DIRECTORATE OF GOVERNMENT EXAMINATIONS

HIGHER SECONDARY SECOND YEAR EXAMINATION - MARCH / APRIL 2018

BUSINESS MATHEMATICS ENGLISH MEDIUM – ANSWER KEY

General Instructions

1. For objective type questions, award 1 mark for "writing the correct option's letter and corresponding option's answer".

2. Award "0 marks" for one who wrote both "option's letter" and "option's answer" with one of them is not correct.

3. Marks should be awarded for suitable alternative method also.

4. Mark(s) should not be reduced for the correct answer / stage, if it is written without formula / properties also.

5. Award full mark directly, if the solution is arrived with nil mistakes without giving weightage for the stages.

6. The stage mark is essential, only if the part of the solution is incorrect.

7. Award marks, if the answer is in decimal value and also approximately equal to the key answer.

8. For a particular stage in which the stage mark is greater than 1 and one who begins with correct step but reaches with incorrect solution, for such cases, the suitable credits should be given by breaking the stage marks.

PART - A

40×1=40

Q.NO	OPTION	ANSWER	Q.NO	OPTION	ANSWER
1	(b)	$\|A\|^2$	21	(b)	$log\|x+1\|+k$
2	(a)	I	22	(a)	3 , 2
3	(b)	n	23	(a)	$y = log(e^x + c)$
4	(c)	2	24	(b)	$\sqrt{1+x^2}$
5	(d)	0.2	25	(d)	$\frac{x^2e^{3x}}{2!}$
6	(b)	36	26	(a)	$f(x+h) - f(x)$
7	(d)	$below\ x - axis$	27	(a)	59
8	(a)	*major axis*	28	(a)	2.4
9	(c)	$(0, -100)$	29	(a)	N (0 , 1)
10	(b)	$P = 30x - 900$	30	(d)	2
11	(c)	3	31	(b)	$\frac{1}{2^{12}}$
12	(c)	$\frac{p}{-p+55}$	32	(a)	principle of statistical regularity
13	(d)	2	33	(d)	$\|Z\| \geq 2.58$
14	(b)	$\frac{7}{2}$	34	(b)	*Type I error*
15	(a)	*concave downward*	35	(a)	45
16	(c)	3	36	(d)	*all the above*
17	(c)	2	37	(d)	*none of these*
18	(b)	$x = 2$	38	(a)	*the current period*
19	(b)	$2\int_0^a f(x)\,dx$	39	(a)	*assignable causes*
20	(b)	log2	40	(c)	*functional relationship*

	PART – B		10×6=60
Q. NO.	KEY STEPS - ANSWER		STEPS MARKS
41	$A \sim \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ -2 & 1 & 3 & 4 \end{pmatrix}$ $R_1 \leftrightarrow R_3$		1
	$\sim \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 7 & 11 & 18 \end{pmatrix}$ $R_3 \to R_3 + 2R_1$		2
	$\sim \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 4 & 4 \end{pmatrix}$ $R_3 \to R_3 - 7R_2$		2
	$\Rightarrow \rho(A) = 3$		1
	Aliter: showing atleast one minor of order 3×3 which is not zero $[\Delta_{3\times3} = -4 \ (or)\ 4 \neq 0]$		3
	$\Rightarrow \rho(A) = 3$		3
42	$\Delta = 19 \neq 0$		1
	$\Delta_x = 38$; $\Delta_y = 19$		1+1
	Cramer's rule formula		1
	$x = 2, y = 1$		1+1
43	Equation of hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$		1
	$(h, k) = (-2, -4)$		1
	$ae = 4$		1
	$a^2 = 9$; $b^2 = 7$		1+1
	Equation of hyperbola: $\frac{(x+2)^2}{9} - \frac{(y+4)^2}{7} = 1$		1
44	Marginal cost $= \frac{dC}{dx}$		2
	$= 0.00015x^2 - 0.12x + 10$		2
	$MC_{x=1000} = 40$		2

45	$A = \pi r^2$	1
	$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ (or) $\frac{k}{r} = 2\pi \frac{dr}{dt}$ $\left[\because \frac{dA}{dt} \text{ is constant}\right]$	1
	$P = 2\pi r$	1
	$\frac{dP}{dt} = 2\pi \frac{dr}{dt}$	1
	$\frac{dP}{dt} = \frac{1}{r}\frac{dA}{dt}$ (or) $\frac{k}{r}$	1
	$\frac{dP}{dt} \propto \frac{1}{r}$	1
46	$f(x)$ (or) $y = x^3 + 8x^2 + 5x - 2$	1
	$f'(x)$ (or) $\frac{dy}{dx} = 3x^2 + 16x + 5$	1
	$= (3x+1)(x+5)$	1
	For $x \in (-\infty, -5)$, $f'(x) > 0$ (or) $+ve$	
	$\therefore f(x)$ *is increasing function in* $(-\infty, -5]$	1
	For $x \in \left(-5, -\frac{1}{3}\right)$, $f'(x) < 0$ (or) $-ve$	
	$\therefore f(x)$ *is decreasing function in* $\left[-5, -\frac{1}{3}\right]$	1
	For $x \in \left(-\frac{1}{3}, \infty\right)$, $f'(x) > 0$ (or) $+ve$	
	$\therefore f(x)$ *is increasing function in* $[-\frac{1}{3}, \infty)$	1
47	$C(x) = \int C'(x)\,dx + k$ (or) $\int_a^b C'(x)\,dx$	2
	$= \int_{15}^{25} \left(85 + \frac{375}{x^2}\right) dx$	2
	$= \left[85x - \frac{375}{x}\right]_{15}^{25}$	1
	$= 860$	1

48	$dC = (ax + b)\, dx$ $C = \frac{a}{2}x^2 + bx + k$ $C = C_0$ *and* $x = 0 \Rightarrow k = C_0$ $C = \frac{a}{2}x^2 + bx + C_0$	2 2 1 1
49	*Auxiliary equation* : $3m^2 + 7m - 6 = 0$ $\Rightarrow m = \frac{2}{3}\ ,\ -3$ $CF = Ae^{m_1x} + Be^{m_2x}$ $= Ae^{\frac{2}{3}x} + Be^{-3x}$ $y = Ae^{\frac{2}{3}x} + Be^{-3x}$	2 1 1 1 1
50	$x_0 = 0, x_1 = 1, x_2 = 3, x_3 = 4, x = 2$ $y_0 = 5, y_1 = 6, y_2 = 50, y_3 = 105$ $y = y_0\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} + y_2\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + y_3\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$ $= -0.8333 + 4.0 + 33.3333 - 17.5$ $y = 19$	1 2 2 1

51	$y = ax + b$, $\sum y = a\sum x + nb$ & $\sum xy = a\sum x^2 + b\sum x$ $\sum x = 18$; $\sum y = 15$; $\sum x^2 = 110$; $\sum xy = 71$ $a = 0.38$ & $b = 1.65$ (or) 1.632 $y = 0.38x + 1.65$ (or) $y = 0.38x + 1.632$	1+1 2 1 1
52	$np = 6$; $\sqrt{npq} = \sqrt{2}$; $npq = 2$ $q = \frac{1}{3}$; $p = \frac{2}{3}$; $n = 9$	1+1+1 1+1+1
53	$n = 50$; $\overline{X} = 67.9$; $S.E(\overline{X}) = \sqrt{0.7}$ $Z_c = 1.96$ $(It\ may\ present\ in\ formula\ substitution)$ $\overline{X} \pm (Z_c)\{S.E(\overline{X})\}$ (or) $\overline{X} \pm (Z_c)\frac{s}{\sqrt{n}}$ $= 67.9 \pm 1.64$ $\approx (66.26\ ,\ 69.54)$ (or) $(66.2\ ,\ 69.54)$	1 1 2 1 1
54	$r = \frac{N\sum XY - \sum X \sum Y}{\sqrt{N\sum X^2 - (\sum X)^2}\sqrt{N\sum Y^2 - (\sum Y)^2}}$ $= \frac{677}{\sqrt{754}\sqrt{4780}}$ $= 0.3566$	2 1+1+1 [Nr & Dr] 1
55	$P = \frac{p_1}{p_0} \times 100$ (It may appear in tabular column) $\sum V = 100$ $\sum PV = 15800$ $CLI = \frac{\sum PV}{\sum V} = 158$	1 1 2 1 + 1

PART - C		10×10=100
Q. NO.	KEY STEPS - ANSWER	STEPS MARKS
56	$adj\ A = \begin{pmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{pmatrix}$	3
	$A(adj\ A) = \begin{pmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{pmatrix}$ (or) $-11\ I$	2
	$(adj\ A)A = \begin{pmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{pmatrix}$ (or) $-11\ I$	2
	$\lvert A \rvert\ I = \begin{pmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{pmatrix}$ (or) $-11\ I$	2
	$A\ (adj\ A) = (adj\ A)\ A = \lvert A \rvert\ I$	1
57	$T = \begin{matrix} & S & C \\ S & 0.9 & 0.1 \\ C & 0.2 & 0.8 \end{matrix}$	2
	After one year: $= \begin{matrix} S & C \\ (0.5 & 0.5) \end{matrix} \begin{matrix} & S & C \\ S & 0.9 & 0.1 \\ C & 0.2 & 0.8 \end{matrix}$	2
	$= \begin{matrix} S & C \\ (0.55 & 0.45) \end{matrix}$ % commuters will be using the transit system after one year = 55%	1
	In a long run, [i.e. At equilibrium] $(S\quad C)\,T = (S\quad C)\quad where\ S + C = 1$ (or) $(S\quad C)\begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix} = (S\quad C)\quad where\ S + C = 1$	2
	$0.9S + 0.2C = S\ (or)\ 0.1S + 0.8C = C$	1
	$0.3S = 0.2$	1
	$S = 66.6\dot{7}\%\ (or)\ 67\%$ 67% of commuters will be using the transit system in a long run.	1

58	$7(x^2 - 2x) + 4(y^2 + 10y) = -79$ $\frac{(x-1)^2}{4} + \frac{(y+5)^2}{7} = 1$ $a^2 = 7$ and $b^2 = 4$ (i) Centre $(0, 0)$: $(1, -5)$ (ii) Vertices $(0, \pm a)$: $(1, -5 \pm \sqrt{7})$ (iii) Eccentricity $\sqrt{\frac{a^2 - b^2}{a^2}}$: $e = \sqrt{\frac{3}{7}}$ (iv) Foci $(0, \pm ae)$: $(1, -5 \pm \sqrt{3})$ (v) Directrices $Y = \pm \frac{a}{e}$: $y = -5 \pm \frac{7}{\sqrt{3}}$ (vi) Latus rectum $\frac{2b^2}{a}$: $\frac{8}{\sqrt{7}}$ Note : If the answers are wrong, award ½ marks to each correct formulae and then rounded off.	1 2 1 1 1 1 1 1 1
59	$Slope\ (m) = \frac{b \sec\theta}{a \tan\theta}$ Equation of Tangent: $(y - y_1) = m(x - x_1)$ $(y - b\tan\theta) = \frac{b\sec\theta}{a\tan\theta}(x - a\sec\theta)$ $\Rightarrow \frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$ Equation of Normal: $(y - y_1) = \frac{-1}{m}(x - x_1)$ $(y - b\tan\theta) = -\frac{a\tan\theta}{b\sec\theta}(x - a\sec\theta)$ $\Rightarrow \frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$	2 2 1 1 2 1 1

60	$\frac{dy}{dx}$ (or) $f'(x) = 15x^4 - 75x^2 + 60$	1
	$\frac{d^2y}{dx^2}$ (or) $f''(x) = 60x^3 - 150x$	1
	$\frac{dy}{dx}$ (or) $f'(x) = 0 \Rightarrow x = \pm 1, x = \pm 2$	1+1
	$x = -2, -1$ *and* $1 \in [-2, 1]$ (or) $2 \notin [-2, 1]$	1
	When $x = -2, f''(x) < 0$ (or) $-ve$, $f(x)$ *is maximum and maximum value* $= -15$	1
	When $x = -1, f''(x) > 0$ (or) $+ve$, $f(x)$ *is minimum and minimum value* $= -37$	1
	When $x = 1, f''(x) < 0$ (or) $-ve$, $f(x)$ *is maximum and maximum value* $= 39$	1
	Absolute (global) maximum value $= 39$	1
	Absolute (global) minimum value $= -37$	1
61	$\frac{\partial q_1}{\partial p_1} = -2p_1$	2
	$\frac{\partial q_1}{\partial p_2} = -3$	2
	$\frac{Eq_1}{Ep_1} = -\frac{p_1}{q_1}\frac{\partial q_1}{\partial p_1}$	2
	At $(3, 1)$, $\frac{Eq_1}{Ep_1} = 6$	1
	$\frac{Eq_1}{Ep_2} = -\frac{p_2}{q_1}\frac{\partial q_1}{\partial p_2}$	2
	At $(3, 1)$, $\frac{Eq_1}{Ep_2} = 1$	1

62	$I = \int_0^{\pi} x \sin^2 x \, dx$ ……. (1)	1
	Property : $\int_0^{a} f(x)\, dx = \int_0^{a} f(a-x)\, dx$	2
	$I = \int_0^{\pi} (\pi - x) \sin^2 x \, dx$ ……. (2)	1
	$(1) + (2) \Rightarrow 2I = \pi \int_0^{\pi} \sin^2 x \, dx$	1
	$= \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} \, dx$	2
	$= \frac{\pi}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$	1
	$2I = \frac{\pi^2}{2}$	1
	$I = \frac{\pi^2}{4}$	1
63	$p_d = p_s$	1
	$x_0 = 4$, $p_0 = 2$	1+1
	$p_0 x_0 = 8$ *(It may appear in formula substitution)*	1
	$C.S = \int_0^{x_0} f(x)\, dx - p_0 x_0$	2
	$C.S = (16 \log 2 - 8)$ *units*	1
	$P.S = p_0 x_0 - \int_0^{x_0} g(x)\, dx$	2
	$P.S = 4$ *units*	1

64	$\frac{dC}{dm}+\frac{2}{m}C=\frac{2}{m^2}$	1
	$P=\frac{2}{m}$ and $Q=\frac{2}{m^2}$	1
	$I.F=e^{\int P\,dm}=m^2$	2
	Solution is $C\,(I.F)=\int Q\,(I.F)\,dm+k$ (or) $C\,e^{\int P\,dm}=\int Q\,e^{\int P\,dm}\,dm+k$	2
	$C\,m^2=\int 2\,dm+k$	1
	$C\,m^2=2m+k$	1
	$C=4$ and $m=2\Rightarrow k=12$	1
	$\therefore C\,m^2=2m+12$ (or) $C\,m^2=2(m+6)$	1

65 By Gregory – Newton's forward formula : **2**

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

where $u = \frac{x - x_0}{h}$ **1**

$u = 3.6$ **1**

$y = 114.84 - 67.248 + 27.3312 - 4.59264 + 0.254592$ **1**

$= 70.59$ **1**

(Table is common to both methods)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	114.84	Forward difference table ↘			
	↑	-18.68			
45	96.16	↑	5.84		
	↑	-12.84	↑	-1.84	
50	83.32	↑	4.00	↑	0.68
	↑	-8.84	↑	-1.16	
55	74.48	↑	2.84		
	↑	-6.00			
60	68.48	Backward difference table ↗			
x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$

Value of Δ's (or) ∇'s

1+1+1+1

Aliter: By Gregory – Newton's backward formula : **2**

$$y = y_4 + \frac{u}{1!}\nabla y_4 + \frac{u(u+1)}{2!}\nabla^2 y_4 + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_4 + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_4$$

where $u = \frac{x - x_4}{h}$ **1**

$u = -0.4$ **1**

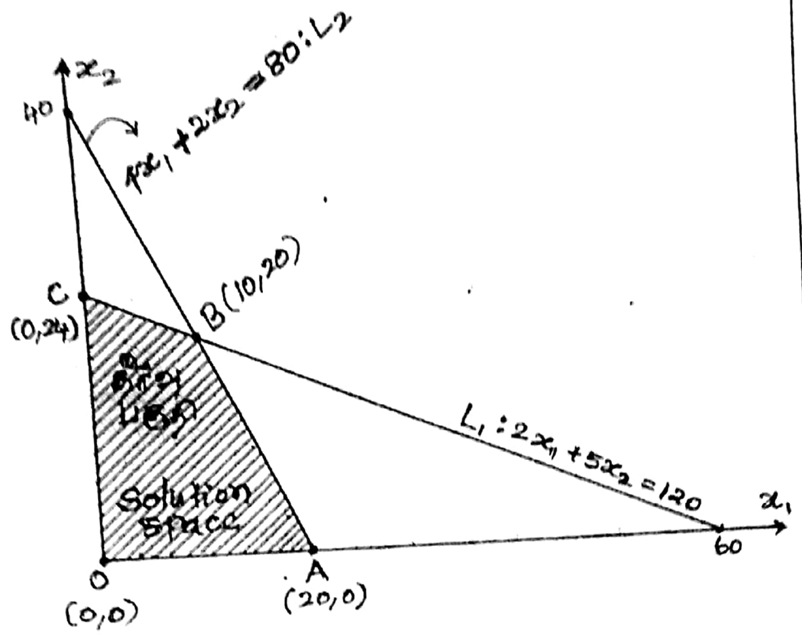
$y = 68.48 + 2.4 - 0.3408 + 0.07424 - 0.028288$ **1**

$= 70.59$ **1**

66	$\sum p(x_i) = 1$, $81a = 1$	1+2
	$a = \frac{1}{81}$ (or) $a = 0.012$	1
	$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$	1
	$= \frac{9}{81}$ (or) $\frac{1}{9}$ (or) 0.111	1
	$P(X > 3) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8)$	1
	$= \frac{65}{81}$ (or) 0.802	1
	$P(0 < X < 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$	1
	$= \frac{24}{81}$ (or) $\frac{8}{27}$ (or) 0.296	1
67	$E(X) = \int_{-\infty}^{\infty} x f(x) dx$	2
	$= \frac{1}{3}$ (or) 0.33	2
	$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$	1
	$= \frac{2}{9}$ (or) 0.22	2
	$var(X) = E(X^2) - [E(X)]^2$	2
	$= \frac{1}{9}$ (or) 0.11	1

68	$\overline{X} = 825, s\ (or)\ \sigma = 110, n = 50$	1
	Null Hypothesis $H_0 : \mu = 900$	1
	Alternate Hypothesis $H_1 : \mu \neq 900$	1
	$z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ (or) $\frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$	2
	$z = -4.82$	1
	$\lvert Z \rvert = 4.82 > 1.96$	1
	$\lvert Z \rvert$ *falls in the critical region*	1
	$\therefore$ *Null Hypothesis* (H_0) *is rejected.*	2

69	**In Graph sheet :**	
	Drawing the lines L_1 and L_2	2+2
	Shading the solution space OABC	1
	Graph:	



	$Z(0,0) = 0$	1
	$Z(20,0) = 60$	1
	$Z(10,20) = 110$	1
	$Z(0,24) = 96$	1
	Maximum of $Z = 110$ at $x_1 = 10$ *and* $x_2 = 20$	1

70	$\sum p_1 q_0 = 1900$	1
	$\sum p_0 q_0 = 1360$	1
	$\sum p_1 q_1 = 1880$	1
	$\sum p_0 q_1 = 1344$	1
	Fisher's Index Number $= \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times 100$	1
	$= 139.793$	1
	Time Reversal Test :	
	Proving the result $P_{01} \times P_{10} = 1$	2
	Factor Reversal Test :	
	Proving the result $P_{01} \times Q_{01} = \frac{1880}{1360} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$	2
	Note :	
	$P_{01} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}}$	
	$P_{10} = \sqrt{\frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}}$	
	$Q_{01} = \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}}$	