SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2018

SUBJECT: STATISTICS **CODE NO. 9020**

Qn No	Scoring key	Split score	Total score
1 a	(iv) 0.5	1	2
1 b	(ii) 20	1	
2	A B (i) No correlation (d) r =0 (ii) $\frac{Cov(x,y)}{\sigma_x \times \sigma_y}$ (c) Correlation coefficient (iii) $1-\frac{6\sum d^2}{n^3-n}$ (a) Rank correlation coefficient (iv) Perfect correlation (b) r = ± 1	½ × 4	2
3	$\sum X = 247, \sum Y = 263, \sum X^2 = 7345, \sum Y^2 = 7537, \sum XY = 7259, n = 10$ $r = \frac{n\sum XY - \sum X \sum Y}{\sqrt{n\sum X^2 - (\sum X)^2} \times \sqrt{n\sum Y^2 - (\sum Y)^2}}$ (**or** $r = \frac{Cov(x,y)}{\sigma_x \times \sigma_y}$) $= \frac{10 \times 7259 - 247 \times 263}{\sqrt{10 \times 7345 - 247^2} \times \sqrt{10 \times 7537 - 263^2}}$ $r = \frac{76.29}{11.15 \times 7.87} = 0.87$ $= \frac{7629}{\sqrt{12441 \times 6201}} = 0.87$	1 ½ ½	2
4	Since f(x) is a pdf, we have $\int_{-\infty}^{+\infty} f(x)dx = 1$ $ie, \int_0^2 kx\,dx = 1$ $\Rightarrow k\left[\frac{x^2}{2}\right]_0^2 = 1 \quad \Rightarrow k = \frac{1}{2}$	½ ½ 1	2
5	Simple AM price index = $\frac{\sum x}{n}$, where x is the price relative. Here $\sum x = 111.11 + 122.86 + 110.42 = 344.39$, n = 3 $\therefore$ Simple AM price index = $\frac{344.39}{3} = 114.797$	1 ½ ½	2
6	$b_{yx} = 0.23,\ \gamma = 0.45,\ \sigma_x = 10,\ \sigma_y = ?$ We have, $b_{yx} = \gamma \frac{\sigma_y}{\sigma_x}$ $\Rightarrow 0.23 = 0.45 \times \frac{\sigma_y}{10}$ $\Rightarrow \sigma_y = 5.11$	1 ½ ½	2
7	Let μ be the average mileage of that particular model automobile. Here, $H_0 : \mu = 23\ and\ H_1 : \mu \neq 23$ (or use $H_1 : \mu < 23$) Given that $\overline{x} = 21.8, s^2 = 7.84\ and\ n = 50$ Since n is large, the test statistic to be used is: $Z = \frac{(\overline{x} - \mu)}{s/\sqrt{n}} \sim N(0,1)$	½ ½	2

<table>
<tr>
<td></td>
<td>

$$ie,\ Z=\frac{21.8-23}{\sqrt{7.84}\big/\sqrt{50}}=-3.03$$

For $\alpha=0.05$, the critical region is $|Z|\ge Z_{\alpha/2},\ ie, |Z|\ge 1.96$ (or $|Z|>2.58$)

Here $|Z|=3.03>1.96$ (or $|Z|=3.03>2.58$)

∴ We reject H_0. The given data does not agree with the claim of the manufacturer.

</td>
<td>½
½</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>

We have the trend equation $y=18.04x+126.55$ with origin 2010. We have to shift the origin to 2015. The trend equation with shifted origin is:

$y=18.04(x+k)+126.55$, where k = 2015 – 2010 = 5

$ie,\ y=18.04(x+5)+126.55$

$or\ y=18.04x+216.75$

</td>
<td>1
1</td>
<td>2</td>
</tr>
<tr>
<td>9.</td>
<td>

Given n = 100, m = 10 and d = 170

$\overline{p}=\frac{d}{mn}=\frac{170}{10\times100}=0.17$ and $\overline{q}=1-0.17=0.83$

The control limits of np chart are:

$CL=n\overline{p}=100\times0.17=17$

$LCL=n\overline{p}-3\sqrt{n\overline{p}\overline{q}}=17-3\sqrt{14.11}=5.73$

$UCL=n\overline{p}+3\sqrt{n\overline{p}\overline{q}}=17+3\sqrt{14.11}=28.27$

</td>
<td>½
½
½
½</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>

$Mean=E(X)=\sum xp(x)=1\times0.5+2\times0.3+3\times0.2=1.7$

$E(X^2)=\sum x^2p(x)=1^2\times0.5+2^2\times0.3+3^2\times0.2=3.5$

$Variance=E(X^2)-[E(X)]^2$

$=3.5-1.7^2=0.61$

</td>
<td>1
1
½
½</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>

The population values are 13, 11, 15, 17 and 18.

The population mean = $\frac{13+11+15+17+18}{5}=\frac{74}{5}=14.8$

The possible number of SRSWORs of size 2 is $5C_2=10$.

<table>
<tr><th>Sample No.</th><th>Sample</th><th>Sample mean</th></tr>
<tr><td>1</td><td>13, 11</td><td>12</td></tr>
<tr><td>2</td><td>13, 15</td><td>14</td></tr>
<tr><td>3</td><td>13, 17</td><td>15</td></tr>
<tr><td>4</td><td>13, 18</td><td>15.5</td></tr>
<tr><td>5</td><td>11, 15</td><td>13</td></tr>
<tr><td>6</td><td>11, 17</td><td>14</td></tr>
<tr><td>7</td><td>11, 18</td><td>14.5</td></tr>
<tr><td>8</td><td>15, 17</td><td>16</td></tr>
<tr><td>9</td><td>15, 18</td><td>16.5</td></tr>
<tr><td>10</td><td>17, 18</td><td>17.5</td></tr>
<tr><td colspan="2">Total</td><td>148</td></tr>
</table>

Mean of sample means = $\frac{148}{10}=14.8$

Mean of sample means = population mean. ∴ Sample mean is unbiased for population mean.

</td>
<td>1
1
½
½</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>

Let X denotes the weight of a particular kind of apple sold at a fruit market. Then X is normally distributed with $\mu=100$ and $\sigma=20$.

</td>
<td></td>
<td></td>
</tr>
</table>

$\therefore Z = \frac{X-\mu}{\sigma} = \frac{X-100}{20}$ follows N(0, 1). ½

a) $P(70 < X < 110) = P\left(\frac{70-100}{20} < \frac{X-\mu}{\sigma} < \frac{110-100}{20}\right)$ ½ 3

$= P(-1.5 < Z < 0.5) = 0.6247$ ½

1

b) $P(X > 110) = P(Z > 0.5) = 0.3085$

$\therefore$ No. of apples weigh greater than 110 gm = $1000 \times 0.3085 = 308$ ½

(or 309)

13

Commodity	p_0	q_0	p_1	q_1	p_0q_0	p_1q_0
A	12	100	15	120	1200	1500
B	6	210	7	240	1260	1470
C	10	110	13	150	1100	1430
Total					3560	4400

1 ½ 3

Laspyre's index number = $\frac{\sum p_1q_0}{\sum p_0q_0} \times 100$ 1

$= \frac{4400}{3560} \times 100 = 123.6$ ½

14 a (ii) Level of significance 1 1

14 b Let X be the thickness of washers produced by the machine and let μ be the average thickness. We have to test $H_0 : \mu = 0.05$ against $H_1 : \mu \neq 0.05$. ½

Given $n = 10, \bar{x} = 0.053$ and $s = 0.003$.

The test statistic is $t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}}$, follows $t_{(n-1)}$. ½

Here, $t = \frac{0.053-0.05}{0.003/\sqrt{9}} = 3$ ½ 2

For $\alpha = 0.01$ and degrees of freedom 9, the critical region is $|t| \geq 3.25$

Here |t| = 3 < 3.25. So we accept H_0 .The machine is working in proper order. ½

15 $\bar{R} = \frac{5+6+\ldots+6}{10} = 5.8$, n = 5 1

The control limits for R – Chart are: ½ 3

$CL = \bar{R} = 5.8$

$LCL = D_3\bar{R} = 0 \times 5.8 = 0$ ½

$UCL = D_4\bar{R} = 2.115 \times 5.8 = 12.267$ ½

All observed values are within the control limits. So the process is in control. ½

16 a (iii) Seasonal variations 1 1

16 b

Month	No. of homes	3 yearly moving total	3 yearly moving average
June	6		
July	7	22	7.33
August	9	24	8
September	8	26	8.67
October	9	27	9
November	10	31	10.33
December	12		

2 2

17 a	$y = x^3 + 7x^2 + 10x + 6$ $\frac{dy}{dx} = 3x^2 + 14x + 10$ $\frac{d^2y}{dx^2} = 6x + 14$	1 1	2
17 b	$\int_0^k x^2 dx = 9 \quad \Rightarrow \left[\frac{x^3}{3}\right]_0^k = 9$ ie, $\frac{k^3}{3} - 0 = 9 \Rightarrow k = 3$	1 1	2
18 a	Let X be the number of candidates selected out of n = 1000 candidates appeared. The probability of selection , $p = 0.2\% = 0.002$ is very small, we can use the Poisson distribution with $\lambda = np = 1000 \times 0.002 = 2$ We have the pmf, $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \; x = 0,1,2,...$ $ie, \; f(x) = \frac{e^{-2}2^x}{x!}, \; x = 0,1,2,...$ P(3 persons selected) = $P(X = 3)$ $= \frac{e^{-2}2^3}{3!} = \frac{0.1353 \times 8}{6} = 0.1804$ (Attempt with Binomial Distribution also consider)	½ ½ ½ ½	2
18 b	Let X be the number of workers suffering from the occupational decease. Then X follows Binomial distribution with n = 6 , p = 0.2 and q = 0.8. The pmf is: $f(x) = {}^nC_x p^x q^{n-x}, \; x = 0,1,...,n$ $ie, \; f(x) = {}^6C_x (0.2)^x (0.8)^{6-x}, \; x = 0,1,...,6$ P(4 will suffer from the decease) = P(X=4) $= {}^6C_4 \times (0.2)^4 \times (0.8)^2 = 15 \times 0.0016 \times 0.64 = 0.01536$	½ ½ ½ ½	2
19 a	(iv) 12	1	1
19 b	**Any 3 of the relations given below. Each carries 1 score.** 1. The square of a Standard Normal variable is a Chi – square variable with 1 degrees of freedom. 2. The sum of squares of n independent Standard Normal variables is Chi – square variable with n degrees of freedom. 3. If X is a Standard Normal variable and Y is a Chi – square variable with degrees of freedom, n. Then $t = \frac{X}{\sqrt{Y/n}}$ is a t variable with degrees of freedom n. 4. If X_1 is a Chi – square variable with degrees of freedom n_1 and X_2 is another independent Chi – square variable with degrees of freedom n_2. Then $F = \frac{X_1/n_1}{X_2/n_2}$ is a F variable with degrees of freedom (n_1, n_2) . 5. The square of a t variable with degrees of freedom n is a F variable with degrees of freedom (1, n).	3 X 1	3

<table>
<tr><td>20 a</td><td>(iii) θ</td><td>1</td><td>1</td></tr>
<tr><td>20 b</td><td>

$$t_1=\frac{x_1+x_2+x_3+x_4}{4}$$ is unbiased.

$$V(t_1)=V\left(\frac{x_1+x_2+x_3+x_4}{4}\right)=\frac{\sigma^2+\sigma^2+\sigma^2+\sigma^2}{16}=\frac{\sigma^2}{4}=0.25\sigma^2$$

$$t_2=\frac{2x_1+x_2+x_3+x_4}{5}$$ is unbiased.

$$V(t_2)=V\left(\frac{2x_1+x_2+x_3+x_4}{5}\right)=\frac{4\sigma^2+\sigma^2+\sigma^2+\sigma^2}{25}=\frac{7\sigma^2}{25}=0.28\sigma^2$$

Here, $V(t_1)<V(t_2)$. So t_1 is efficient than t_2 .

</td><td>1
1
1</td><td>3</td></tr>
<tr><td>21</td><td>

Let *x* denotes the rank of number of years of smoking and *y* denotes the rank of lung damage grade.

<table>
<tr><td>x</td><td>y</td><td>$d=x-y$</td><td>d^2</td></tr>
<tr><td>10</td><td>10</td><td>0</td><td>0</td></tr>
<tr><td>9</td><td>7</td><td>2</td><td>4</td></tr>
<tr><td>8</td><td>6</td><td>2</td><td>4</td></tr>
<tr><td>7</td><td>9</td><td>-2</td><td>4</td></tr>
<tr><td>6</td><td>5</td><td>1</td><td>1</td></tr>
<tr><td>5</td><td>8</td><td>-3</td><td>9</td></tr>
<tr><td>4</td><td>4</td><td>0</td><td>0</td></tr>
<tr><td>3</td><td>2</td><td>1</td><td>1</td></tr>
<tr><td>2</td><td>3</td><td>-1</td><td>1</td></tr>
<tr><td>1</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td colspan="3">Total</td><td>$\sum d^2=24$</td></tr>
</table>

Rank correlation coefficient, $\rho=1-\frac{6\sum d^2}{n^3-n}$

$$=1-\frac{6\times 24}{10^3-10}=1-\frac{144}{990}=0.8545$$

(or attempt using Karl Pearsons formula, give 2 scores)

</td><td>2
1
1</td><td>4</td></tr>
<tr><td>22 a</td><td>Correlation coefficient.</td><td>1</td><td>1</td></tr>
<tr><td>22 b</td><td>

The two regression lines are:

$3x+2y=26$ --------(1) and $6x+y=31$---------(2)

(i) Let (1) denotes the regression line of Y on X and (2) denotes the regression line of X on Y.

Equation (1) becomes, $2y=-3x+26$ or $y=\frac{-3}{2}x+13 \quad \Rightarrow b_{yx}=\frac{-3}{2}$

Equation (2) becomes, $6x=-y+31$ or $x=\frac{-1}{6}y+\frac{31}{6} \quad \Rightarrow b_{xy}=\frac{-1}{6}$

Now, $b_{yx}\times b_{xy}=\frac{-3}{2}\times\frac{-1}{6}=\frac{1}{4}<1$. Hence our assumption is correct

$$\therefore r=\pm\sqrt{b_{yx}\times b_{xy}}=\pm\sqrt{\frac{1}{4}}=\frac{-1}{2}=-0.5$$

(ii) Let $\overline{x}$ and $\overline{y}$ be the averages. The equations (1) and (2) become:

$3\overline{x}+2\overline{y}=26$ ---------(3)

$6\overline{x}+\overline{y}=31$ ---------(4)

$(3)-2\times(4)\Rightarrow -9\overline{x}=-36\Rightarrow \overline{x}=4$, average price = 4

$(4)-2\times(3)\Rightarrow -3\overline{y}=-21\Rightarrow \overline{y}=7$, average demand =7

</td><td>½
½
1
1
½
½</td><td>4</td></tr>
</table>

23 a (iv) Equality of more than two means — 1 — 1

23 b H_0 : The yields are equal and H_1 : The yields are not equal.

A	B	C
20	18	25
21	20	28
23	17	22
$T_1 = 64$	$T_2 = 55$	$T_3 = 75$

½

Grand Total, $G = 64 + 55 + 75 = 194$

Correction factor, $CF = \frac{G^2}{n} = \frac{194^2}{9} = 4181.78$ ½

Total Sum of Squares, TSS = Sum of squares of all observations – CF

$$= 20^2 + 21^2 + ... + 22^2 - 4181.78 = 94.22$$ ½

Between Sum of Squares, SSB = $\sum \frac{T_i^2}{n_i} - CF$

$$= \frac{64^2}{3} + \frac{55^2}{3} + \frac{75^2}{3} - 4181.78 = 66.89$$ ½

4

ANOVA Table

Source	*df*	*SS*	*MSS*	*F*	$F_{0.5}$
Between Within	2 6	66.89 27.33	33.445 4.555	7.34	5.14
Total	8	94.22			

1 ½

Here $F > F_{0.5}$. So we reject H_0 . That is the yields are not equal for the three varieties of seeds. ½

24 The hypotheses are:

H_0 : Drinking habit and the favour in local sale of liquor are independent.

H_1 : Drinking habit and the favour in local sale of liquor are not independent. 1

We are given a 2X2 contingency table. The contingency table along with the expected frequencies is:

Question 1	**Question 2**		Total
	Yes	No	
Yes	56 (58)	31 (29)	87
No	18 (16)	6 (8)	24
Total	74	37	111

1 ½

5

The test statistics is $\chi^2 = \sum \frac{(O-E)^2}{E}$ follows Chi – square distribution with degrees of freedom $(R-1)(C-1) = (2-1)(2-1) = 1$. 1

The value of test statistic is:

$$\chi^2 = \frac{(56-58)^2}{58} + \frac{(31-29)^2}{29} + \frac{(18-16)^2}{16} + \frac{(6-8)^2}{8} = 0.96$$ 1

The critical region with df=1 is, $\chi^2 > 3.84$.

Here $\chi^2 = 0.96 < 3.84$. So we accept H_0. Hence the drinking habit and the favour in local sale of liquor are independent. ½

Scheme finalisation - Participants.

1. Biju. G.V, GHSS Puthoor, Kollam - 9447584301
2. Parameswarajan. P.K. 14079 - GHSS Ravaneshwaram.
3. Sudhi Kumar. T.V Govt: HSS Kannadiparamba (13101)
4. Juliet Sebastian SAVHSS Kothamangalam Juliet.
5. Manoj K, HSS Panangad 9447235515
6. SAJIDA. N.K Nochad H.S.S 9946027843
7. Sreeja. T.N. G.H.S.S. Pulamanthole. Sreeja
8. Ambily. A. HSST, St. Peter's HSS Kolenchery
9. Jyothi. T.J, SMV HSS Poonjar Jyothi
10. Deepa Koshy, PSVPM HSS, Aypravon, Konni Pathanamthitta
11. Santhosh V C, Govt HSS Poovar, Trivandrum 9497017823