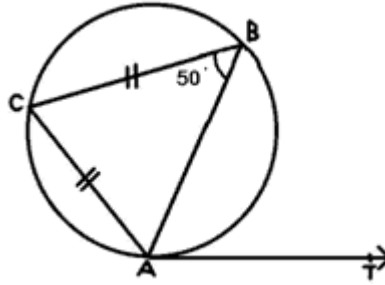
Fig. 4

Section – B

(11) A train covered a certain distance at a uniform speed. If the train had been 6 km/hr faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr, it would have taken 6 hours more than scheduled time. Find the length of the journey.

(12) 17 cards numbered 1,2,3,4…….,16,17 are put in a box and mixed thoroughly. One person draws a card from the box. Find the probability that the number on the card is:-
(i) odd   **(ii) a prime**
(iii) divisible by 3   **(iv) divisible by 3 and 2 both.**

(13) Prove that $\dfrac{1+\cos\Theta}{1-\cos\Theta} = \dfrac{2-\sin^2\Theta+2\cos\Theta}{\sin^2\Theta}$

OR

Evaluate: $\dfrac{-\tan\Theta\ \cot(90-\Theta)+\sec\Theta\ \text{cosec}(90-\Theta)+\sin^2 35^\circ+\sin^2 35^\circ}{\tan10^\circ\ \tan20^\circ\ \tan45^\circ\ \tan70^\circ\ \tan80^\circ}$

(14) Find the point on x-axis which is equidistant from(-2,5) and (2,-3).

(15) Determine the ratio in which the point P(m,6) divides the join of A(-4,3) and B(2,8). Also find the value of m.

(16)Construct a quadrilateral ABCD with AB=3cm, AD= 2.7cm, BD= 3. 6cm, $\angle$B= 110° and BC= 4 .2cm. Construct a quadrilateral A'B'C'D' similar to ABCD so that diagonal BD' =4.8cm.

(17) Find the missing frequencies(f_1 and f_2) in the following frequency- distribution table, it being given that the mean of this distribution is 50 and the sum of all the frequencies is 120.

Class Interval	Frequency
0 – 20	17
20 – 40	f_1
40 – 60	32
60 – 80	f_2
80 – 100	19

(18) Solve the following system of equations graphically:
$$2x + y – 6 = 0$$
$$2x - y + 2 = 0$$
Shade the region bounded by these lines and the x-axis.
Find the area of the shaded region.

(19) A cone of height 24cm and radius of base 6cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

(20) The number of students admitted in different faculties of a college are given below:

Faculty	No. of students
Science	1000
Arts	1200
Commerce	650
Law	450
Education	300
Total	3600

Draw a pie- chart to represent the above information.

Section – C

(21) Prove that in a right – angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Using the above, prove that altitude of an equilateral triangle is $\frac{\sqrt{3}}{2}a$ if each side of the equilateral triangle is a cm.

(22) Prove that if a chord is drawn through the point of contact of a tangent to a circle then the angles which this chord makes with the given tangent are equal respectively to the angles formed in the corresponding alternate segments.

Using the above theorem, prove the following:

In the given figure, $\triangle$ABC is isosceles in which AB=AC. Prove that the tangent PAT to the circumcircle of $\triangle$ABC is parallel to BC.
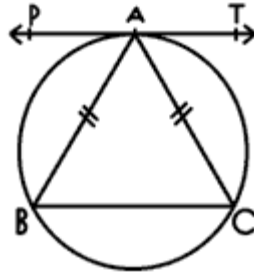

Fig. 5

(23) From the top of a lighthouse, the angles of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the lighthouse be h metres and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is

$$\frac{h(\tan\alpha + \tan\beta)}{\tan\alpha \ \tan\beta} \text{metres}$$

OR

An aeroplane when flying at a height of 4000m from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60^0 and 45^0 respectively. Find the vertical distance between the aeroplanes at that instant.

(24) A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2cm and the diameter of the base is 4cm. If the right circular cylinder circumscribes the solid, find how much more space it will cover.
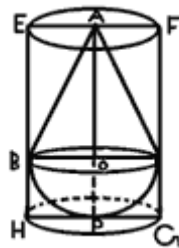

Fig. 6

(25) Savita, aged 30 is a computer executive and has an annual income of Rs. 3,00,000 (excluding HRA). In addition, she gets a performance incentive of 7. 5% of her annual salary at the end of the year. She contributes Rs. 2500 per month towards Company Provident Fund. In addition, she has donated Rs. 10,000 towards P.M.'s National Relief Fund (deduction of 100% on donation). She pays Rs. 1850 per quarter as premium on her LIC policies besides investing Rs. 30,000 in her PPF

account. What amount should be deducted as income tax from the salary per month so that the deduction in March is not more than Rs.1000?

A. Rate of Tax on individual taxable income:	
Taxable Income	Income Tax
(i) Upto Rs. 1,00,000	Nil
(ii) Rs. 1,00,001 to 1,50,000	10% of the amount exceeding Rs. 1,00,000
(iii) Rs. 1,50,001 to 3,00,000	Rs. 5,000 + 20% of the amount exceeding Rs. 1,50,000
(iv) Above Rs. 3,00,000	Rs. 35,000 + 30% of the amount exceeding Rs. 3,00,000
B. Rebate In Taxable Income: Upto Rs. 1,00,000 from taxable Income for the amount Invested in the company Provident Fund, Public Provident Fund, National Saving Certificates, LIC Premium, Unit Linked Insurance Policy, Infrastructure Bonds	
C. Educational Cess	2% of the Income tax due
D. Special Rebate	In case of women a deduction of Rs.5000 from income tax
E. Surcharge	Surcharge on income tax @ 5% if Income exceeds Rs. 3,00,000