DEPARTMENT OF GOVERNMENT EXAMINATIONS, CHENNAI – 600 006

HIGHER SECONDARY EXAMINATION - MARCH 2018

MATHEMATICS KEY ANSWER SECOND YEAR - ENGLISH MEDIUM

GENERAL INSTRUCTIONS

1. The answers given in the marking scheme are TEXT BOOK, SOLUTION BOOK and COME BOOK bound.

2. If a student has given any answer which is different from one given in the marking scheme, but carries the prescribed content meaning (rigorous) such answers should be given full credit with suitable distribution.

3. Follow the foot notes which are given under certain answer schemes.

4. If a particular stage is wrong and if the candidate writes the appropriate formula then award 1 mark for the formula(for the stage mark 2*) and 2 marks (for the stage mark 3*). This mark (*) is attached with that stage. This is done with the aim that a student who did the problem correctly without writing the formula should not be penalized.

5. In the case of Part B and Part C , if the solution is correct then award full mar directly. The stage mark is essential only if the part of the solution is incorrect.

6. Answers written only in Black and Blue ink should be evaluated.

PART-A

A

1. One mark to write correct option and corresponding answer.
2. If one of them (option or answer) is wrong ,then award zero mark only .
3. Instead of 1,2,3,4 if one writes a,b,c,d then marks should be awarded .

Q. No	Option	Answer	Q.No	Option	Answer
1	1	2,1	21	4	1
2	4	1	22	3	$e^9, \frac{-3\pi}{4}$
3	3	$\frac{3\pi}{8}$	23	4	$x^2 + y^2 = a^2$
4	4	xoy plane	24	2	$8\sqrt{5}\pi$
5	1	(i), (iii),(iv)	25	4	2
6	3	no solution	26	2	$\log \frac{1}{k}$
7	3	$\vec{x} = \vec{0}$ or $\vec{y} = \vec{0}$ or $\vec{x}$ and $\vec{y}$ are $parallel$	27	3	$2^7 \lfloor 6$
8	2	3	28	1	-z
9	4	$\frac{1}{2}(x^4 + x^2 + 1)^{-\frac{1}{2}}(4x^3 + 2x)dx$	29	2	$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$
10	3	$-\tan x$	30	3	$\left(25, \frac{1}{5}\right)$
11	1	$xdx + ydy = 0$	31	2	$(\vec{r}.\vec{n_1} - q_1) + \lambda(\vec{r}.\vec{n_2} - q_2) = 0$
12	3	is a perpendicular bisector of the line joining z_1 and z_2	32	3	$x^2 - y^2 - 2xy = c$
13	3	60°	33	1	$a = \frac{1}{\lvert m \rvert}$
14	4	an asymptote parallel to y-axis	34	3	maximum height of the curve is $\frac{1}{\sqrt{2\pi}}$
15	3	has only trivial solution only if rank of the coefficient matrix is equal to the number of unknowns	35	1	$\frac{1}{k} I$
16	2	$\frac{9}{4}$	36	1	(Z , .)
17	2	$(6t^2, 8t)$	37	4	Fermat's theorem
18	1	-16	38	2	(4, 4) , (-4,-4)
19	3	if p and q are two statements then $p \leftrightarrow q$ is a tautology	39	1	6
20	3	9π	40	1	(1, 1, 2)

PART-A

B

1. one mark to write correct option and corresponding answer.
2. If one of them (option or answer) is wrong ,then award zero mark only .
3. Instead of 1,2,3,4 if one writes a,b,c,d then marks should be awarded .

Q. No	Option	Answer	Q. No	Option	Answer
1	3	maximum height of the curve is $\frac{1}{\sqrt{2\pi}}$	21	3	$\vec{x} = \vec{0}$ or $\vec{y} = \vec{0}$ or $\vec{x}$ and $\vec{y}$ are parallel
2	3	no solution	22	3	if p and q are two statements then $p \leftrightarrow q$ is a tautology
3	3	has only trivial solution only if rank of the coefficient matrix is equal to the number of unknowns	23	2	3
4	3	is a perpendicular bisector of the line joining z_1 and z_2	24	4	2
5	3	$x^2 - y^2 - 2xy = c$	25	3	9π
6	1	(Z , .)	26	1	$\frac{1}{k}$ I
7	1	(i), (iii), (iv)	27	2	$\log \frac{1}{k}$
8	2	(4, 4), (-4, -4)	28	4	1
9	3	$e^9, \frac{-3\pi}{4}$	29	1	$xdx + ydy = 0$
10	2	$(6t^2, 8t)$	30	1	-16
11	2	$\frac{9}{4}$	31	3	$-\tan x$
12	1	2,1	32	1	6
13	3	$2^7\lfloor 6$	33	4	1
14	1	(1, 1, 2)	34	4	$x^2 + y^2 = a^2$
15	4	$\frac{1}{2}(x^4 + x^2 + 1)^{-\frac{1}{2}}(4x^3 + 2x)dx$	35	4	Fermat's theorem
16	4	xoy plane	36	2	$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$
17	3	$\frac{3\pi}{8}$	37	4	an asymptote parallel to y-axis
18	2	$(\vec{r}.\vec{n_1} - q_1) + \lambda(\vec{r}.\vec{n_2} - q_2) = 0$	38	1	$a = \frac{1}{\lvert m \rvert}$
19	2	$8\sqrt{5}\pi$	39	3	$(25, \frac{1}{5})$
20	1	-z	40	3	60^0

Note: In an answer to a question, between any two particular stages of marks (greater than one) if a student starts from a stage with correct step but reaches the next stage with a wrong result then suitable credits should be given to the related steps instead of denying the entire marks meant for the stage.

	PART B	Mark
41	$\|A\| = -11$	1
	$adjA = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}$	1
	$A(adjA) = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$	1
	$(adjA)A = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$	1
	$\|A\|I = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$	1
	$A(adjA) = (adjA)A = \|A\| I$	1
42	$\Delta = 0$	2
	$\Delta x \neq 0$	2
	The system is inconsistent (or) no solution	2
43 i)	$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$	1
	They form a triangle	1
	To write, dot product between any two vectors = 0	1
	Note: One can take the vectors in a different way and solve the problem	
ii)	centre : $\left(\frac{3}{2}, 1, -1\right)$	2*
	the coordinate of B is (4, -2,1)	1

44	$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0 = 0$	2
	$P(z) = a_n z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0 = 0$	1
	$\overline{P(z)} = \overline{a_n z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0} = \bar{0}$	1
	$P(\bar{z})=0$	2
45	$x^4 = -4$	1
	$x = \sqrt{2}(\cos\pi + i\sin\pi)^{1/4}$	2
	$= \sqrt{2}\, cis\, (2k\pi + \pi)^{1/4}$	1
	$= \sqrt{2}\, cis\, (2k+1)\frac{\pi}{4}$, $k = 0,1,2,3$	2
46	Eqn. of the tangent is y=2x+1 or any other form	3*
	Eqn. of the normal is 2x+4y-9=0 or any other form	3*
	Note : This problem can be done by using either Cartesian form or parametric form.	

47	$\frac{dy}{dx} = 3x^2 - 3$	1
	$\frac{d^2y}{dx^2} = 6x$	1
	$\frac{d^2y}{dx^2} = 0 \Rightarrow x = 0$	1
	concave down or convex up in $(-\infty, 0)$	1
	concave up or convex down in $(0, \infty)$	1
	(0,1) is the point of inflection	1
48	The degree of the function is 0	3*
	$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$	3*
	Note : one can solve this problem either by using Euler's theorem or by direct differentiation method.	
49	$I_5 = \frac{1}{5}\cos^4x \sin x + \frac{4}{5}I_3$	3*
	$I_3 = \frac{1}{3}\cos^2x \sin x + \frac{2}{3}I_1$	1
	$I_1 = \sin x$	1
	$I_5 = \frac{1}{5}\cos^4x \sin x + \frac{4}{15}\cos^2x \sin x + \frac{8}{15}\sin x$	1
	Note : one can do this problem by substitution method also.	

Q.No.	Answer	Marks
50	$C.F = Ae^{-\frac{1}{2}x} + Be^{-2x}$	3*
	$P.I = \frac{1}{2D^2 + 5D + 2} e^{-\frac{1}{2}x}$	1
	$y = Ae^{-\frac{1}{2}x} + Be^{-2x} + \frac{1}{3} xe^{-\frac{1}{2}x}$	2*

51

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Each column or row - one mark — 4

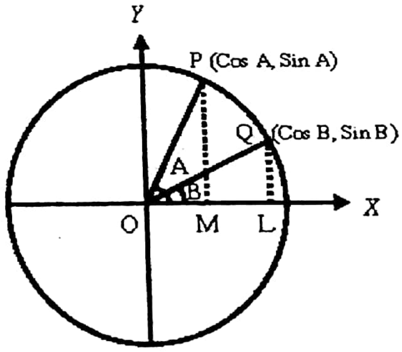
To write , $p \to q$ & $q \to p$ *are* not equivalent — 2

Note: 1) Instead of T & F one may use 0 and 1 (or) 1 and 0

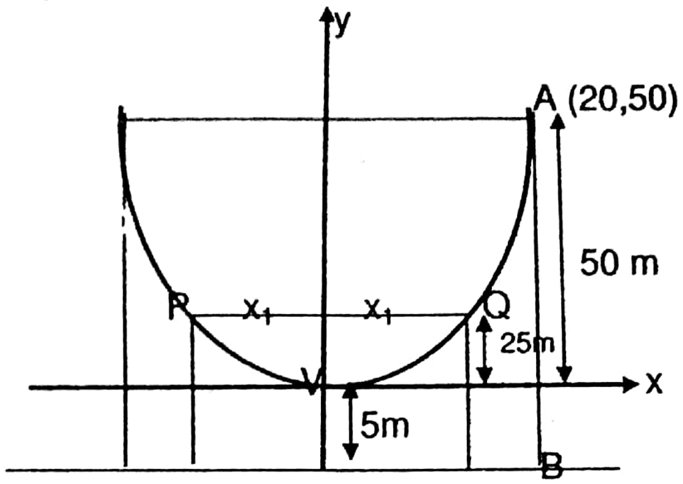
2) the order of rows and columns need not be same as in the scheme.

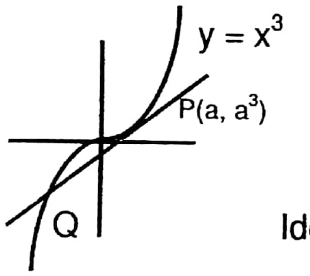
Q.No.	Answer	Marks
52 i)	$e_1 * e_2 = e_2$	1
	$e_1 * e_2 = e_1$	1
	$e_1 = e_2$	1
ii)	$a * a^{-1} = a^{-1} * a = e$	1
	$a^{-1} * (a^{-1})^{-1} = (a^{-1})^{-1} * a^{-1} = e$	1
	$a = (a^{-1})^{-1}$	1

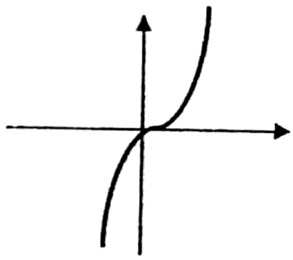
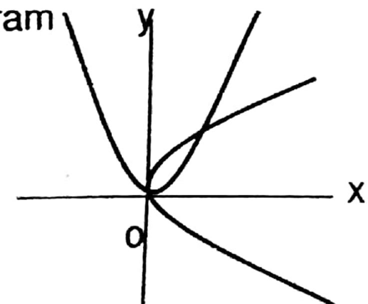
Q.No	Answer	Marks
53	Table	3
		x	2	3	4	5	6	7	8	9	10	11	12 | p(X=x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	
	$E(x) = \frac{252}{36}$ (or) 7	3*
54	$\mu = 64.5$, $\sigma = 3.3$	1
	$p(-\infty<z<c) = 0.99$ or any other form	1
	$c = 2.33$	2
	$x = 72.19$	2*
55 (a)	$\vec{a} = \overrightarrow{4i} - \overrightarrow{2j} - \overrightarrow{5k}$	1
	$\vec{n} = \overrightarrow{4i} - \overrightarrow{2j} - \overrightarrow{5k}$	1
	$\vec{r}.(\overrightarrow{4i} - \overrightarrow{2j} - \overrightarrow{5k}) = 45$	2*
	$4x - 2y - 5z = 45$ or any other form	2*
(b)	f(x) is continuous on [1,4]	1
	f(x) is differentiable on (1,4)	1
	$f'(x)$ or $f'(c) = \frac{f(4)-f(1)}{4-1}$	2*
	$f(4) \geq 16$	2

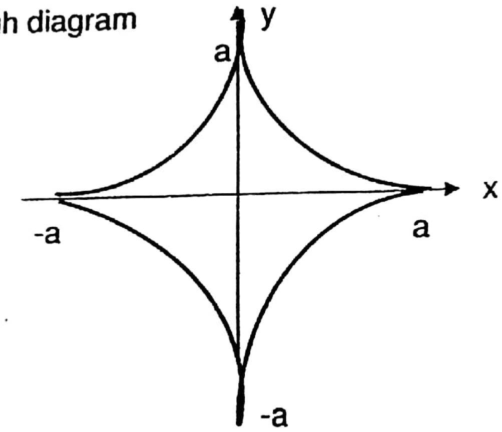
	PART C	Mark
56	$[A,B]=\begin{pmatrix}1 & 1 & 1 & 2\\ 2 & 1 & -2 & 2\\ \lambda & 1 & 4 & 2\end{pmatrix}$	2
	$\sim\begin{pmatrix}1 & 1 & 1 & 2\\ 0 & -1 & -4 & -2\\ 0 & 1-\lambda & 4-\lambda & 2-2\lambda\end{pmatrix}$ $\begin{matrix}R_1\\ R_2\to R_2-2R_1\\ R_3\to R_3-\lambda R_1\end{matrix}$	2
	$\sim\begin{pmatrix}1 & 1 & 1 & 2\\ 0 & -1 & -4 & -2\\ 0 & 0 & 3\lambda & 0\end{pmatrix}$ $R_3\to R_3-\lambda R_2$	2
	<u>case(i)</u> $\lambda=0$ $\rho(A)=\rho(A,B)=2$	1
	The system is consistent and has infinitely many solutions.	1
	<u>case(ii)</u> $\lambda\neq 0$ $\rho(A)=\rho(A,B)=3$	1
	The system is consistent and has unique solution **Note** : one may use different type of transformation to get the echelon form	1
57	Rough diagram  	3
	$\overrightarrow{OP}=cosA\vec{i}+\sin A\vec{j}$	2
	$\overrightarrow{OQ}=cosB\vec{i}+\ sinB\vec{j}$	2
	$\overrightarrow{OP}\,.\,\overrightarrow{OQ}=\ \cos AcosB+sinAsinB$	1
	$\overrightarrow{OP}\,.\,\overrightarrow{OQ}=\ \cos(A-B)$	1
	$\cos(A-B)=\cos AcosB+sinAsinB$	1

58		1
	$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$	1
	$\vec{b} = 2\vec{i} + 3\vec{j} + \vec{k}$	1
	$\vec{v} = 3\vec{i} - 2\vec{j} + 4\vec{k}$	
	Vector Equation $\vec{r} = (1-s)(\vec{i} + 2\vec{j} + 3\vec{k})$ + s$(2\vec{i} + 3\vec{j} + \vec{k})$+t$(3\vec{i} - 2\vec{j} + 4\vec{k})$ (or) $\vec{r} = (\vec{i} + 2\vec{j} + 3\vec{k})$ + s$(\vec{i} + \vec{j} - 2\vec{k})$+t$(3\vec{i} - 2\vec{j} + 4\vec{k})$	2*
	Cartesian Equation $\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 3 & -2 & 4 \end{vmatrix} = 0$	3*
	2y+z-7=0 or any other form	2
59	$z = x + iy$	1
	$\frac{z-1}{z+i} = \frac{(x-1)+iy}{x+i(y+1)}$	2
	$= \frac{(x-1)+iy}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$	2
	$\frac{x(x-1)+y(y+1)}{x^2+(y+1)^2} = 1$	3
	$x + y + 1 = 0$	2
	Note : one may simplify the result in different way.	

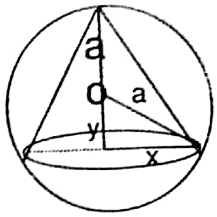
60	Rough Diagram 	3
	To write , $x^2 = 4ay$	1
	Identifying the Point (20,50)	1
	$x^2 = 8y$	2
	Identifying the point Q (x_1 , 25)	1
	$x_1 = 10\sqrt{2}$	1
	$PQ = 2\,x_1 = 20\sqrt{2}$ mts.	1
61	centre of hyperbola = (-1, -2)	2
	centre of ellipse = (-1,-2)	1
	$a^2 = 9$	1
	$b^2 = 1$	1
	Equation of ellipse is	
	$$\frac{(x+1)^2}{1} + \frac{(y+2)^2}{9} = 1$$	3*
	$$e = \frac{2\sqrt{2}}{3}$$	2*

62	other asymptote is $2x - y + k = 0$	2
	combined equation is $(x + 2y - 5)(2x - y + k) = 0$	2
	Equation of rectangular hyperbola is of the form	
	$(x + 2y - 5)(2x - y + k) + C = 0$	2
	$k = 4, \; c = -16$	1+1
	Equation of rectangular hyperbola is	
	$(x + 2y - 5)(2x - y + 4) - 16 = 0$	2
	Note: Name of the unknowns and the process may be different	
63	Rough diagram	3
		
	Identifying $P(a, a^3)$	1
	$\dfrac{dy}{dx} = 3x^2$	1
	Slope at P = $3a^2$	1
	Equation of tangent at P is $y - a^3 = 3a^2(x - a)$	2*
	$x = a$ (or) $x = -2a$	1
	Slope at Q = 4 (Slope at P)	1

64	**Domain, Extent , Intercepts, origin**	1
	Domain $(-\infty,\infty)$ Horizontal Extent $(-\infty,\infty)$ Vertical Extent $(-\infty,\infty)$	1
	x Intercept $=0$ y Intercept $=0$	1
	It passes through the origin	1
	Symmetry : it is symmetrical about the origin	1
	Asymptotes : the curve does not admit any asymptote	1
	Monotonicity: the curve is increasing in $(-\infty,\infty)$	
	Special points: Concave upward in $(0,\infty)$ Concave downward in $(-\infty,0)$ Point of inflection $(0,0)$ →	1
	Diagram 	3
65	Rough diagram 	3
	Point of intersection : (0,0) & (4,3)	2
	Required Area $=\int_0^4\left(\frac{3}{2}x^{1/2}-\frac{3}{16}x^2\right)dx$	3*
	Required Area $= 4.$	2
	Note : This problem can be solved by using y- axis also.	

66	rough diagram 	2
	$x = a\cos^3 t \ , y = a\sin^3 t$	2
	$\frac{dx}{dt} = -3a\cos^2 t \sin t$	1
	$\frac{dy}{dt} = 3a\sin^2 t \cos t$	1
	Length of the curve $= 4\int_0^{\frac{\pi}{2}} 3a\sin t \cos t \, dt$ (or) any other form	2*
	= 6a	2
	Note: One can use Cartesian form also	
67	$\frac{dy}{dx} = -\frac{x^3 + 3xy^2}{y^3 + 3x^2y}$	2
	put y = vx	1
	$\frac{dy}{dx} = v + x\frac{dv}{dx}$	1
	$x\frac{dv}{dx} = -\frac{v^4+6v^2+1}{v^3+3v}$	2
	$\frac{dx}{x} = -\frac{v^3+3v}{v^4+6v^2+1}dv$	2
	$y^4 + 6x^2y^2 + x^4 = c$	2
	Note : This problem can also be done in variable seperable method .	

68	$A = c\,e^{kt}$	3*
	c = 130000	1
	A = 130000 e^{kt}	1
	$e^{30k} = \frac{16}{13}$	2
	when t = 60, A = 130000 e^{60k}	2
	A = 197000	1
	Note: All approximate values (100,10,1) are also correct.	
69	i) $\lambda = 4.5$	1
	$P(X = 9) = \frac{e^{-4.5} \times (4.5)^9}{\lfloor 9}$	3*
	ii) $\lambda = 7.2$	1
	Required probability = $\sum_{x=0}^{9} \frac{e^{-7.2} \times (7.2)^x}{\lfloor x}$	2
	iii) $\lambda = 9.9$	1
	Required probability = $1 - \sum_{x=0}^{13} \frac{e^{-9.9} \times (9.9)^x}{\lfloor x}$	2

70 a	rough diagram 	2
	volume of cone v $=\frac{1}{3}\pi x^2 (a+y)$	2*
	$=\frac{1}{3}\pi(a^2-y^2)(a+y)$	1
	$v'=0 \Rightarrow y=\frac{a}{3}$	2
	when $y=\frac{a}{3}$, $v''<0$	2
	$v=\frac{8}{27}$ (volume of the sphere)	1
70 b	i) **closure axiom:**	1
	To prove closure axiom	2
	ii) **Associative axiom:**	1
	To write : addition modulo 'n' is always associative	1
	iii) **Identity axiom :**	1
	To write : identity element is [0]	1
	iv) **Inverse axiom :**	1
	To write : the inverse of $[l]$ is $[n-l]$	1
	$(Z_n, +_n)$ is a group	1