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GATE 2018

PAPER SOLUTIONS

for

MECHANICAL ENGINEERING

NOTE

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Section-I: General Ability

1. The perimeters of a circle, a square and an equilateral triangle are equal. Which one of the following statements is true?
(A) The circle has the largest area
(B) The square has the largest area
(C) The equilateral triangle has the largest area.
(D) All the three shapes have the same area.

Key: (A)

Sol: Let, side of equilateral triangle as 'a'.
Then perimeter of equilateral triangle = 3a.
We know that, perimeter of circle = $2\pi r$
Perimeter of square = $4x$; [where x is side of square]

Given, $3a = 2\pi r$

$\Rightarrow r = \dfrac{3a}{2\pi}$

$\therefore$ Radius of circle $= r = \dfrac{3a}{2\pi}$

$3a = 4x \Rightarrow x = \dfrac{3a}{4}$

$\Rightarrow$ side of square $= \dfrac{3a}{4}$

$\therefore \quad x = \dfrac{3a}{4}$

$$\therefore \text{Area of circle} = \pi r^2 = \left(\frac{9a^2}{4\pi^2}\right) = \frac{9a^2}{4\pi} \simeq 0.716a^2$$

$$\therefore \text{Area of square} = x \times x = \frac{3a}{4} \times \frac{3a}{4} = \frac{9a^2}{16} \simeq 0.563a^2$$

$$\therefore \text{Area of equilateral triangle} = \frac{\sqrt{3}a^2}{4} \cong 0.433a^2$$

$\therefore$ The Circle has the largest area.

2. Find the missing group of letters in the following series:
BC, FGH, LMNO, ________
(A) UVWXY
(B) TUVWX
(C) STUVW
(D) RSTUV

Key: (B)

Sol: $\text{BC} \overset{+3}{\frown} \text{FGH} \overset{+4}{\frown} \text{LMNO} \overset{+5}{\frown} \text{TUVWX}$

3. “The judge’s standing in the legal community, though shaken by false allegations of wrongdoing, remained _______.”
The world that best fills the blank in the above sentence is
(A) Undiminished
(B) damaged
(C) illegal
(D) uncertain

Key: (A)

4. The value of the expression $\frac{1}{1+\log_u vw}+\frac{1}{1+\log_v wu}+\frac{1}{1+\log_w uv}$ is ______

(A) –1 (B) 0 (C) 1 (D) 3

Key: (C)

Sol: $\frac{1}{1+\frac{\log vw}{\log u}}+\frac{1}{1+\frac{\log wu}{\log v}}+\frac{1}{1+\frac{\log uv}{\log w}}$

$= \frac{\log u}{\log u+\log vw}+\frac{\log v}{\log v+\log wu}+\frac{\log w}{\log w+\log uv}$

$= \frac{\log u}{\log(uvw)}+\frac{\log v}{\log(uvw)}+\frac{\log w}{\log(uvw)}=\frac{\log(uvw)}{\log(uvw)}=1$

5. "The dress ______ her so well that they all immediately ______ her on her appearance."

The words that best fill the blanks in the above sentence are

(A) complemented, complemented (B) complimented, complemented

(C) complimented, complimented (D) complemented, complimented

Key: (D)

6. Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films?

(A) 0 (B) 2 (C) 4 (D) 8

Key: (C)

Sol: Given, Total no. of students who watched films A, B and C over a week = $n(s)=40.$

Also given that, each student watched either only one film or all three.

i.e., $n(A\cap B\cap \bar{C})=n(B\cap C\cap \bar{A})=n(C\cap A\cap \bar{B})=0;$

Where $\left[\bar{A}, \bar{B}, \bar{C} \text{ are complementatry events of } A, B, C \text{ respectively}\right]$

Given, $n(A)=13; n(B)=16; n(C)=19$

Assume that no. of students who watch all three films = $n(A\cap B\cap C)=x$

Given, $n(A\cap\bar{B}\cap\bar{C})=13-x$

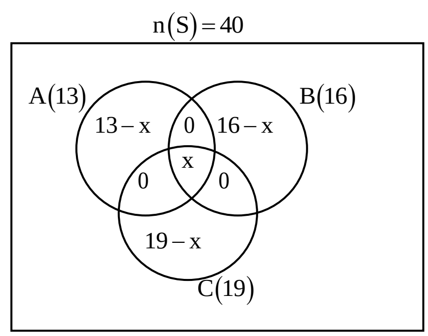
$n(B\cap\bar{C}\cap\bar{A})=16-x$

$n(C\cap\bar{B}\cap\bar{A})=19-x$

$n(s)=13-x+16-x+19-x+x$

$\Rightarrow 40=48-2x\Rightarrow 2x=48-40$

$\Rightarrow 2x=8\Rightarrow x=\frac{8}{2}=4$



∴ No. of students who watched all the three films = x=4

7. A house has a number which needs to be identified. The following three statements are given that can help in identifying the house number.

i. If the house number is a multiple of 3, then it is a number from 50 to 59.

ii. If the house number is NOT a multiple of 4, then it is a number from 60 to 69.

iii. If the house number is NOT a multiple of 6, then it is a number from 70 to 79.

What is the house number?

(A) 54 (B) 65 (C) 66 (D) 76

Key: (D)

Sol: From Statement-i, we have

if house no. is multiple of 3, then house no. $\in[50,59]$

From Statement-ii, we have

if house no. is not a multiple of 4, then house no. $\in[60,69]$

From Statement-iii, we have

if house no. is not a multiple of 6, then house no $\in[70,79]$

$\therefore$ The required house number, is 76 among {54, 65, 66, 76};

Since (i) 76 is not multiple of 3, so house no. $\notin[50,59]$

(ii) 76 is not multiple of 6, so house no. $\in[70, 79]$

(iii) 76 is multiple of 4, so house no. $\notin[60, 69]$

$\therefore$ The required house no. is 76.

8. A contract is to be completed in 52 days and 125 identical robots were employed, each operational for 7 hours a day. After 39 days, five-seventh of the work was completed. How many additional robots would be required to complete the work on time, if each robot is now operational for 8 hours a day?

(A) 50 (B) 89 (C) 146 (D) 175

Key: (0)

Sol: $1\text{Robot}\to 1\text{hr}\to 1\text{ work}$

$7\text{hrs}\to 7\text{work}$

$\Rightarrow 125\text{ Robots}\to 7\text{hrs / day}\to 125\times 7$

$\Rightarrow 125\text{Robots}\to 7\text{hrs / day}\to 52\text{days}\to 125\times 7\times 52=45,500\to \text{Total work}$

Given, After 39 days, $5/7^{th}$ of work was completed.

$$\text{i.e., } 45,500\times\frac{5}{7}=32,500.\ [\text{Actually, in 39 days 34125 work has to be completed}]$$

$$\text{Remaining work}=45,500-32,500=13,000\text{ work}$$

After 39 days, each robot is working 8hrs/day (given)

$13\text{days}\times 8\text{hrs/day}\times 125$ robots can work $=13,000$ work.

i.e. Addtional robots not required, if each robot work 8hrs/day on 13days.

9. An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials:

(1) HTHTHT (2) TTHHHT (3) HTTHHT (4) HHHT __ __.

Which statement describing the last two coin tosses of the fourth trial has the highest probability of being correct?

(A) Two T will occur (B) One H and one T will occur
(C) Two H will occur (D) One H will be followed by one T

Key: (B)

Sol: In this, we are talking about 4^{th} trial, ie., nothing but one trial

One trial $\rightarrow$ Tossing a coin six times (or) six coins tossed at a time

$$(1) \Rightarrow P[H\ T\ H\ T\ H\ T] = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \left(\frac{1}{2}\right)^6$$

$$(2) \Rightarrow P[T\ T\ H\ H\ H\ T] = \left(\frac{1}{2}\right)^6$$

$$(3) \Rightarrow P[H\ T\ T\ H\ H\ T] = \left(\frac{1}{2}\right)$$

$$(4) \Rightarrow H\ H\ H\ T\ \_\ \_$$

Option(A):
$H\ H\ H\ T\ \underline{T}\ \underline{T}$
$\therefore$ Required probability
$= \left(\frac{1}{2}\right)^6$

Option(B):
$H\ H\ H\ T\ \underline{H}\ \underline{T}$
(or)
$H\ H\ H\ T\ \underline{T}\ \underline{H}$
$\therefore$ Required probability
$= \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 = 2\left(\frac{1}{2}\right)^6$

Option(C):
$H\ H\ H\ T\ \underline{H}\ \underline{H}$
$\therefore$ Required probability
$= \left(\frac{1}{2}\right)^6$

Option(D):
$H\ H\ H\ T\ T\ \underline{H}$
$\therefore$ Required probability
$= \left(\frac{1}{2}\right)^6$

10. A wire would enclose an area of 1936 m^2, if it is bent into a square. The wire is cut into two pieces. The longer piece is thrice as long as the shorter piece. The long and the short pieces are bent into a square and a circle, respectively. Which of the following choices is closest to the sum of the areas enclosed by the two pieces in square meters?

(A) 1096 (B) 1111 (C) 1243 (D) 2486

Key: (C)

Given that, a wire bent over square has area of $1936 m^2$

Let us assume that, the side of square as 'x'.

Then $x^2 = 1936 \Rightarrow x = 44$ meters.

$\therefore$ Length of wire= perimeter of square = $4x = 176m$

Again given $a + b = 176$ such that $a = 3b$

$\Rightarrow 3b + b = 17b \Rightarrow 4b = 17b$

$\Rightarrow b = 44 \therefore a = 132 [\because a = 3b]$

Given 'a' is bent over square and 'b' bent over circle.

i.e,. $4x' = a = 132 \Rightarrow x' = \frac{132}{4} = 33$ $[x' = \text{length of the side of square}]$

$\Rightarrow x' = 33$

$\because$ Area of square $= 33 \times 33 = 1089$ ...(1)

$\therefore 2\pi r = b [\because \text{'b' bent over circle}]$

$\Rightarrow r = \frac{44}{2\pi} (\because b = 44) \Rightarrow r = 7$

$\therefore$ Area of circle $= \pi(7^2) = 154$ ...(2)

$\therefore$ From (1) & (2) $\Rightarrow$ sum of area's $= 1089 + 154 = 1243 m^2$

Section-II: Mechanical Engineering

1. The Fourier cosine series for an even function f(x) is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$$

The value of the coefficient a_2 for the function $f(x) = \cos^2(x)$ in $[0, \pi]$ is

(A) –0.5 (B) 0.0 (C) 0.5 (D) 1.0

Key: (C)

Exp: We have, Fourier expression for even function:

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{L}$; if f(x) is defined over the range –L to L. [i.e., period 2L]

But given $f(x) = \cos^2 x = \frac{1 + \cos 2x}{2}$

$$= \frac{1}{2} + \frac{1}{2}\cos 2x = \frac{a_0}{2} + a_2.\cos 2x$$

$$\therefore a_0 = 1; a_2 = 1/2; a_1 = 0$$

2. Select the correct statement for 50% reaction stage in a steam turbine.

(A) The rotor blade is symmetric.

(B) The stator blade is symmetric.

(C) The absolute inlet flow angle is equal to absolute exit flow angle.

(D) The absolute exit flow angle is equal to inlet angle of rotor blade.

Key: **(D)**

Exp: For 50% reaction turbine $\theta=\beta, \alpha=\phi$

3. During solidification of a pure molten metal, the grains in the casting near the mould wall are
   (A) coarse and randomly oriented
   (B) fine and randomly oriented
   (C) fine and ordered
   (D) coarse and ordered

Key: **(B)**

Exp: Cooling at wall is faster so, grain size is fine & randomly oriented

4. The peak wavelength of radiation emitted by a black body at a temperature of 2000 K is 1.45 μm. If the peak wavelength of emitted radiation changes to 2.90 μm, then the temperature (in K) of the black body is
   (A) 500
   (B) 1000
   (C) 4000
   (D) 8000

Key: **(B)**

Exp: $\lambda_1\left(T_{max}\right)_1=\lambda_2\left(T_{max}\right)_2$

$1.45\times2000=2.90\times\left(T_{max}\right)_2$

$\Rightarrow\left(T_{max}\right)_2=1000K$

5. Metal removal in electric discharge machining takes place through
   (A) ion displacement
   (B) melting and vaporization
   (C) corrosive reaction
   (D) plastic shear

Key: **(B)**

Exp: In EDM, electric spark is used to melt the metal & vaporization of metal takes place

6. The preferred option for holding an odd-shaped work piece in a centre lathe is
   (A) Live and dead centres
   (B) three jaw chuck
   (C) lathe dog
   (D) four jaw chuck

Key: **(D)**

7. The arrival of customers over fixed time intervals in a bank follow a Poisson distribution with an average of 30 customers / hour. The probability that the time between successive customer arrival is between 1 and 3 minutes is _____ (Correct to two decimal places).

Key: **(0.3834)**

Exp: $\lambda=30/\text{hr}$

$\lambda=0.5/\text{min}$

$\therefore P_{(t)}=1-e^{-\lambda t}$

$P(1)=1-e^{-0.5\times1}=0.3934$

$P(3)=1-e^{-0.5\times3}=0.7768$

So, $P(1\le T\le3)=0.7768-0.3934=0.3834$

8. For an ideal gas with constant properties undergoing a quasi-static process, which one of the following represents the change of entropy (Δs) from state 1 to 2?

(A) $\Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$

(B) $\Delta s = C_v \ln\left(\frac{T_2}{T_1}\right) - C_p\ln\left(\frac{V_2}{V_1}\right)$

(C) $\Delta s = C_p \ln\left(\frac{T_2}{T_1}\right) - C_v\ln\left(\frac{P_2}{P_1}\right)$

(D) $\Delta s = C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_1}{V_2}\right)$

Key: **(A)**

Exp: For an ideal gas undergoing a quasi static process,

Change in entropy, $\Delta s = C_p \ell n\left(\frac{T_2}{T_1}\right) - R\ell n\left(\frac{P_2}{P_1}\right)$

Proof:- $dQ = dU + pdV = C_V dT + pdV$

$$\Delta s = \frac{dQ}{T} = C_V\frac{dT}{T} + \frac{P}{T}dV$$

$$= C_V\frac{dT}{T} + \frac{R}{V}dV \qquad [PV = RT] \rightarrow 'v'\text{ is specific volume}$$

On Integration,

$$s_2 - s_1 = C_V \ell n\left(\frac{T_2}{T_1}\right) + R\ell n\left(\frac{V_2}{V_1}\right) \quad ...(i)$$

We know that

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2}\times\frac{T_2}{T_1} \qquad ...(2)$$

Substituting (2) in (1)

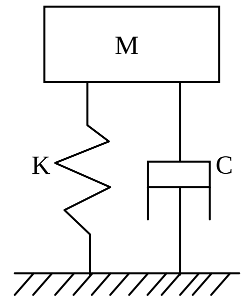
$$s_2 - s_1 = C_V \ell n\left(\frac{T_2}{T_1}\right) + R\ell n\left(\frac{P_1}{P_2}\times\frac{T_2}{T_1}\right)$$

$$= C_V \ell n\left(\frac{T_2}{T_1}\right) + R\ell n\left(\frac{P_1}{P_2}\right) + R\ell n\left(\frac{T_2}{T_1}\right)$$

$$= \ell n\left(\frac{T_2}{T_1}\right)(C_v + R) + R\ell\left(\frac{P_1}{P_2}\right)$$

$$\therefore s_2 - s_1 = C_p \ell n\left(\frac{T_2}{T_1}\right) - R\ell n\left(\frac{P_2}{P_1}\right) \qquad \left[\begin{array}{l}\because C_p - C_v = R\\ C_p = C_v + R\end{array}\right]$$

9. In a single degree of freedom under damped spring-mass-damper system as shown in the figure, an additional damper is added in parallel such that the system still remains underdamped. Which one of the following statements is ALWAYS true?



(A) Transmissibility will increase
(B) Transmissibility will decrease
(C) Time period of free oscillations will increase.
(D) Time period of free oscillations will decrease.

Key: **(C)**

Exp: $\omega_d \propto \sqrt{1-\xi^2}$

If additional damper used, then ξ increases and ω_d decrease

$\because T_d = \dfrac{2\pi}{\omega_d}$

so, T_d increases

10. The divergence of the vector field $\vec{u} = e^x\left(\cos y\,\hat{i} + \sin y\,\hat{j}\right)$ is

(A) 0 (B) $e^x \cos y + e^x \sin y$ (C) $2e^x \cos y$ (D) $2e^x \sin y$

Key: **(C)**

Exp: We have; Div $\vec{u} = \nabla.\vec{u}$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).\left(e^x \cos y\hat{i} + e^x \sin y\hat{j}\right)$$

$$= \frac{\partial}{\partial x}\left[e^x \cos y\right] + \frac{\partial}{\partial y}\left[e^x \sin y\right]$$

$$= e^x \cos y + e^x \cos y = 2e^x \cos y$$

$$\therefore \text{Div}\,\vec{u} = 2e^x \cos y$$

11. Match the following products with the suitable manufacturing process

	Product		Manufacturing Process
P	Toothpaste tube	1	Centrifugal casting
Q	Metallic pipes	2	Blow moulding
R	Plastic bottles	3	Rolling
S	Threaded bolts	4	Impact extrusion

(A) P-4, Q-3, R-1, S-2 (B) P-2, Q-1, R-3, S-4
(C) P-4, Q-1, R-2, S-3 (D) P-1, Q-3, R-4, S-2

Key: **(C)**

12. A hollow circular shaft of inner radius 10mm, outer radius 20mm and length 1m is to be used as a torsional spring. If the shear modulus of the material of the shaft is 150 GPa, the torsional stiffness of the shaft (in kN-m/rad) is _______ (correct to two decimal places).

Key: **(35.34)**

Exp: R_i =10mm. D_i=20mm

R_0=20mm, D_0=40mm

L=1mts =1000mm

G=150GPa

$$\frac{T}{\theta}=\frac{GJ}{\ell}=\frac{150\times10^3\times\frac{\pi}{32}\left(40^4-20^4\right)}{(1000)}$$
$$=35.34\,\text{kN}-\text{m}/\text{rad}$$

13. If y is the solution of the differential equation $y^3\frac{dy}{dx}+x^3=0,\ y(0)=1,$ the value of y(-1) is

(A) -2 (B) -1 (C) 0 (D) 1

Key: **(C)**

Exp: Given D.E

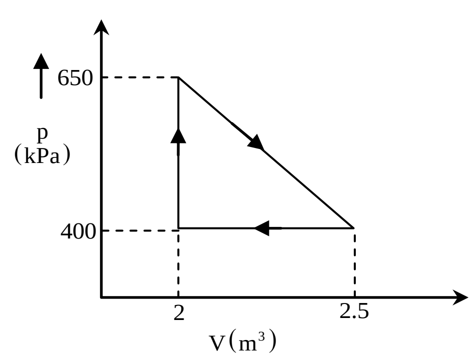
$$y^3\frac{dy}{dx}+x^3=0$$
$$\Rightarrow y^3dy+x^3dx=0\rightarrow \text{variable}-\text{separable D.E}$$
$$\Rightarrow \int y^3dy+\int x^3dx=c$$
$$\Rightarrow \frac{y^4}{4}+\frac{x^4}{4}=c\rightarrow(1)$$

Given $y(0)=1$; i.e., $y=1$ at $x=0$

From (1); $c=\frac{1}{4}$

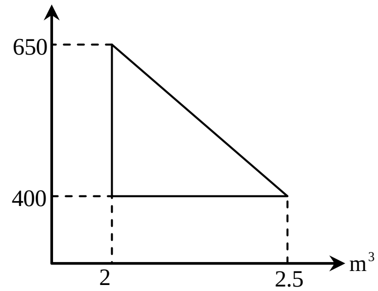
$$\therefore \text{From }(1);\ \frac{x^4}{4}+\frac{y^4}{4}=\frac{1}{4}$$
$$\Rightarrow x^4+y^4=1$$
$$\Rightarrow y^4=1-x^4$$
$$\Rightarrow y=\sqrt[4]{1-x^4}$$
$$\therefore y(-1)=\sqrt[4]{1-(-1)^4}=0$$

14. An engine operates on the reversible cycle as shown in the figure.
The work output from the engine (in kJ/cycle) is ________ (correct to two decimal places).



Key: **(62.5)**

Exp: Work done = Area $=\frac{1}{2}\times 0.50\times 250$
$=62.5\,\text{kN}-\text{m}$

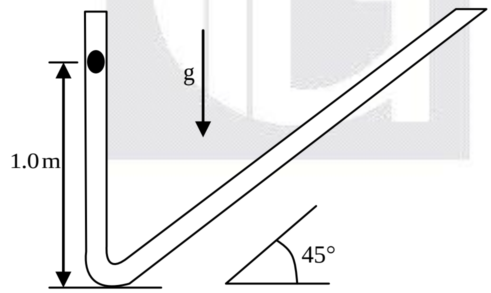


15. Pre-tensioning of a bolted joint is used to
(A) strain harden the bolt head
(B) decrease stiffness of the bolted joint
(C) increase stiffness of the bolted joint
(D) prevent yielding of the thread root

Key: **(C)**

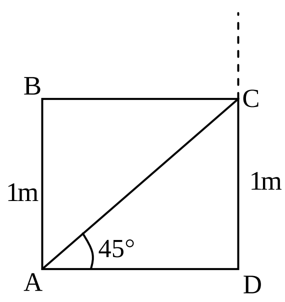
Exp: Pre-tensioning increases the stiffness of the bolts.

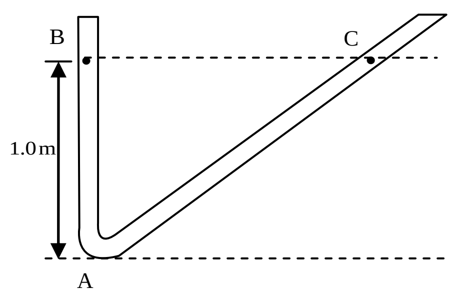
16. A ball is dropped from rest from a height of 1 m in a frictionless tube as shown in the figure. If the tube profile is approximated by two straight lines (ignoring the curved portion), the total distance travelled (in m) by the ball is ______ (correct to two decimal places).



Key: **(2.414)**

Exp: Ball will start moving from point B to point A & then reach at point C.
$\because$ Since ignoring friction,
i.e, potential energy at point B and C will be same.





$=1+AC, \qquad AC=\frac{1}{\sin 45°}$

$=1+\sqrt{2}=2.414\text{mts}$

17. The viscous laminar flow of air over a flat plate results in the formation of a boundary layer. The boundary layer thickness at the end of the plate of length L is δ_L. When the plate length is increased to twice its original length. The percentage change in laminar boundary layer thickness at the end of the plate (with respect to δ_L) is _______ (correct to two decimal places).

Key: (41.42)

Exp: For laminar boundary layer

Where $\delta \propto \sqrt{x}$

δ =Boundary Layer thickness of location 'x' from leading edge

So, $\dfrac{\delta_2}{\delta_1} = \sqrt{\dfrac{2L}{L}}$

$\delta_2 = 1.4142\delta_1$

So, % change $= \dfrac{\delta_2 - \delta_1}{\delta_1} \times 100$

$= \dfrac{1.4142\delta_1 - \delta_1}{\delta_1} \times 100 = 41.42\%$

18. The minimum axial compressive load, P, required to initiate buckling for a pinned-pinned slender column with bending stiffness EI and length L is

(A) $P = \dfrac{\pi^2 EI}{4L^2}$ (B) $P = \dfrac{\pi^2 EI}{L^2}$ (C) $P = \dfrac{3\pi^2 EI}{4L^2}$ (D) $P = \dfrac{4\pi^2 EI}{L^2}$

Key: (B)

Exp: $P_E = \dfrac{\pi^2 EI}{L_e^2}$

for both ends pinned,

$L_e = L$

$P_E = \dfrac{\pi^2 EI}{L^2}$

19. Consider a function u which depends on position x and time t. The partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

is known as the

(A) Wave equation (B) Heat equation

(B) Laplace's equation (D) Elasticity equation

Key: (B)

Exp: Clearly; $\dfrac{du}{dt} = \dfrac{\partial^2 u}{\partial x^2}$ is known as the heat equation

20. Feed rate in slab milling operation is equal to

(A) rotation per minute (rpm)

(B) product of rpm and number of teeth in the cutter

(C) product of rpm, feed per tooth and number of teeth in the cutter

(D) product of rpm, feed per tooth and number of teeth in contact

Key: (C)

Exp: $\because$ feed rate in milling,

$f_m = f_t \cdot Z \cdot N$

$f_t =$ feed per tooth

$Z =$ No. of teeth

$N =$ rpm

21. Denoting L as liquid and M as solid in a phase-diagram with the subscripts representing different phases, a *eutectoid* reaction is described by

(A) $M_1 \to M_2 + M_3$

(B) $L_1 \to M_1 + M_2$

(C) $L_1 + M_1 \to M_2$

(D) $M_1 + M_2 \to M_3$

Key: (A)

Exp: At Eutectoid point one solid converts into the another solid

$M_1 \to M_2 + M_3$

M = Solid

i.e. $\gamma \to \alpha + Fe_3C$

22. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A^{-1})$ is _______ (correct to two decimal places).

Key: (0.25)

Exp: Given,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Clearly; A is upper triangular matrix

$\det\left(A^{-1}\right) = \left|A^{-1}\right| = \dfrac{1}{|A|}$ ($\because$ From the properties of determinant)

$\Rightarrow \left|A^{-1}\right| = \dfrac{1}{1 \times 4 \times 1} = \dfrac{1}{4} = 0.25$

($\because$ The determinant of upper triangular matrix is the product of diagonal elements).

23. A local tyre distributor expects to sell approximately 9600 steel belted radial tyres next year. Annual carrying cost is Rs. 16 per tyre and ordering cost is Rs. 75. The economic order quantity of the tyres is

(A) 64 (B) 212 (C) 300 (D) 1200

Key: (C)

Exp: D = 9600units/year

C = Rs.16/unit/year

F = Rs. 75/order

$$EOQ = \sqrt{\frac{2DF}{C}}$$

$$= \sqrt{\frac{2\times 9600\times 75}{16}}$$

$$= 300\,\text{units}$$

24. Fatigue life of a material for a fully reversed loading condition is estimated from

$\sigma_a = 1100N^{-0.15}$

Where σ_a is the stress amplitude in MPa and N is the failure life in cycles. The maximum allowable stress amplitude (in MPa) for a life of 1×10^5 cycles under the same loading condition is ______ (correct to two decimal places).

Key: (195.61)

Exp: For completely reversed loading,

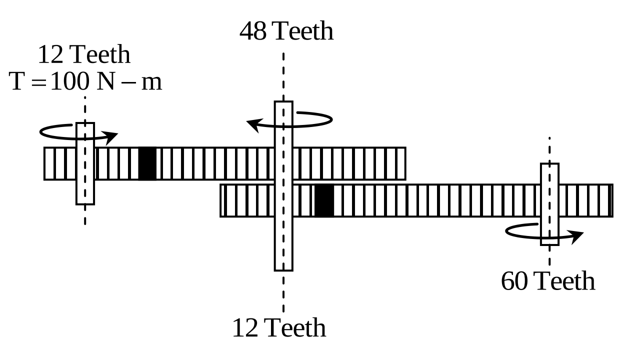
$$\sigma_{max} = \sigma_a$$

$$\sigma_a = 1100N^{-0.15}$$

$$\sigma_a = 1100\times\left(1\times10^5\right)^{-0.15}$$

$$= 195.61\text{MPa}$$

25. A frictionless gear train is shown in the figure. The leftmost 12-teeth gear is given a torque of 100N-m. The output torque from the 60-teeth gear on the right in N-m is



(A) 5 (B) 20 (C) 500 (D) 2000

Key: (D)

Exp: $\frac{T_1}{Z_1} = \frac{T_2}{Z_2}$

$$T_2 = \frac{100 \times 48}{12} = 400\text{N} - \text{m}$$

$$T_2 = T_3 = 400\text{N} - \text{m}$$

$$\frac{T_3}{Z_3} = \frac{T_4}{Z_4}$$

$$\frac{400}{12} = \frac{T_4}{60}$$

$$T_4 = \frac{400 \times 60}{12} = 2000\text{N} - \text{m}$$

26. A circular hole of 25 mm diameter and depth of 20mm is machined by EDM process. The material removal rate (in mm^3/min) is expressed as

$$4\times10^4 IT^{-1.23}$$

Where I=300 A and the melting point of the material, T=1600°C. The time (in minutes) for machining this hole is _________ (correct to two decimal places)

Key: (7.143)

Exp: $\text{MRR} = 4 \times 10^4\, IT^{-1.23}$

$$= 4 \times 10^4 \times 300 \times 1600^{-1.23}$$

$$= 1374.4\text{mm}^3 / \text{min}$$

Volume to be removed $= \frac{\pi}{4} D^2 L$

$$= \frac{\pi}{4}(25)^2 \times 20 = 9817.4\text{mm}^3$$

Time required $= \frac{9817.4}{1374.4} = 7.143\text{min}$

27. Following data correspond to an orthogonal turning of a 100mm diameter rod on a lathe. Rake angle: +15°; Uncut chip thickness: 0.5 mm; nominal chip thickness after the cut:1.25 mm. The shear angle (in degrees) for this process is ________ (correct to two decimal places).

Key: (23.31)

Exp: d=100mm

$$\alpha = 15°, t_1 = 0.5\text{mm}$$

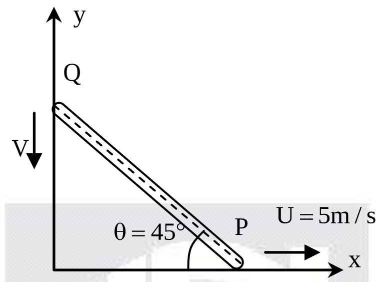
$$t_c = 1.25\text{mm}$$

$$r = \frac{t_1}{t_c} = 0.4$$

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha} \Rightarrow \tan\phi = \frac{0.4\times\cos 15}{1 - 0.4\times\sin 15}$$

$$\tan\phi = 0.4309 \therefore \phi = 23.31$$

28. A rigid rod of length 1 m is resting at an angle $\theta = 45°$ as shown in the figure. The end P is dragged with a velocity of U=5 m/s to the right. At the instant shown, the magnitude of the velocity *V* (in m/s) of point Q as it moves along the wall without losing contact is



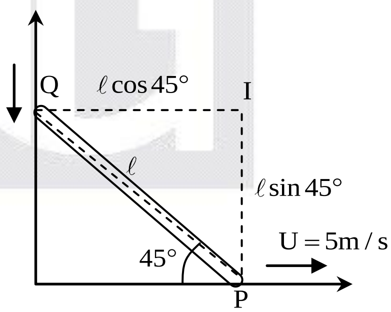
(A) 5 (B) 6 (C) 8 (D) 10

Key: (A)

Exp: $V_P = (IP)\omega$

$V_Q = (IQ)\omega \qquad IP = IQ = \ell\cos 45°$

$V_P = V_Q = 5\text{m/sec}$



29. The true stress (in MPa) versus true stain relationship for a metal is given by

$$\sigma = 1020\varepsilon^{0.4}$$

The cross-sectional area at the start of a test (when the stress and strain values are equal to zero) is 100mm^2. The cross – sectional area at the time of necking (in mm^2) is __________ (correct to two decimal places)

Key: (67.032)

Exp: $\sigma = 1020 \in^{0.4}$

$\sigma = K\in^n \qquad A_i = 100\text{mm}^2$

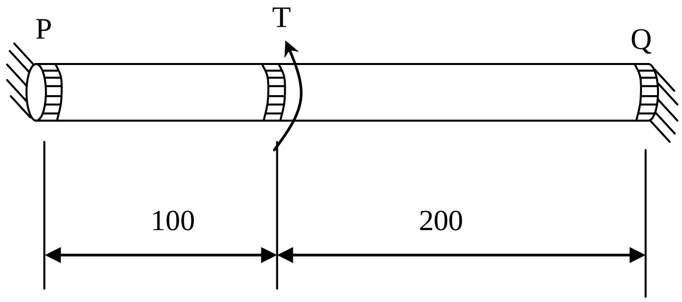
at necking $n = \in = 0.4$

$$\in = \ell n\left(\frac{\ell_f}{\ell_i}\right) = \ell n\left(\frac{A_i}{A_f}\right)$$

$$0.4 = \ell n\left(\frac{A_i}{A_f}\right) \Rightarrow e^{0.4} = \frac{100}{A_f}$$

$$A_f = \frac{100}{e^{0.4}} \therefore A_f = 67.032\text{mm}^2$$

30. A bar of circular cross section is clamped at ends P and Q as shown in the figure. A torsional moment T=150 Nm is applied at a distance of 100mm from end P. The torsional reactions (T_P,T_Q) in Nm at the ends P and Q respectively are

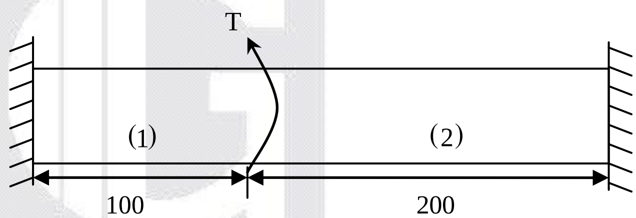


(All dimensions are in mm)

(A) (50, 100) (B) (75, 75) (C) (100, 50) (D) (120, 30)

Key: **(C)**

Exp: $T = 150N - m$



$T_P + T_Q = 150N - m$

$\theta_1 = \theta_2$

$\frac{T_P \ell_1}{GJ} = \frac{T_Q \ell_2}{GJ}$

$T_P(100) = T_Q(200)$

$T_P = 2T_Q \Rightarrow T_Q = 0.5T_P$

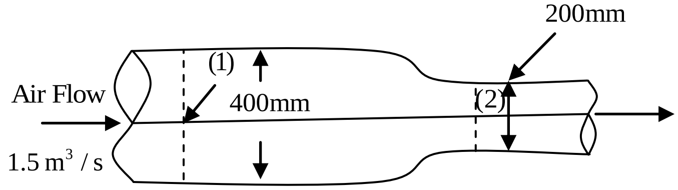
$T_P + T_Q = 150$

$T_P + 0.5T_P = 150$

$T_P = 100N - m,\ T_Q = 50N - m$

$(T_P, T_Q) = (100, 50)$

31. Air flows at the rate of $1.5m^3/s$ through a horizontal pipe with a gradually reducing cross-section as shown in the figure. The two cross-sections of the pipe have diameters of 400mm and 200 mm. Take the air density as 1.2 kg/m^3 and assume inviscid incompressible flow. The change in pressure (P_2-P_1) (in kPa) between sections 1 and 2 is



(A) –1.28 (B) 2.56 (C) -2.13 (D) 1.28

Key: **(A)**

Exp: Q=1.5m^2/s

Apply Bernoulli's equation b/w (1) & (2)

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2 \quad \Bigg| \quad V=\frac{Q}{A_1};\ V_1=\frac{1.5\times4}{\pi(0.4)^2}=11.936\,m/s$$

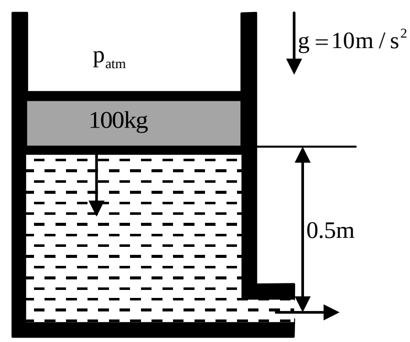
$$\frac{P_1-P_2}{\rho g}=\frac{V_2^2-V_1^2}{2g} \quad \Bigg| \quad V_2=\frac{1.5\times4}{\pi\times(0.2)^2}=47.746\,m/s$$

$$\text{So, } P_1-P_2=1.2\times\left(\frac{47.746^2-11.936^2}{2}\right)$$

$$P_1-P_2=1.28\,kPa$$

$$\text{or } P_2-P_1=-1.28\,kPa$$

32. A frictionless circular piston of area 10^{-2} m^2 and mass 100kg sinks into a cylindrical container of the same area filled with water of density 1000 kg/m^3 as shown in the figure. The container has a hole of area 10^{-3} m^2 at the bottom that is open to the atmosphere. Assuming there is no leakage from the edges of the piston and considering water to be incompressible, the magnitude of the piston velocity (in m/s) at the instant shown is ______ (correct to three decimal places).



Key: **(1.456)**

Exp:

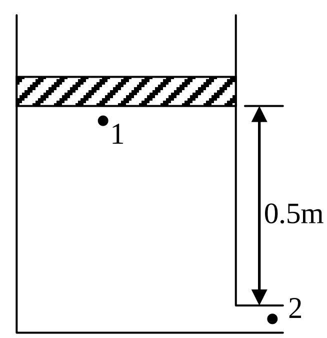
$$A_1V_1=A_2V_2$$

$$10^{-2}V_1=10^{-3}V_2$$

$$V_2=10V_1$$

$$P_1=\frac{100\times10}{10^{-2}}\,N/m^2$$

$$P_2=0$$



Apply Bernoulli's equation between (1) & (2)

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2$$

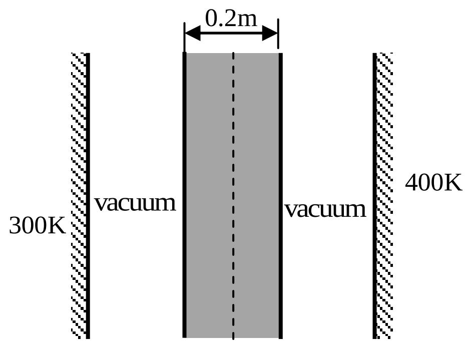
$$\frac{10^5}{10^3\times10}+\frac{V_1^2}{2\times10}+0.5=0+\frac{100V_1^2}{2\times10}+0$$

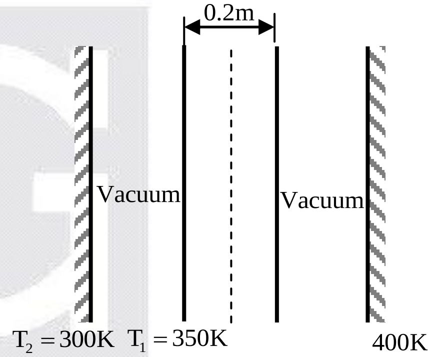
$$\frac{99V_1^2}{20}=10+0.5$$

$$V_1=1.456\,m/s$$

33. A 0.2 m thick infinite black plate having a thermal conductivity of 3.96 W/m-K is exposed to two infinite black surfaces at 300 K and 400 K as shown in the figure. At steady state, the surface

temperature of the plate facing the cold side is 350K. The value of Stefan-Boltzmann constant, σ, is 5.67×10^{-8} W/m^2 K^4. Assuming 1-D heat conduction, the magnitude of heat flux through the plate (in W/m^2) is _______ (correct to two decimal places).





Key: (391.58)

Exp: 1-D study state & Black Surfaces $(\varepsilon = 1)$

$$\text{Heat flux}(q) = \sigma\left(T_1^4 - T_2^4\right)$$

$$= 5.67\times10^{-8}\left(350^4 - 300^4\right)$$

$$= 391.58\,{}^{W}\!/_{m^2}$$

34. A steel wire is drawn from an initial diameter (d_i) of 10 mm to a final diameter (d_f) of 7.5mm. The half cone angle (α) of the die is 5° and the coefficient of friction (μ) between the die and the wire is 0.1. The average of the initial and final yield stress $\left[(\sigma_Y)\text{avg}\right]$ is 350 MPa. The equation for drawing stress σ_f,(in MPa) is given as:

$$\sigma_f = (\sigma_Y)_{avg}\left\{1+\frac{1}{\mu\cot\alpha}\right\}\left[1-\left(\frac{d_f}{d_i}\right)^{2\mu\cot\alpha}\right]$$

The drawing stress (in MPa) required to carry out this operation is _______ (correct to two decimal places).

Key: (316.24)

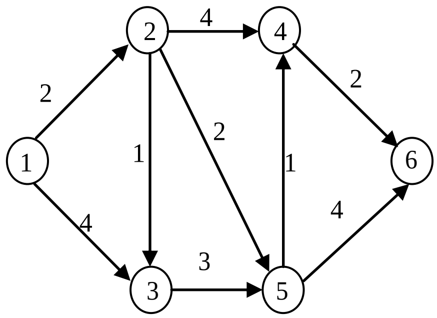
Exp: $d_i = 10\text{mm},\ \alpha = 5°,\ \sigma_y = 350\text{MPa}$

$d_f = 7.5\text{mm} \qquad \mu = 0.1$

$$\sigma_f = (\sigma_y)_{avg}\left[1+\frac{1}{\mu\cot\alpha}\right]\left[1-\left(\frac{d_t}{d_i}\right)^{2\mu\cot\alpha}\right]$$

$$= 350\left[1+\frac{1}{0.1\cot5°}\right]\left[1-\left(\frac{7.5}{10}\right)^{2\times0.1\times\cot5°}\right] = 316.24\text{MPa}$$

35. The arc lengths of a directed graph of a project are as shown in the figure. The shortest path length from node 1 to node 6 is ____________



Key: **(7)**

Exp: Shortest Path is

(1) – (2) – (5) – (4) – (6)

& length of shortest path = 2+2+1+2 = 7

36. The problem of maximizing z=x_1-x_2 subject to constraints $x_1 + x_2 \le 10,\ x_1 \ge 0,\ x_2 \ge 0$ and $x_2 \le 5$ has

(A) no solution
(B) one solution
(C) two solutions
(D) more than two solutions

Key: **(B)**

Exp: $\because$ slope of objective function is not equal to slope of any one constraint so, unique optional solution

37. Ambient air is at a pressure of 100kPa, dry bulb temperature of 30°C and 60% relative humidity. The saturation pressure of water at 30^0 C is 4.24 kPa. The specific humidity of air (in g/kg of dry air) is ___________ (correct to two decimal places).

Key: **(16.24)**

Exp: $P_t = 100\text{kPa}, \phi = 60\%$

$t_{db} = 30°\text{C}, P_{vs} = 4.424\text{kPa}$

$$\phi = \frac{P_v}{P_{vs}} \Rightarrow P_v = \phi \times P_{vs}$$

$$= 0.6 \times 4.24 = 2.544\text{kPa}$$

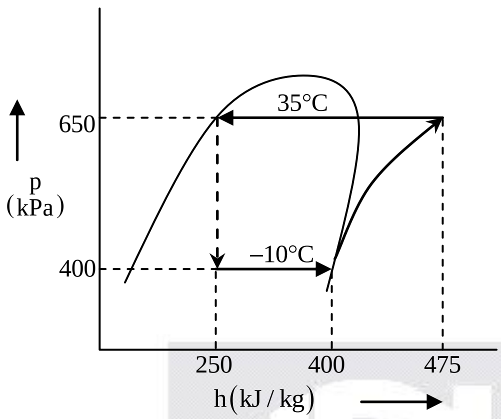
$$\text{Specific humidity}(w) = 0.622\frac{P_v}{P_t - P_v}$$

$$= 0.622 \times \frac{2.544}{100 - 2.544}$$

$$= 0.016236 \,{}^{\text{kgw.v}}\!/\!_{\text{kgd.a}}$$

$$= 16.24 \,{}^{\text{gmw.v}}\!/\!_{\text{kgd.a}}$$

38. A standard vapor compression refrigeration cycle operating with a condensing temperature of 35°C and an evaporating temperature of -10°C develops 15 kW of cooling. The *p-h* diagram shows the enthalpies at various states. If the isentropic efficiency of the compressor is 0.75, the magnitude of compressor power (in kW) is _____ (correct to two decimal places).

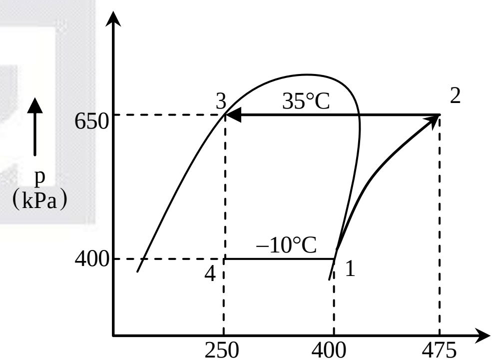


Key: **(10)**

Exp: $R.E = 15kW$ | $h_1 = 400kJ/kg$

$\eta_{isent_C} = 0.75$ | $h_2 = 475kJ/kg$

$h_4 = 250kJ/kg$



$$C.O.P = \frac{h_1 - h_4}{h_2 - h_1} = \frac{400-250}{475-400} = \frac{150}{75} = 2$$

$$\text{But } C.O.P = \frac{R.E}{W_{in}}$$

$$W_{in} = \frac{15}{2} = 7.5kW$$

$\because$ Isotropic efficiency of compressor is given, actual compressor power required

$$= \frac{W_{in}}{\eta_{isent_C}} = \frac{7.5}{0.75} = 10kW$$

39. For sand-casting a steel rectangular plate with dimensions 80mm×120mm×20mm, a cylindrical riser has to be designed. The height of the riser is equal to its diameter. The total solidification time for the casting is 2 minutes. In Chvorinov's law for the estimation of the total solidification time, exponent is to be taken as 2. For a solidification time of 3 minutes in the riser, the diameter (in mm) of the riser is _______ (correct to two decimal places).

Key: **(51.84)**

Exp: Given, $t_c = 2\,min$ $\quad A_r = \frac{\pi}{2}d^2 + \pi d^2$

$t_r = 3\,min$ $\quad A_r = \frac{3}{2}\pi d^2$

$V_c = 80 \times 120 \times 20 = 192000 mm^3$

$V_r = \frac{\pi}{4} d^3$

$A_c = 2\left(\left(80 \times 120\right) + \left(120 \times 20\right) + \left(20 \times 80\right)\right) = 27200 mm^2$

$\frac{t_c}{t_r} = \left(\frac{V_c}{A_c} \times \frac{A_r}{V_r}\right)^2$

$\frac{2}{3} = \left(\frac{192000}{27200} \times \frac{\frac{3}{2}\pi d^2}{\frac{\pi}{4} d^3}\right)^2$

$\frac{2}{3} = \left(\frac{42.352}{d}\right)^2 \Rightarrow d = 51.84 mm$

40. A welding operation is being performed with voltage =30V and current =100A. The cross sectional area of the weld bead is $20mm^2$. The work-piece and filler are of titanium for which the specific energy of melting is 14 $J/mm^{3.}$ Assuming a thermal efficiency of the welding process 70% the welding speed (in mm/s) is ______ (correct to two decimal places).

Key: **(7.5)**

Exp: $H_m.A.V = \eta.V.I$

14×20×V=0.7×30×100

V=7.5m/s

41. For a position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ the norm of the vector can be defined as $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$. Given a function $\phi = \ln|\vec{r}|$, its gradient $\nabla\phi$ is

(A) $\vec{r}$ (B) $\frac{\vec{r}}{|\vec{r}|}$ (C) $\frac{\vec{r}}{\vec{r}.\vec{r}}$ (D) $\frac{\vec{r}}{|\vec{r}|^3}$

Key: **(C)**

Exp: Given $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

Also given; $\phi = \ell n|\vec{r}|$

$= \ell n\left[\sqrt{x^2 + y^2 + z^2}\right]$

$= \frac{1}{2}\ell n\left(x^2 + y^2 + z^2\right)$

$\therefore$ gradient of $\phi = \nabla\phi$

$= \Sigma\hat{i}\frac{\partial}{\partial x}\left[\frac{1}{2}\ell n\left(x^2 + y^2 + z^2\right)\right]$

$$=\Sigma\hat{i}\frac{1}{2}\left(\frac{1}{x^2+y^2+z^2}\right)(2x)$$

$$=\Sigma\hat{i}\left[\frac{x}{x^2+y^2+z^2}\right]$$

$$=\hat{i}\left[\frac{x}{x^2+y^2+z^2}\right]+\hat{j}\left[\frac{y}{x^2+y^2+z^2}\right]+\hat{k}\left[\frac{z}{x^2+y^2+z^2}\right]$$

$$=\frac{x\hat{i}+y\hat{j}+z\hat{k}}{x^2+y^2+z^2}=\frac{\vec{r}}{\vec{r}.\vec{r}}$$

42. A bar is subjected to a combination of a steady load of 60kN and a load fluctuating between -10kN and 90 kN. The corrected endurance limit of the bar is 150 MPa, the yield strength of the material is 480 MPa and the ultimate strength of the material is 600MPa. The bar cross-section is square with side a. If the factor of safety is 2, the value of a (in mm), according to the modified Goodman's criterion, is _____ (correct to two decimal places).

Key: (31.62)

Exp: $P_{static}=60kN,\quad \sigma_{yt}=480MPa$

$P_{max}=90kN,\quad \sigma_{ut}=600MPa$

$P_{min}=-10kN,\quad \sigma_e=150MPa, F.S=2$

Considering Static varying load

$P_{max}=90+60=150kN$

$P_{min}=60-10=50kN$

$$P_{mean}=\frac{150+50}{2}=100kN$$

$$P_V=\frac{150-50}{2}=50kN$$

$$\sigma_{mean}=\frac{P_{mean}}{a^2}$$

$$\sigma_v=\frac{P_V}{a^2}$$

Good man equation

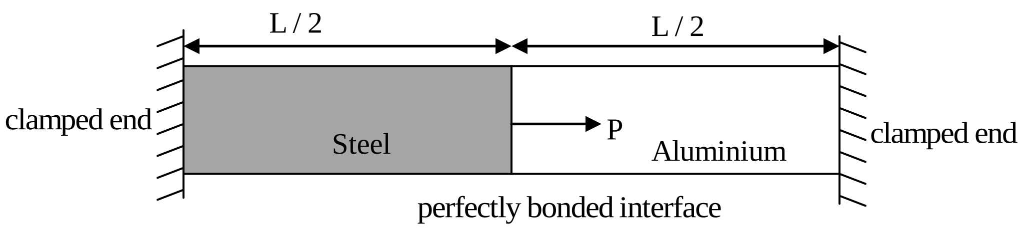
$$\frac{\sigma_{mean}}{\sigma_{ut}}+\frac{\sigma_v}{\sigma_e}=\frac{1}{F.S}$$

$$=\frac{\frac{100\times10^3}{a^2}}{600}+\frac{\frac{50\times10^3}{a^2}}{150}=\frac{1}{2}$$

$a^2=1000$

$a=31.62mm$

43. A bimetallic cylindrical bar of cross sectional area 1 m^2 is made by bonding steel (Young's modulus =210 GPa) and Aluminium (Young's modulus =70GPa) as shown in the figure. To maintain tensile axial strain of magnitude 10^{-6} in Steel bar and compressive axial strain of magnitude 10^{-6} in Aluminium bar, the magnitude of the required force P (in kN) along the indicated direction is



(A) 70 (B) 140 (C) 210 (D) 280

Key: (D)

Exp: $\epsilon_{al} = \epsilon_{st} = 10^{-6}$

$\epsilon_{st} = \epsilon_{al}$

$$\frac{P_{st}}{A_{st}.E_{st}} = \frac{P_{al}}{A_{al}.E_{al}}$$

$$P_{st} = \frac{210}{70} P_{al} \Rightarrow P_{st} = 3P_{al}$$

$\sigma_{st} = 3\sigma_{al}$

$\sigma_{st} = \epsilon_{st} \times \epsilon_{st}$

$= 210 \times 10^3 \times 10^{-6} = 0.21\text{MPa}$

$\sigma_{al} = 70 \times 10^3 \times 10^{-6} = 0.07$

$P = (\sigma_{st} + \sigma_{al})A$

$= (0.21 + 0.07) \times 10^6 = 280\,\text{kN}$

44. A vehicle powered by a spark ignition engine follows air standard Otto cycle $(\gamma = 1.4)$. The engine generates 70kW while consuming 10.3kg/hr of fuel. The calorific value of fuel is 44,000kJ/kg. The compression ratio is _____ (correct to two decimal places).

Key: (7.61)

Exp: γ = 1.4, W=70kW

m_f = 10.3kg/h

C.V=44000KJ/kg

r_c =?

For Otto cycle

$$\eta = 1 - \frac{1}{(r_c)^{\gamma-1}}$$

$$\text{But } \eta = \frac{W.D}{H.S}$$

$$\Rightarrow \frac{W.D}{H.S} = 1 - \frac{1}{(r_c)^{\gamma-1}}$$

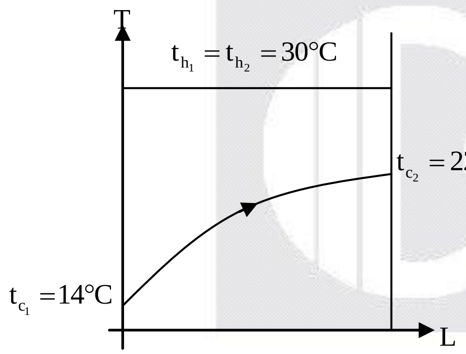
$$\frac{70}{\frac{10.3}{3600} \times 44000} = 1 - \frac{1}{(r_c)^{1.4-1}}$$

$$r_c = 7.61$$

45. Steam in the condenser of a thermal power plant is to be condensed at a temperature of 30^0 C with cooling water which enters the tubes of the condenser at 14°C and exits at 22°C. The total surface area of the tubes is 50 m^2, and the overall heat transfer coefficient is 2000 W/m^2 K. The heat transfer (in MW) to the condenser is ______ (correct to two decimal places).

Key: (1.15)

Exp:



$$\theta_1 = t_{h_1} - t_{c_1} = 30 - 14 = 16°C \quad A = 50m^2$$

$$\theta_2 = t_{h_2} - t_{c_2} = 30 - 22 = 8°C \quad U = 2000\, W/m^2K$$

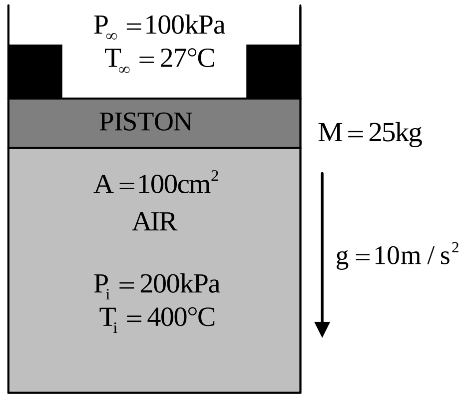
$$\text{LMTD}(\theta_m) = \frac{\theta_1 - \theta_2}{\ell n\left(\frac{\theta_1}{\theta_2}\right)}$$

$$= \frac{16-8}{\ell n\left(\frac{16}{8}\right)} = 11.54°C$$

$$\therefore \text{Heat Transfer } (Q) = UA\theta_m$$

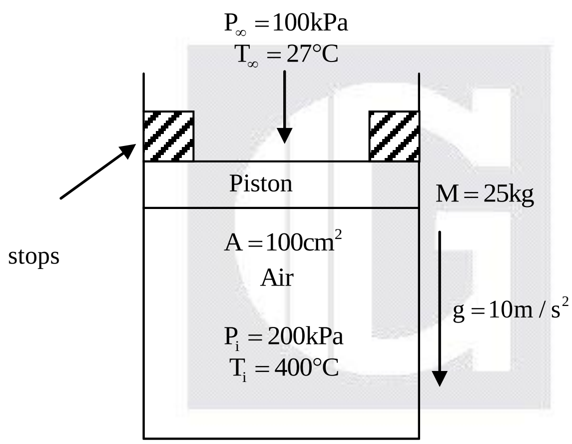
$$= 2000 \times 50 \times 11.54 = 1154156W = 1.15MW$$

46. Air is held inside a non insulated cylinder using a piston (mass M=25kg and area A=100 cm^2) and stoppers (of negligible area), as shown in the figure. The initial pressure P_i and temperature T_i of air inside the cylinder are 200kPa and 400°C, respectively. The ambient pressure P_∞ and temperature T_∞ are 100kPa and 27°C, respectively. The temperature of the air inside the cylinder (°C) at which the piston will begin to move is ________ (correct to two decimal places).



Key: (147.63)

Exp:



Total pressure on the piston $= P_\infty +$ pressure due to weight of piston

$$= P_\infty + \left(\frac{mg}{A}\right)$$

$$= 100 + \left(\frac{25 \times 10 \times 10^{-3}}{100 \times 10^{-4}}\right) = 125kPa$$

Internal pressure of air =200kPa

The pressure at which the piston can move is 125kPa

Let T_2 the temperature corresponding to P_2=125KPa

$\therefore$ From $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\frac{200}{673} = \frac{125}{T_2} \qquad (\because \text{volume of air constant})$$

$T_2 = 420.625K$

$= 147.625°C \cong 147.63°C$

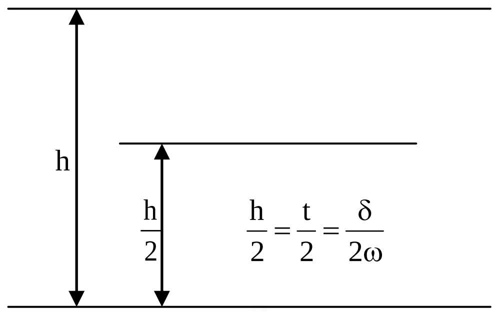
47. In a cam-follower, the follower rises by h as the cam rotates by δ (radians) at constant angular velocity ω(radians/s). The follower is uniformly accelerating during the first half of the rise

period and it is uniformly decelerating in the latter half of the rise period. Assuming that the magnitudes of the acceleration and deceleration are same, the maximum velocity of the follower is

(A) $\frac{4h\omega}{\delta}$ (B) $h\omega$ (C) $\frac{2h\omega}{\delta}$ (D) $2h\omega$

Key: (C)

Exp: $S=h,\ \theta=\delta$



$$t=\frac{\theta}{\omega}=\frac{\delta}{\omega}$$

$$V=u+at \Rightarrow V=a\times\frac{\delta}{2\omega}$$

$$S=ut+\frac{1}{2}at^2$$

$$\frac{h}{2}=\frac{1}{2}\times a\times\left(\frac{t}{2}\right)^2$$

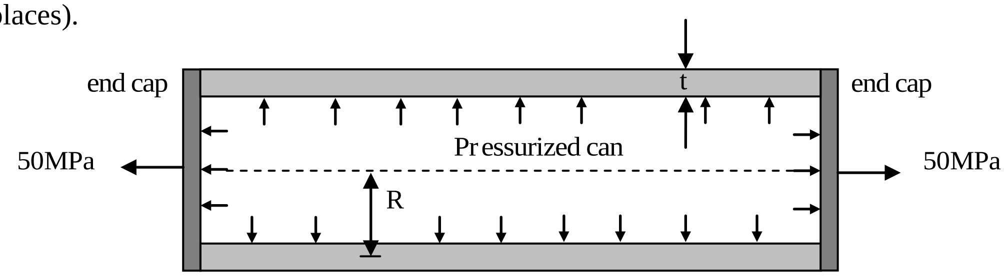
$$\frac{h}{2}=\frac{1}{2}\times a\times\left(\frac{\delta}{2\omega}\right)^2$$

$$a=\frac{4\omega^2h}{\delta^2}$$

$$V=a\times\frac{\delta}{2\omega}=\frac{4\omega^2h}{\delta^2}\times\frac{\delta}{2\omega}$$

$$\therefore V=\frac{2\omega h}{\delta}$$

48. A thin-walled cylindrical can with rigid end caps has a mean radius R=100 mm and a wall thickness of t=5 mm. The can is pressurized and an additional tensile stress of 50MPa is imposed along the axial direction as shown in the figure. Assume that the state of stress in the wall is uniform along its length. If the magnitudes of axial and circumferential components of stress in the can are equal, the pressure (in MPa) inside the can is __________(correct to two decimal places).



Key: (5)

Exp: t=5mm

R=100m $\Rightarrow D=200mm$

Additional $\sigma=50\,MPa$

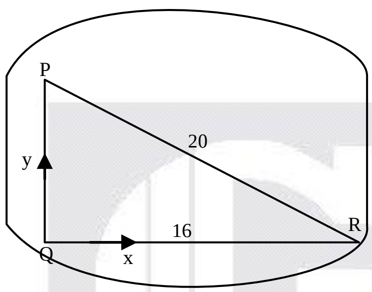
$$\sigma_t = \sigma_L + 50$$

$$\frac{PD}{2t} = \frac{PD}{4t} + 50$$

$$\frac{PD}{4t} = 50$$

$$P = \frac{50 \times 4 \times 5}{200} = 5MPa$$

49. In a rigid body in plane motion, the point R is accelerating with respect to point P at $10\angle 180°$ m/s^2. If the instantaneous acceleration of point Q is zero, the acceleration (in m/s^2) of point R is



(A) $8\angle 233°$ (B) $10\angle 255°$ (C) $10\angle 217°$ (D) $8\angle 217°$

Key: **(D)**

Exp: $a_{PR} = \vec{a}_R - \vec{a}_P, \quad PR = 20$

$$10 = (PR)\omega^2$$

$$\omega^2 = \frac{10}{20} \Rightarrow \omega = \frac{1}{\sqrt{2}}$$

$$a_{QR} = (QR)\omega^2$$

$$= 16 \times \left(\frac{1}{\sqrt{2}}\right)^2 = 8m/sec^2$$

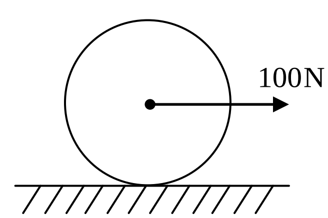
$$\Delta PQR \Rightarrow \sin\theta = \frac{120}{20}$$

$$\theta = \sin^{-1}(0.6) = 36.86°.$$

$$180 + 36.86 = 216.86 = 217°$$

$$8\angle 217°$$

50. A force of 100N is applied to the centre of a circular disc, of mass 10 kg and radius 1m, resting on a floor as shown in the figure. If the disc rolls without slipping on the floor, the linear acceleration (in m/s^2) of the centre of the disc is _______ (correct to two decimal places).



Key: (6.66)

Exp: r =1m

$\Sigma F_x = ma \Rightarrow (100 - F) = ma$

$(100 - F) = 10 \times a$

$(100 - 10a) = F$

$\text{Torque, } I\alpha = F \times r$

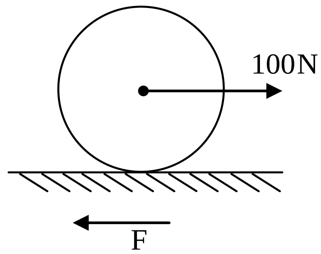
$\left[\frac{1}{2} mr^2\right]\alpha = (100 - 10a) \times 1$

$\left[\frac{1}{2} \times 10 \times 1^2\right]\alpha = (100 - 10a)$

$5\alpha = 100 - 10\alpha \Rightarrow 15\alpha = 100$

$\alpha = 6.66 \text{rad} / \sec^2$

$a = r\alpha = 6.66 \times 1 = 6.66 \text{m} / \sec^2$



51. A test is conducted on a one-fifth scale model of a Francis turbine under a head of 2m and volumetric flow rate of 1m^3/s at 450 rpm. Take the water density and the acceleration due to gravity as 10^3 kg/m^3 and 10 m/s^2, respectively. Assume no losses both in model and prototype turbines. The power (in MW) of a full sized turbine while working under a head of 30 m is ___________ (correct to two decimal places).

Key: (29.025)

Exp: $\frac{D_1}{D_2} = \frac{D_m}{D_p} = \frac{1}{5}$

(1)	(2)
$H_1 = 2\text{m}$	$H_2 = 30\text{m}$
$Q_1 = 1\text{m}^3/\text{s}, \; N_1 = 450\text{rpm}$	$P_2 = ?$

$H \propto N^2 D^2$

$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \times \left(\frac{D_2}{D_1}\right)^2$

$\frac{30}{2} = \left(\frac{N_2}{N_1}\right)^2 \times 5^2 \Rightarrow \frac{N_2}{N_1} = 0.774$

$\frac{Q_2}{Q_1} = \left(\frac{N_2}{N_1}\right)\left(\frac{D_2}{D_1}\right)^3$

$\frac{Q_2}{1} = 0.774 \times 5^3$

$Q_2 = 96.75 = m^3 / s$

$P_2 = \rho g Q_2 H_2 = 1000 \times 10 \times 96.75 \times 30 = 29.025 \text{MW}$

52. Given the ordinary differential equation

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$$

With $y(0)=0$ and $\frac{dy}{dx}(0)=1$, the value of y(1) is _____ (correct to two decimal places).

Key: **(1.47)**

Exp: Given D.E

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$$

$$\Rightarrow\left(D^2+D-6\right)y=0$$

The Auxiliary equation is $m^2+m-6=0$

$$\Rightarrow m^2+3m-2m-6=0$$

$$\Rightarrow m(m+3)-2(m+3)=0$$

$$\Rightarrow (m+3)(m-2)=0$$

$$\Rightarrow m=2,-3. \quad [\text{Roots are real and distinct}]$$

The solution is

$$y=C_1e^{-3x}+C_2e^{2x} \qquad ...(i)$$

given $y(0)=0$

$$\text{from}(i)\Rightarrow 0=C_1+C_2 \qquad ...(ii)$$

$$\text{from }(i)\Rightarrow\frac{dy}{dx}=-3C_1e^{-3x}+2C_2e^{2x}$$

$$\therefore\frac{dy}{dx}=1 \text{ at } x=0$$

$$1=-3C_1+2C_2 \quad \rightarrow(iii)$$

$$\Rightarrow 1=-3C_1+2\left(-C_1\right) \qquad [\because \text{From}(ii)]$$

$$\Rightarrow C_1=-\tfrac{1}{5}\Rightarrow C_2=\tfrac{1}{5}\ [\because C_2=-C_1]$$

$$\therefore \text{The required solution is } y=\frac{-1}{5}e^{-3x}+\frac{1}{5}e^{2x}$$

$$\Rightarrow y(1)=\frac{-1}{5}e^{-3}+\frac{1}{5}e^{2}$$

$$\Rightarrow y(1)=\frac{1}{5}\left[e^2-e^{-3}\right]$$

$$\therefore y(1)\cong 1.47$$

53. Let z be a complex variable. For a counter-clockwise integration around a unit circle C centred at origin,

$$\oint_C \frac{1}{5z-4}dz = A\pi i,$$

the value of A is

(A) 2/5 (B) 1/2 (C) 2 (D) 4/5

Key: (A)

Exp: Singular point is z = 4/5, which lies inside the unit circle C: |z|=1

By Cauchy's integral formula:

$$\oint_C \frac{1}{5z-4}dz = \oint_C \frac{1}{5\left(z-\frac{4}{5}\right)}dz = \frac{1}{5}[2\pi i(1)]$$

$$= \frac{2}{5}\pi i \qquad ...(i)$$

But given $\oint_C \frac{1}{5z-4}dz = A\pi i$

$\Rightarrow \frac{2}{5}\pi i = A\pi i \qquad (\because \text{from (i)})$

$\Rightarrow A = \frac{2}{5}$

54. Let X_1 and X_2 be two independent exponentially distributed random variables with means 0.5 and 0.25, respectively. Then Y=min (X_1, X_2) is

(A) exponentially distributed with mean 1/6

(B) exponentially distributed with mean 2

(C) normally distributed with mean 3/4

(D) normally distributed with mean 1/6

Key: (A)

Exp: We know that, if X_1 and X_2 are independent and exponential R. V's with parameters λ_1 and λ_2 then X = min(X_1, X_2) is exponential R.V with parameter $\lambda = \lambda_1 + \lambda_2$.

Given $E(X_1) = 0.5$	$E(X_2) = 0.25$
$\Rightarrow$ parameter of $X_1 = \lambda_{X_1} = \frac{1}{0.5}$	$\Rightarrow \lambda_{X_2} = \frac{1}{0.25} = 4$
$\Rightarrow \lambda_{X_1} = 2$	

$\therefore$ parameter of $Y = \lambda_{X_1} + \lambda_{X_2} = 2 + 4 = 6$

Mean of $Y = E(Y) = \frac{1}{6}$

55. Taylor's tool life equation is used to estimate the life of a batch of identical HSS twist drills by drilling through holes at constant feed in 20mm thick mild steel plates. In test 1, a drill lasted 300 holes at 150 rpm while in test 2, another drill lasted 200 holes at 300 rpm. The maximum number of holes that can be made by another drill from the above batch at 200 rpm is _______ (correct to two decimal places).

Key: (254)

Exp:

(1)	(2)
$T_1 = 300\text{holes}$	$T_2 = 200\text{holes}$
$N_1 = 150\text{rpm}$	$N_2 = 300\text{rpm}$

At N_3=200rpm, T_3=?

$$\because VT^n = C \Rightarrow V_1T_1^n = V_2T_2^n$$

$$\frac{N_1}{N_2} = \left(\frac{T_2}{T_1}\right)^n \Rightarrow \frac{150}{300} = \left(\frac{200}{300}\right)^n$$

$$0.5 = \left(\frac{2}{3}\right)^n \Rightarrow 0.67^n = 0.5$$

$$\log_{0.67}^{0.5} = n \Rightarrow n = 1.73$$

$$V_1T_1^n = V_3T_3^n$$

$$\left(\frac{T_3}{T_1}\right) = \left(\frac{N_1}{N_3}\right)^{1/n} \Rightarrow \left(\frac{T_3}{300}\right) = \left(\frac{150}{200}\right)^{1/1.73}$$

$$T_3 = 254\text{holes}$$