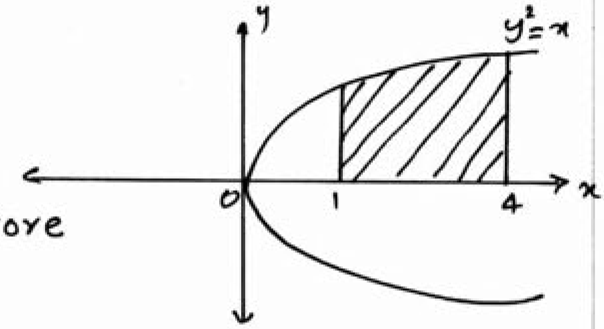
SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2018

SUBJECT : MATHEMATICS (SCIENCE) CODE. NO: 9018

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
1.	(a)	$f\circ f(x) = f(f(x))$	½	
		$= f\left(\frac{x}{x-1}\right)$	½	
		$= \dfrac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$	½	2
		$= \dfrac{x}{x-(x-1)} = x$	½	
	(b)	f	1	1
		Remark: $y = \frac{x}{x-1}$ $\quad xy - y = x$ $xy - x = y \quad x(y-1) = y$ $x = \frac{y}{y-1}$ $\therefore f^{-1}(y) = \frac{y}{y-1}$ or $f^{-1}(x) = \frac{x}{x-1}$	1	
2.		$A = IA$	½	
		$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$	½	
		$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ $\quad R_2 \to R_2 - 2R_1$	½	
		$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ +\frac{2}{5} & \frac{-1}{5} \end{bmatrix} A$ $R_2 \to \frac{-1}{5} R_2$	1	
		$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} A.$ $R_1 \to R_1 - 2R_2$	½	3
		Remark: Using matrix method $A^{-1} = \frac{Adj\,A}{\lvert A\rvert} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ give (1)		

www.shenischool.in

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
3.	(a)	(i) positive	1	1
	(b)	$f'(x) = 3x^2 - 6x + 4$	1	
		$= 3(x^2 - 2x + 1) + 1 > 0$	½	
		$\therefore f(x)$ is strictly increasing in $\mathbb{R}$	½.	2.
		Remark: If for $f'(x) = 3x^2 - 6x + 4 > 0$ $f(x)$ is increasing (1½)		
4.	(a)	$\int_0^a f(a-x)\,dx = \int_0^a f(x)\,dx$ (iii)	1	1
	(b)	$I = \int_0^{\pi/2} \frac{\sin^4 x\,dx}{\sin^4 x + \cos^4 x}$		
		$= \int_0^{\pi/2} \frac{\sin^4(\pi/2 - x)\,dx}{\sin^4(\pi/2 - x) + \cos^4(\pi/2 - x)}$	1	
		$I = \int_0^{\pi/2} \frac{\cos^4(x)\,dx}{\sin^4 x + \cos^4 x}$	½	
		$2I = \int_0^{\pi/2} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x}\,dx$		
		$= \int_0^{\pi/2} dx = \left[x\right]_0^{\pi/2}$	½	
		$= \pi/2$		2
		$\therefore I = \pi/4$.		
5		Area $= \int_a^b f(x)\,dx$	1	
		$= \int_1^4 \sqrt{x}\,dx$	1	
		$= \left(\frac{x^{3/2}}{3/2}\right)_1^4$	½	3
		$= \frac{14}{3}$ sq. units	½	

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
		Remark: For this figure only give (1) score 		
6		$\frac{dy}{dx} + \frac{2}{x}y = x\log x$	½	
		$P = \frac{2}{x} \quad Q = x\log x.$	½	
		$I.F = e^{\int P dx}$	½	
		$= e^{\int \frac{2}{x} dx} = e^{2\log x}$		
		$= x^2$	½	
		Solutions are		
		$y.(I.F) = \int Q(I.F)dx$	½	
		$yx^2 = \int x^2 . x\log x\, dx$		
		$= \int \log x . x^3 dx$		
		$= \log x . \frac{x^4}{4} - \int \frac{x^4}{x . 4} dx.$		
		$= \frac{x^4}{4}\log x - \frac{x^4}{16} + C$	½	3
7.		Let x packets of nuts and y packets of bolts		
		maximise $Z = 17.5x + 7y$	1½	
		Subject to $x + 3y \le 12$	½	
		$3x + y \le 12$	½	
		$x, y \geq 0$	½.	3

Remark:

	Nut	Bolts	
A	1	3	12
B	3	1	12

For this table only give (1) score

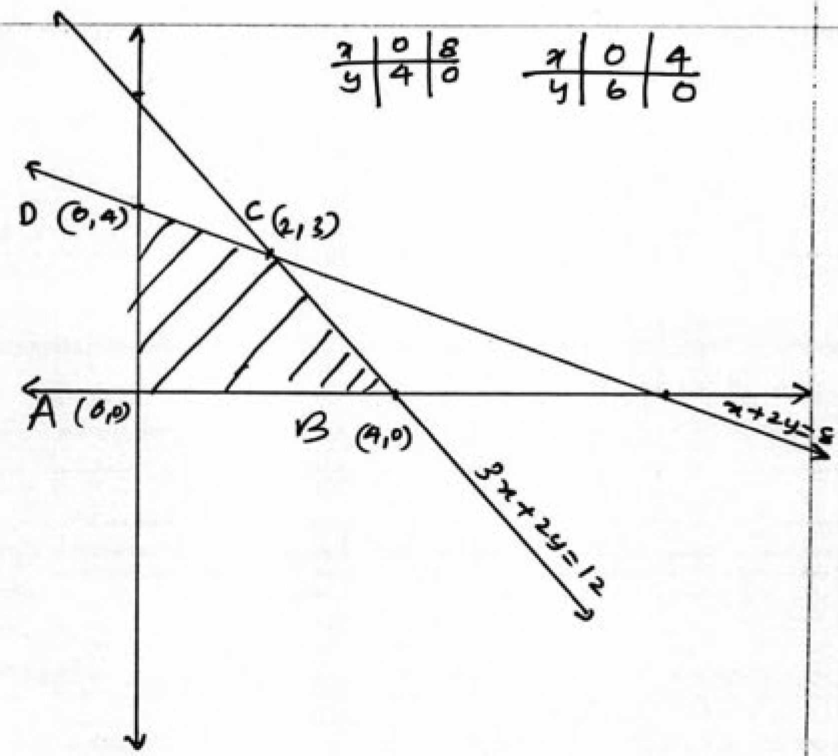
Qn No	Sub Qns	Answer Key/Value Points	Score	Total
8	(a)	$(1,2) \star (2,3) = (1+2,\ 2+3)$	½	1
		$= \underline{\underline{(3,5)}}$	½	
	(b)	$(c,d) \star (a,b) = (c+a,\ d+b)$	½	
		$= (a+c,\ b+d)$		1
		$= (a,b) \star (c,d)$	½	
	(c)	$(a,b) \star [(c,d) \star (e,f)] = (a,b) \star (c+e, d+f)$	½	
		$= (a+c+e,\ b+d+f)$	½	
		$[(a,b) \star (c,d)] \star (e,f) = (a+c,\ b+d) \star (e,f)$	½	2.
		$= (a+c+e,\ b+d+f)$	½	
		$\therefore (a,b) \star [(c,d) \star (e,f)] = [(a,b) \star (c,d)] \star (e,f)$		
		<u>Remarks</u>:		
		(i) If $a \star b = b \star a$ then $\star$ is commutative give (½) score		
		(ii) If $a \star (b \star c) = (a \star b) \star c$ then $\star$ is associative give (½) score.		
		(iii) Proving 'b' and 'c' using numbers give (3) score		
9	(a)	(ii) $\sin^{-1} x$	1	1
	(b)	Domain $[-1, 1]$	½	1
		Range $[-\pi/2, \pi/2]$	½	
	(c)	$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	½	
		$\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2}\times\frac{2}{11}}\right)$	1	
		$= \tan^{-1}\left(\frac{15}{20}\right)$	½	2
		$= \underline{\underline{\tan^{-1}\left(\frac{3}{4}\right)}}$		

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
10	(a)	(iii) $a^x \log a$	1	1
	(b)	$\log x^y = \log y^x$	½	
		$y \log x = x \log y$	½	
		$y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} = x \cdot \frac{1}{y}\frac{dy}{dx} + \log y$	1	
		$\left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = \log y - \frac{y}{x}$	½	
		$\therefore \frac{dy}{dx} = \dfrac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}$	½	3
11	(a)	$\frac{dy}{dx} = 2(x-2)$	½	1
		Slope $= -2$	½	
	(b)	Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1}$	½	
		$= \frac{4-0}{4-2} = 2$	½	
		Here $2(x-2) = 2$	½	
		$x = 3 \quad y = 1$	½	2
		Point $(3, 1)$		
	(c)	Equation of tangent is $y - y_0 = m(x - x_0)$	½	
		$y - 1 = 2(x-3)$	½	1
		$2x - y - 5 = 0$		
12.		$\int_a^b f(x)dx = \lim_{h \to 0} h[f(a) + f(a+h) + \cdots + f(a + \overline{n-1}h)]$	1	
		$a = 0 \quad b = 2 \quad f(x) = x^2 + 1$	½	
		$h = \frac{2}{n} = \frac{b-a}{n}$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$\int_0^2 x^2+1\,dx = \lim_{h\to 0} h\left[1+(h^2+1)+((2h)^2+1) + \cdots + [((n-1)h)^2+1]\right]$	½	
		$= \lim_{h\to 0} h\left[n + h^2(1^2+2^2+3^2+\cdots+(n-1)^2)\right]$	½	
		$= \lim_{h\to 0}\left[nh + \frac{h^3 n(n-1)(2n-1)}{6}\right]$	½	
		$= 2 + \frac{2(2-0)(4-0)}{6}$		
		$= 2 + \frac{8}{3} = \frac{14}{3}$	½	4
		Remark:		
	i	For Direct method $\int_0^2 (x^2+1)dx = \left(\frac{x^3}{3}+x\right)_0^2 = \frac{14}{3}$ give (1) score		
	ii	$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+\cdots + f(a+(n-1)h)\right]$ and $\int_0^2 x^2+1\,dx = \frac{14}{3}$ give (4) score		
13	(a)	point of intersection		
		$x^2+x^2=50 \quad x^2=25$		
		$x=\pm 5 \quad y=\pm 5$		
		$\therefore$ Point $P(5,5)$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
13	b	Required Area $= \int_0^5 x\,dx + \int_5^{\sqrt{50}} \sqrt{50-x^2}\,dx$	1	
		$= \left(\frac{x^2}{2}\right)_0^5 + \left[\frac{x}{2}\sqrt{50-x^2} + \frac{50}{2}\sin^{-1}\frac{x}{\sqrt{50}}\right]_5^{\sqrt{50}}$	1	
		$= \frac{25}{2} + 25\frac{\pi}{2} - \frac{25}{2} - 25.\frac{\pi}{4}$	½	
		$= \frac{25\pi}{4}$	½	3
		<u>Remark.</u>		
		(i) Area $= \frac{\pi r^2}{8} = \frac{\pi \times 50}{8} = \frac{25\pi}{4}$ (3)		
		(ii) Area $= \frac{1}{2}\int_0^{\sqrt{50}} \sqrt{50-x^2}\,dx$.		
		$= \frac{25\pi}{4}$ (3)		
		(iii) Area $= \int_a^b f(x)dx$ (1)		
14.	(a)	(iii) 2	1	1
	(b)	$\frac{\sec^2 x}{\tan x}dx = \frac{-\sec^2 y}{\tan y}dy$	1	
		$\int \frac{\sec^2 x}{\tan x}dx = -\int \frac{\sec^2 y}{\tan y}dy$		
		$\log \tan x = -\log \tan y + c$	2	3
		$\log \tan x + \log \tan y = c$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
15	a.	$[\bar{a}+\bar{b}, \bar{b}+\bar{c}, \bar{c}+\bar{a}] = (\bar{a}+\bar{b}) \cdot [(\bar{b}+\bar{c}) \times (\bar{c}+\bar{a})]$	½	
		$= (\bar{a}+\bar{b}) \cdot [\bar{b}\times\bar{c} + \bar{b}\times\bar{a} + \bar{c}\times\bar{c} + \bar{c}\times\bar{a}]$	½	
		$= (\bar{a}+\bar{b}) \cdot [\bar{b}\times\bar{c} + \bar{b}\times\bar{a} + \bar{c}\times\bar{a}]$	½	
		$= \bar{a}\cdot\bar{b}\times\bar{c} + \bar{a}\cdot\bar{b}\times\bar{a} + \bar{a}\cdot\bar{c}\times\bar{a}$ $+ \bar{b}\cdot\bar{b}\times\bar{c} + \bar{b}\cdot\bar{b}\times\bar{a} + \bar{b}\cdot\bar{c}\times\bar{a}$	1	
		$= 2[\bar{a}\ \bar{b}\ \bar{c}]$	½	3
	b.	$[\bar{a}\ \bar{b}\ \bar{c}] = 0$ Hence $\bar{a}, \bar{b}, \bar{c}$ are coplanar	1	1
16.	a.	$\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$	2.	2
		$6x+3y+2z = 6$		
	b.	Plane parallel to given plane is $6x+3y+2z = k.$	1	
		Since it passes through 1, 2, 3 $k = 18$	½	
		$\therefore$ equation of plane is $6x+3y+2z = 18$	½	2
		Remark. Equation of plane is $6(x-1)+3(y-2)+2(z-3)=0$ $6x+3y+2z-18=0$ (2) score		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
17.		$\begin{array}{c\|c\|c} x & 0 & 8 \\ \hline y & 4 & 0 \end{array}$ $\quad$ $\begin{array}{c\|c\|c} x & 0 & 4 \\ \hline y & 6 & 0 \end{array}$ 	2.	
		At ~~A(~~ $\quad z = {}^{-}3x+4y$ A (0,0) $\quad z = 0$ B (4,0) $\quad z = -12$ minimum C (2,3) $\quad z = 6$ D (0,4) $\quad z = 16$ z is minimum at B (4,0)	2	4
18.	(a)	$2x + {}^{-}y = 10$ $3x + y = 5$	1½	2
		$x = 3 \quad y = {}^{-}4$	½	
	(b)	$A^T = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$P = \frac{1}{2}\left[A + A^T\right]$		
		$= \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$, symmetric	1	
		$Q = \frac{1}{2}\left[A - A^T\right]$		
		$= \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ skew, symmetric	1	
		$A = P + Q$	1	4
19.	a	$\begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix} = 0$	2.	2
		Remark.		
		$\Delta = \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix}$ $R_2 \to R_2 - R_1$		
		$= 0$ (2) score		
		For direct expansion give (1) score		
	b (i)	Proving $AB = I$	$1\frac{1}{2}$	
		$\therefore B = A^{-1}$	$\frac{1}{2}$	
		Remark: $A^{-1} = \frac{\text{Adj } A}{\lvert A \rvert} = B$ give (2) score.		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
	ii	$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$	½	
		$X = A^{-1}B$	½	
		$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$	½	
		$= \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$	½	4
		$x = 0 \quad y = 5 \quad z = 3$		
20	(a)	$f(x) = \cos x \quad g(x) = x^2$, both are continuous	1	
		Composition of two continuous functions are continuous	½	
		$\therefore f(g(x)) = fog(x) = \cos(x^2)$ is continious	½	2
	(b)(i)	$\frac{dy}{dx} = a e^{a\cos^{-1}x} \times \frac{-1}{\sqrt{1-x^2}}$		
		$= \frac{-ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$	1	1
	ii	$\sqrt{1-x^2}\,\frac{dy}{dx} = -ay$.	½	
		$(1-x^2)\left(\frac{dy}{dx}\right)^2 = a^2y^2$.	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$(1-x^2)\, 2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \times -2x = 2a^2y\frac{dy}{dx}$	1	
		$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$	1	3
21	a	$-\frac{\cos mx}{m} + c$	1	1
	b	$x^2+2x+2 = (x+1)^2+1$	1	
		$\int\frac{dx}{\sqrt{x^2+2x+2}} = \int\frac{dx}{\sqrt{(x+1)^2+1}}$	1	
		$= \log\lvert(x+1)+\sqrt{x^2+2x+2}\rvert + c$	1	3
	c	$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$	½	
		$A = -1 \qquad B = 2$	½	
		$\int\frac{x}{(x+1)(x+2)}dx = \int\frac{-1}{x+1}dx + \int\frac{2}{x+2}dx$	½	
		$= -\log(x+1) + 2\log(x+2) + c$	½	2
22	(a) i	$\bar{a}+\bar{b} = 4\bar{i}+4\bar{j}$	1	2
		$\bar{a}-\bar{b} = 2\bar{i}+4\bar{k}$	1	
	(ii)	unit vector $\perp^r$ to both $= \dfrac{\begin{vmatrix} i & j & k \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}}{\sqrt{16^2+16^2+8^2}}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$= \dfrac{16\hat{i} - 16\hat{j} - 8\hat{k}}{\sqrt{576}}$	½	
		$= \dfrac{2\hat{i} - 2\hat{j} - k}{3}$	½	2
		Remark: Unit vector $\perp^r$ to $\bar{a}$ and $\bar{b}$ $= \dfrac{\bar{a} \times \bar{b}}{\lvert \bar{a} \times \bar{b} \rvert}$ give (1) score		
	(b)(i)	$\bar{AB} = \hat{i} + 4\hat{j} - 4\hat{k}$	½	1
		$\bar{Bc} = \hat{i} + 4\hat{j} - 4\hat{k}$	½	
	(ii)	$\therefore \bar{AB} = \bar{Bc}$	½	1
		A, B, C are collinear	½	
		Remark: Using distance formula or Section formula for proving collinearity give (1) score.		
23	(a)	Cos $a_1 = 2 \quad b_1 = 5 \quad c_1 = -3$ $a_2 = -1 \quad b_2 = 8 \quad c_2 = +4$	½	
		$\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$	½	
		$= \dfrac{-2 + 40 - 12}{\sqrt{4+25+9}\sqrt{1+64+16}}$	½	
		$= \dfrac{26}{9\sqrt{38}}$	½	2.

$\theta = \cos^{-1}\left(\dfrac{26}{9\sqrt{38}}\right)$

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		Remark:		
		$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{\lvert a\rvert \lvert b\rvert}$ give $(\frac{1}{2})$ score.		
	(b)	$a_1 = \hat{i}+2\hat{j}+3\hat{k}$ $\quad b_1 = \hat{i}-3\hat{j}+2\hat{k}$	1	
		$a_2 = 4i+5j+6k$ $\quad b_2 = 2\hat{i}+3\hat{j}+\hat{k}$		
		Shortest Distance $= \left\lvert \frac{(a_2-a_1)\cdot(b_1\times b_2)}{\lvert b_1\times b_2\rvert} \right\rvert$	1	
		$a_2-a_1 = 3\hat{i}+3\hat{j}+3\hat{k}$.	½	
		$b_1\times b_2 = -9\hat{i}+3\hat{j}+9\hat{k}$	1	
		$SD = \frac{-27+9+27}{\sqrt{171}}$	½	
		$= \frac{9}{\sqrt{171}} = \frac{3}{\sqrt{19}}$		4.
24	(a)	$\Sigma P(x) = 1$	1	1
	b (i)	$k+2k+3k+4k+5k+7k+8k+9k+10k+11k+12k = 1$	1	
		$72k = 1$	½	2
		$k = 1/72$	½	
	(ii)	$P(x>3) = P(x=4)+P(x=5)$	½	
		$= \frac{11}{72}+\frac{12}{72} = \frac{23}{72}$	½	1

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
	(iii)	$P(-3 < x < 4)$		
		$= P(x=2) + P(x=-1) + P(x=0)$		
		$+ P(x=1) + P(x=2) + P(x=3)$	½	1
		$= 43k = \dfrac{43}{72}$	½	
	(iv)	$P(x < -3)$	½	
		$= P(x=-5) + P(x=-4)$		1
		$= 3k = \dfrac{3}{72}$	½	
		<u>Remark:</u> For any value of 'k' if (ii), (iii) and (iv) are correct give corresponding scores.		

List of Teachers Participated in
Scheme finalisation Camp

1. Abdul Gafoor. K
JDT Islam VHSS, Kozhikode

2. Raghunathan. K.V.
Chattanchal HSS, Kasargod

3. Lali John
AMMHSS, Edayaranmula
Pathanamthitta (Dist)

4 Shaji mathew
M.S.M. H.S.S. Kallingaparamba
Malappuram (Dist)

5. Vinod M. J
St Mary's H.S.S Edoor
Kannur dt.

6. S. Sujadevi
St. Sebastian's H.S.S
Cheenthikulam
Idukki

7. K. Sree Mary. K.
Leo XIII HSS, Pulluvila,
Thiruvananthapuram Dist.

8 FRAGGY P JOSEPH
Kadavathur VHSS
KANNUR (Dt.)

9. Resh Paul
St. Peter's VHSS kolenchery
EKM (Dt)

10. Sreelatha. M.
KRGPM VHSS, Odanavattom
Kollam (Dt.)

11. Manju. V.P.
Chempakassery HSS, Poothakulam
Kollam (Dist)

12. Mini. K.L
NVT in mathematics
EVHSS, Elamannoor
Pathanamthitta

13 VALSA. P. P
AKMHSS Poochatty
Eravimangalam P.O
~~Thrissur - 680751~~

14 Geetha M
RMVHSS Perinjanam
Thrissur

15. NARAYANA IYER. R
N. S. H. S. S
Nedumudy
Alappuzha

16 SHAFEEQ RAHIMAN. M
9046 KALLADI HSS
KUMARAMPUTHUR
PALAKKAD

17 Ambika. S. Menon
NUT in mathemati
H M VHS thuvalath
kodunlu (po)
palakkad Ph: 9946555918

18. Biji B.
HSST, KPM HSS, Poothotta.
Ernakulam
Ph. 9495509535

19. Mini Das
CMS HSS
Melukavu
Kottayam
Ph 9446859975

20 Shameem Beegam. A. N
HGHMKM VHSS.
Vallakkadavu
Trivandrum.
Ph: 9446157436