XII CBSE - BOARD - MARCH - 2017

CODE (65/1) SET -1

Date: 20.03.2017 **MATHEMATICS - SOLUTIONS**

SECTION - A

1. Given $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

we know that $A(\text{adj } A) = |A| \cdot I$

$$|A| \cdot I = 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow |A| = 8$

Topic:Matrices; Sub-Topic:Adjoint_L-1_XII-CBSE Board Exam-2017_Mathematics.

2. Given f(x) is continuous at x = 3

$$\therefore \lim_{x\to 3} f(x) = k$$

$$\lim_{x\to 3} \frac{(x+3)^2 - 36}{x-3} = k$$

$$\lim_{x\to 3} \frac{(x+3)^2 - 6^2}{x-3} = k$$

$$\lim_{x\to 3} \frac{(x+3+6)(x+3-6)}{x-3} = k$$

$3 + 3 + 6 = k$

$k = 12$

Topic:Continuity & Differentiability ; Sub-Topic:Continuity_L-1_XII-CBSE Board Exam-2017_Mathematics.

3. $\int \frac{\sin^2 x}{\sin x \cos x} dx - \int \frac{\cos^2 x}{\sin x \cos x} dx$

$= \int \tan x dx - \int \cot x dx$

$= \ln|\sec x| - \ln|\sin x| + C$

$= \ln\left|\frac{\sec x}{\sin x}\right| + C$

$= \ln\left|\frac{1}{\sin x \cos x}\right| + C$

$= \ln\left|\frac{2}{2\sin x \cos x}\right| + C$

$= \ln|2\operatorname{cosec} 2x| + C$

Topic:_Integrals_ Sub-Topic:_Indefinite Integration_L-1_XII-CBSE Board Exam-2017_Mathematics.

4. $\frac{2}{5} = -\frac{1}{-2.5} = \frac{2}{5}$

$\frac{2}{5} = \frac{2}{5} = \frac{2}{5}$

$\Rightarrow$ parallel planes.

$5x - \frac{5}{2}y + 5z = 20 \qquad \times\frac{2}{5}$

$\Rightarrow 2x - y + 2z = 8$

$\&\ 2x - y + 2z = 5$

$\Rightarrow d = \left|\frac{8-5}{\sqrt{4+1+4}}\right| = \frac{3}{\sqrt{9}} = 1\,unit$

Topic:3D Sub-Topic:_Plane_L-1_Target-2017_ XII-CBSE Board Exam_Mathematics

SECTION - B

5. If A is skew symmetric matrix then $A^T = -A$

$\therefore\ |A| = -|A^T|$

$|A| = -|A|$

$\Rightarrow\ 2|A| = 0$

$\Rightarrow\ |A| = 0$

Topic:Matrices; Sub-Topic:Skew symmetric_L-2_ _XII-CBSE Board Exam-2017_Mathematics.

6. $f(x)=x^3-3x$

(i) $f(-\sqrt{3})=(-\sqrt{3})^3-3(-\sqrt{3})=-3\sqrt{3}+3\sqrt{3}=0$

$f(0)=0$

Also $f(x)$ continuous in $[-\sqrt{3},0]$ & differentiable in $(-\sqrt{3},0)$

$f'(c)=0$

$\Rightarrow 3x^2-3=0$

$\therefore 3c^2-3=0$

$c^2=1$

$c=\pm 1$

$\Rightarrow c=-1$

Topic:_AOD_Sub-Topic:_Rolle's theorem_L-2_XII-CBSE Board Exam-2017_Mathematics.

7. $\frac{dv}{dt}=9cm^3/s$

$\frac{dA}{dt}=?$

$l=10\,cm$

$\frac{dv}{dt}=\frac{d}{dt}(l)^3=9\Rightarrow 3l^2\frac{dl}{dt}=9$

$\frac{dl}{dt}=\frac{3}{l^2}....(i)$

Now $\frac{dA}{dt}=\frac{d}{dt}(4l^2)=8l\frac{dl}{dt}=8l\times\frac{3}{l^2}\quad(from\ i)$

$=\frac{24}{l}=\frac{24}{10}=2.4cm^2/\sec$

Topic:_AOD_ Sub-Topic:_Rate of change L-1_Target-2017_XII-CBSE Board Exam-2017_Mathematics.

8. $f(x)=x^3-3x^2+6x-100$

$f'(x)=3x^2-6x+6$

$=3(x^2-2x+1)+3$

$=3(x-1)^2+3>0$

For all values of x, $(x-1)^2$ is always positve

$\therefore\ f'(x)>0$

So, $f(x)$ is increasing function.

Topic: AOD_ Sub-Topic:_Increasing & Decreasing_L-1_XII-CBSE Board Exam-2017_Mathematics.

9. Given $P(2,2,1)$ and $Q(5,1,-2)$

Let line divide PQ in the ratio $k:1$ and

given $x-$co-ordinate of point on the line is 4

so by section formula

$$x=\frac{5k+2}{k+1}$$

$$4=\frac{5k+2}{k+1}$$

$$k=2$$

Now, z-co-ordinate

$$z=\frac{-2k+1}{k+1}=\frac{-2\times 2+1}{2+1}=-\frac{3}{3}=-1$$

$$z=-1$$

z-co-ordinate = –1

Topic: 3D_Sub-Topic:Section formula_L-1_XII-CBSE Board Exam-2017_Mathematics.

10. S = {1, 2, 3, 4, 5, 6}

Let A : The number is even = {2, 4, 6}

$$\Rightarrow \text{P(A)} = \frac{3}{6} = \frac{1}{2}$$

B : The number in Red = {1, 2, 3}

$$\Rightarrow \text{P(B)} = \frac{3}{6} = \frac{1}{2}$$

and A $\cap$ B = {2}

$$\Rightarrow \text{P}(\text{A} \cap \text{B}) = \frac{1}{6}$$

$$\text{So P(A)} \cdot \text{P(B)} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

then $\text{P(A)} \cdot \text{P(B)} \neq \text{P}(\text{A} \cap \text{B})$

So A and B are not independent

Topic:Probability; Sub-Topic:Independent events_L-2_XII-CBSE Board Exam-2017_Mathematics.

11.

Tailors/Product	$A(x)$	$B(y)$	Avl
Shirts	6	10	60
Trousers	4	4	32

Let A work x days and B work y days.

$\therefore x \geq 0, y \geq 0$

So L.P.P., objective function Min. Z = 300 x + 400 y

Subject to.

$6x + 10y \geq 60$

$4x + 4y \geq 32$

$\therefore$ L.P.P

Min. Z = 300 x + 400 y

Subject to.

$3x + 5y \geq 30$

$x + y \geq 8$

$x \geq 0, y \geq 0$

Topic:L.P.P; Sub-Topic:Formulation_ L-1_XII-CBSE Board Exam-2017_Mathematics.

12. Let $I = \int \frac{dx}{5-8x-x^2} dx$

$$= \int \frac{dx}{5+16-16-8x-x^2} dx$$

$$= \int \frac{dx}{21-(x+4)^2} dx$$

$$= \int \frac{dx}{\left(\sqrt{21}\right)^2 - (x+4)^2} dx$$

$$= \frac{1}{2\times\sqrt{21}} \log\left|\frac{\sqrt{21}+x+4}{\sqrt{21}-x-4}\right| + C$$

$$= \frac{1}{2\sqrt{21}} \log\left|\frac{\sqrt{21}+4+x}{\sqrt{21}-4-x}\right| + C$$

Topic:Integrals; Sub-Topic:Indefinite_L-1_XII-CBSE Board Exam-2017_Mathematics.

SECTION - C

13. $\tan^{-1}\left(\frac{x-3}{x-4}\right) + \tan^{-1}\left(\frac{x+3}{x+4}\right) = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-3}{x-4}+\frac{x+3}{x+4}}{1-\frac{(x+3)(x-3)}{(x-4)(x+4)}}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{(x-3)(x+4)+(x+3)(x-4)}{(x^2-16)-(x^2-9)}\right] = \frac{\pi}{4}$$

$$\Rightarrow \frac{x^2+x-12+x^2-x-12}{x^2-16-x^2+9} = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-24}{-7}=1$$

$$2x^2-24=-7$$

$$2x^2=17$$

$$x^2=\frac{17}{2}$$

$$x=\pm\sqrt{\frac{17}{2}}$$

Topic: ITF_Sub-Topic:_ITF_L-2_XII-CBSE Board Exam-2017_Mathematics.

14. $$L.H.S.=\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$=\begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$=(a-1)\begin{vmatrix} a+1 & 1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$R_2 \to R_2 - R_3$

$$=(a-1)\begin{vmatrix} a+1 & 1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$$=(a-1)^2\begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

expanding along C_3

$$=(a-1)^2(a+1-2)=(a-1)^2(a-1)$$

$$=(a-1)^3=R.H.S.$$

Topic:Determinants_ Sub-Topic:_Properties_L-_2_XII-CBSE Board Exam-2017_Mathematics.

(OR)

14. Given

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

A should be 2×2 order

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Now

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

By solving and comparing, we get

$2a - c = -1 ....(i)$

$2b - d = -8 ....(ii)$

$a = 1 ....(iii)$

$b = -2 ......(iv)$

sub $a = 1$ in (i)

$c = 3$

and sub. $b = -2$ in (iii)

$d = 4$

so, A matrix

$$A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$$

Topic:Matrices_ Sub-Topic:_Multiplication_L-1_XII-CBSE Board Exam-2017_Mathematics.

15. Given that,

$$x^y + y^x = a^b$$

$$\Rightarrow \quad e^{y \log x} + e^{x \log y} = a^b$$

On differentiating both sides with respect to x, we get

$$\frac{d}{dx}\left(e^{y \log x}\right) + \frac{d}{dx}\left(e^{x \log y}\right) = \frac{d}{dx} a^b$$

$$\Rightarrow \quad e^{y \log x} \frac{d}{dx}(y \log x) + e^{x \log y} \frac{d}{dx}(x \log y) = 0$$

$$\Rightarrow \quad x^y \left\{ \frac{dy}{dx} \times \log x + y \times \frac{1}{x} \right\} + y^x \left\{ 1 \times \log y + x \times \frac{1}{y} \frac{dy}{dx} \right\} = 0$$

$$\Rightarrow \quad \left\{x^y \log x + y^x \frac{x}{y}\right\}\frac{dy}{dx} + \left\{x^y \times \frac{y}{x} + y^x \times \log y\right\} = 0$$

$$\Rightarrow \quad \left\{x^y \log x + xy^{x-1}\right\}\frac{dy}{dx} + \left\{y\,x^{y-1} + y^x \log y\right\} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left\{\frac{y\,x^{y-1} + y^x \log y}{x^y \log x + x\,y^{x-1}}\right\}$$

Topic:Continuity&Differentiability_Sub-Topic:Implicite_L-2_XII-CBSE Board Exam-2017_Mathematics.

(OR)

15. Given that

$e^y.(x+1)=1$

diff w.r.t x

$$e^y \frac{d}{dx}(x+1) + (x+1)\frac{d}{dx}e^y = \frac{d}{dx}1$$

$$e^y + (x+1)\,e^y\,\frac{dy}{dx} = 0$$

$$e^y\left[1 + (x+1)\frac{dy}{dx}\right] = 0$$

$$\therefore \; (x+1)\frac{dy}{dx} = -1$$

$$\frac{dy}{dx} = \frac{-1}{x+1} \qquad ...(1) \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{(x+1)^2}$$

Again diff.w.r.t. x equation (1)

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} \qquad ...(2)$$

$\therefore$ By equation (1)&(2)

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

Topic:Continuity&Differentiability_Sub-Topic:Implicite_L-2_XII-CBSE Board Exam-2017_Mathematics.

16. $I=\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}d\theta$

$=\int \frac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)}d\theta$

Let $\sin\theta=t \Rightarrow \cos\theta\, d\theta=dt$

$\therefore I=\int \frac{1}{(4+t^2)(1+4t^2)}dt$

Consider

$\frac{1}{(4+t^2)(1+4t^2)}=\frac{At+B}{4+t^2}+\frac{Ct+D}{1+4t^2}$

$1=(At+B)(1+4t^2)+(Ct+D)(4+t^2)$

$=At+B+4At^3+4Bt^2+4Ct+Ct^3+4D+Dt^2$

$=(4A+C)t^3+(4B+D)t^2+(A+4C)t+(B+4D)$

$4A+C=0 \Rightarrow C=-4A$

$4B+D=0 \Rightarrow D=-4B$

$A+4C=0 \Rightarrow A=-4C$

$B+4D=1$

By solving we get $A=0,\ B=-\frac{1}{15},\ C=0,\ D=\frac{4}{15}$

$\therefore \frac{1}{(4+t^2)(1+4t^2)}=\frac{-1/15}{4+t^2}+\frac{4/15}{1+4t^2}$

$\therefore I=-\frac{1}{15}\int\frac{1}{4+t^2}dt+\frac{4}{15}\times\frac{1}{4}\int\frac{1}{\frac{1}{4}+t^2}dt$

$=-\frac{1}{15}\times\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+\frac{1}{15}\times\frac{1}{\frac{1}{2}}\tan^{-1}\left(\frac{t}{1/2}\right)+C$

$=-\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right)+\frac{2}{15}\tan^{-1}(2t)+C$

$=\frac{2}{15}\tan^{-1}(2\sin\theta)-\frac{1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right)+C$

Topic:Integrals; Sub-Topic:Indefinite_L-3_XII-CBSE Board Exam-2017_Mathematics.

17. $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

$$= \int_0^{\pi} \frac{x \frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx$$

$$I = \int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx \qquad \text{...(i)}$$

By using the property $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$$= \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \sin(\pi - x)} dx$$

$$I = \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \sin x} dx \qquad \text{...(ii)}$$

Adding (i) and (ii)

$$\therefore 2I = \int_0^{\pi} \pi \frac{\sin x}{1 + \sin x} dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \sin x} dx$$

$$= \frac{\pi}{2} \int_0^{\pi} \left(1 - \frac{1}{1 + \sin x}\right) dx$$

$$= \frac{\pi}{2} \left[\int_0^{\pi} dx - \int_0^{\pi} \frac{1}{1 + \sin x} dx\right]$$

$$= \frac{\pi}{2} \left[|x|_0^{\pi} - \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx\right]$$

$$= \frac{\pi}{2} \left[\pi - (\tan x - \sec x)_0^{\pi}\right]$$

$$= \frac{\pi}{2} [\pi - [(0-(-1)-(0-1)]]$$

$$I = \frac{\pi}{2} [\pi - 2]$$

Topic: Integrals; Sub-Topic: Definite_L-2_XII-CBSE Board Exam-2017_Mathematics.

(OR)

17. $I=\int_1^4 \left(|x-1|+|x-2|+|x-4|\right)dx$

Let $f(x)=|x-1|+|x-2|+|x-4|$

We have three critical points $x=1,2,4$

(i) when $x<1$

(ii) when $1\le x<2$

(iii) when $2\le x<4$

(iv) when $x\ge 4$

$$\begin{aligned} f(x) &= -(x-1)-(x-2)-(x-4) && \text{if} \quad x<1 \\ &= (x-1)-(x-2)-(x-4) && \text{if} \quad 1\le x<2 \\ &= (x-1)+(x-2)-(x-4) && \text{if} \quad 2\le x<4 \\ &= (x-1)+(x-2)+(x-4) && \text{if} \quad x\ge 4 \end{aligned}$$

$$\begin{aligned} \therefore\ f(x) &= -3x+7 && \text{if} \quad x<1 \\ &= -x+5 && \text{if} \quad 1\le x<2 \\ &= x+1 && \text{if} \quad 2\le x<4 \\ &= 3x-7 && \text{if} \quad x\ge 4 \end{aligned}$$

$$\therefore\ I=\int_1^4 f(x)\,dx$$

$$=\int_1^2 f(x)\,dx+\int_2^4 f(x)dx$$

$$\therefore\ I=\int_1^2(-x+5)\,dx+\int_2^4(x+1)\,dx$$

$$=\left[-\frac{x^2}{2}+5x\right]_1^2+\left[\frac{x^2}{2}+x\right]_2^4$$

$$=\left(-\frac{4}{2}+10\right)-\left(-\frac{1}{2}+5\right)+\left(\frac{16}{2}+4\right)-\left(\frac{4}{2}+2\right)$$

$$=8-\frac{9}{2}+12-4$$

$$=\frac{23}{2}$$

Topic:Integrals; Sub-Topic:Definite_L-3_XII-CBSE Board Exam-2017_Mathematics.

18. $\frac{dy}{dx}=\frac{\left(\tan^{-1}x-y\right)}{1+x^2}$

$\frac{dy}{dx}+\frac{y}{1+x^2}=\frac{\tan^{-1}x}{1+x^2}$

$P=\frac{1}{1+x^2}\qquad Q=\frac{\tan^{-1}x}{1+x^2}$

$IF = e^{\int Pdx}$

$= e^{\int\frac{1}{1+x^2}dx}=e^{\tan^{-1}x}$

Solution of D.F.

$y.(IF)=\int Q(IF)\,dx$

$y\,e^{\tan^{-1}x}=\int\frac{\tan^{-1}x}{1+x^2}.e^{\tan^{-1}x}dx$

Let $\tan^{-1}x=t$

$\frac{dx}{1+x^2}=dt$

$\therefore\ y\,e^{\tan^{-1}x}=\int t.e^t\,dt$

$\therefore\ y\,e^{\tan^{-1}x}=\int t\,e^t\,dt$

$=t\int e^t\,dt-\int\left(\frac{d}{dt}t\int e^t dt\right)dt$

$=t\,e^t-e^t+C$

$=e^t\,(t-1)+C$

$y\,e^{\tan^{-1}x}=e^{\tan^{-1}x}(\tan^{-1}x-1)+C$

$y=\tan^{-1}x-1+C\,e^{-\tan^{-1}x}$

$\therefore y=\tan^{-1}x-1+C.e^{-\tan^{-1}x}$

Topic:Differential equation; Sub-Topic:LDE_ L-2_XII-CBSE Board Exam-2017_Mathematics.

19. Given

$\vec{a}=2\hat{i}-\hat{j}+\hat{k}$

$\vec{b}=\hat{i}-3\hat{j}-5\hat{k}$

$\vec{c}=3\hat{i}-4\hat{j}-4\hat{k}$

then

$\overrightarrow{AB}=\vec{b}-\vec{a}$

$\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$

$\overrightarrow{AC} = \vec{c} - \vec{a} = \hat{i} - 3\hat{j} - 5\hat{k}$

and $\overrightarrow{BC} = \vec{c} - \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

Now angle between $\overrightarrow{AC}$ and $\overrightarrow{BC}$

$$\cos\theta = \frac{(\overrightarrow{AC})\cdot(\overrightarrow{BC})}{|AC|^2|\overrightarrow{BC}|} = \frac{2+3-5}{\sqrt{1+9+25}.\sqrt{4+1+1}}$$

$\cos\theta = 0 \Rightarrow BC \perp AC$

So, A, B, C are vertices of right angle triangle.

Now area of ΔABC

$$= \frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BC}\right|$$

$$= \frac{1}{2}\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -5 \\ 2 & -1 & 1 \end{matrix}\right\| = \frac{1}{2}\left|8\hat{i} - 11\hat{j} + 5\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{64+121+25} = \frac{\sqrt{210}}{2} \text{ sq unit.}$$

Topic:Vectors; Sub-Topic:Product of vectors_L-3_XII-CBSE Board Exam-2017_Mathematics.

20. Let

$A:(3,6,9)$

$B:(1,2,3)$

$C:(2,3,1)$

$D:(4,6,\lambda)$

$[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

DR of $\overrightarrow{AB} \equiv -2, -4, -6$

DR of $\overrightarrow{AC} \equiv -1, -3, -8$

DR of $\overrightarrow{AD} \equiv 1, 0, \lambda - 9$

$$\therefore \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & \lambda-9 \end{vmatrix} = 0 \qquad \Rightarrow \begin{vmatrix} 2 & 4 & 6 \\ 1 & 3 & 8 \\ 1 & 0 & \lambda-9 \end{vmatrix} = 0$$

$1(3\lambda - 27) - 4(\lambda - 9 - 8) + 6(0-3) = 0$

$3\lambda - 27 - 4\lambda + 68 - 18 = 0$

$-\lambda + 23 = 0 \Rightarrow \lambda = 23$

Topic:Vectors; Sub-Topic:Triple product_L-2_XII-CBSE Board Exam-2017_Mathematics.

21. X can be 4, 6, 8, 10, 12

P(X = 4) = {1, 3} or {3, 1}

$$= \left(\frac{1}{4}\times\frac{1}{3}\right)+\left(\frac{1}{4}\times\frac{1}{3}\right)=\frac{2}{12}=\frac{1}{6}$$

P(X = 6) = {1, 5} or {5, 1}

$$= \frac{1}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{3}=\frac{1}{6}$$

P(X = 8) = {3, 5} or {5, 3} or {1, 7} or {7, 1}

$$= \frac{1}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{3}$$

$$=\frac{4}{12}=\frac{1}{3}$$

P(X = 10) = {3, 7} or {7, 3}

$$=\frac{1}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{3}=\frac{2}{12}=\frac{1}{6}$$

P(X = 12) = {5, 7} or {7, 5}

$$=\frac{1}{6}$$

X	$P(X)$	$XP(X)$	X^2	$X^2P(X)$
4	1/6	4/6	16	16/6
6	1/6	1	36	6
8	1/3	8/3	64	64/3
10	1/6	10/6	100	100/6
12	1/6	2	144	144/6

$\therefore$ Mean = E(X) = $\sum xP(x)$

$$= \frac{4}{6}+1+\frac{8}{3}+\frac{10}{6}+2$$

$$= \frac{2}{3}+1+\frac{8}{3}+\frac{5}{3}+2$$

$$= \frac{2+3+8+5+6}{3}$$

$$= \frac{25}{3}=8.33$$

Variance $= E\left(X^2\right)-\left[E\left(X\right)\right]^2$

$$= \sum x^2 P(x) - \left(\frac{25}{3}\right)^2$$

$$= \frac{16}{6} + 6 + \frac{64}{3} + \frac{100}{6} + \frac{144}{6} - \left(\frac{25}{3}\right)^2$$

$$= \frac{8}{3} + 6 + \frac{64}{3} + \frac{50}{3} + \frac{72}{3} - \frac{625}{9}$$

$$= \frac{24 + 54 + 192 + 150 + 216 - 625}{9}$$

$$= \frac{11}{9} = 1.22$$

Topic:Probability; Sub-Topic:Distribution_L-3_XII-CBSE Board Exam-2017_Mathematics.

22. Let E_1 is event of students which have 100% attendance and E_2 is event of students which are Irregular.

then $P(E_1) = 0.3$

$P(E_2) = 0.7$

Let A : Event of students which attendance A grade.

then $P(A/E_1) = 0.7$

and $P(A/E_2) = 0.1$

So By Bays theorem

P(Probability that student has 100% Attendance)

$$= P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2).P(A/E_2)}$$

$$= \frac{0.3 \times 0.7}{0.3 \times 0.7 + 0.7 \times 0.1}$$

$$= \frac{0.3 \times 0.7}{0.7\,(0.3 + 0.1)} = \frac{0.3}{0.4}$$

$$= \frac{3}{4} = 0.75$$

As per answer, the probability of regular students is more than 50%. So the regularty is required. (Also answer may be different according to student logic.)

Topic:Probability; Sub-Topic:Bays theorem_L-3_XII-CBSE Board Exam-2017_Mathematics.

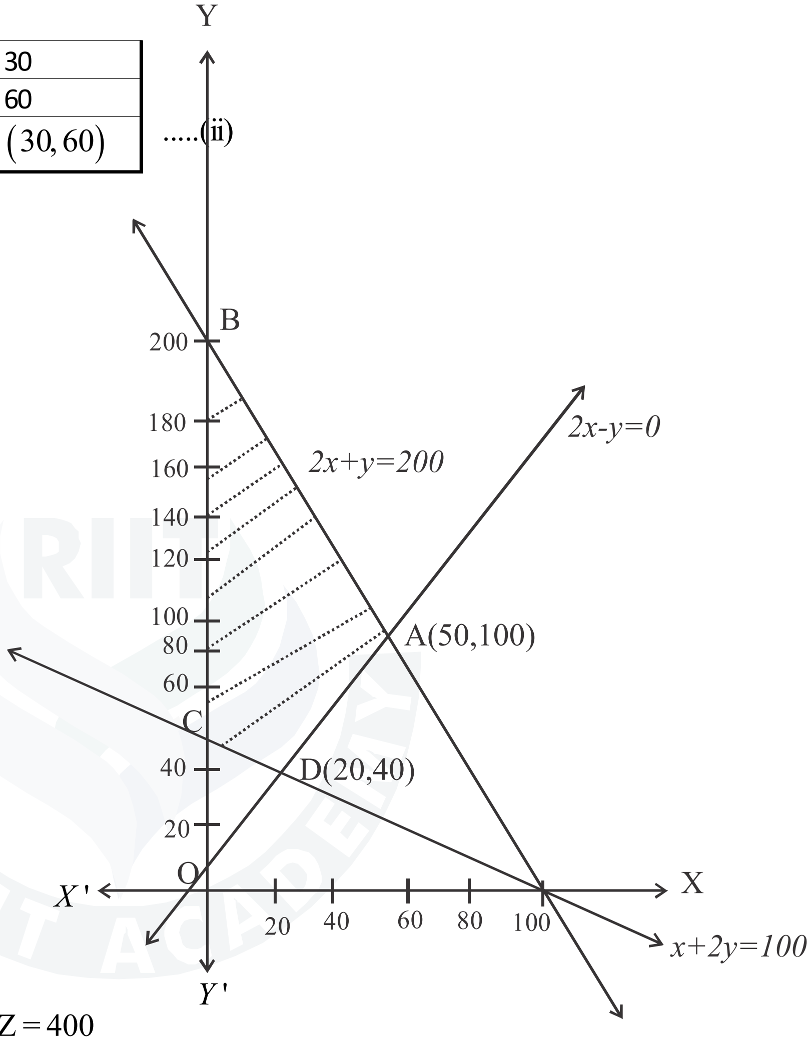
23. $x+2y\ge 100$

$\frac{x}{100}+\frac{y}{50}\ge 1.....(i)$

$2x-y\le 0$

x	10	20	30
y	20	40	60
(x,y)	$(10,20)$	$(20,40)$	$(30,60)$

.....(ii)



$2x+y\le 200$

$\frac{x}{100}+\frac{y}{200}\le 1.....(iii)$

$A(50,100)$

$B(0,200)$

$C(0,50)$

$D(20,40)$

$\therefore Z=x+2y$

$Z_A=50+2(100)=250$

$Z_B=0+2(200)=400$

$Z_C=0+2(50)=100$

$\therefore Z_D=20+2(40)=100$

$\therefore$ Z is maximum at $(0,200)$ & max $Z=400$

Topic:LPP_Sub-Topic:Graphical solution_L-2_XII-CBSE Board Exam-2017_Mathematics.

SECTION - D

24. **Bonus (Wrong printing)**

$$\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-4+4+8 & 4-8+4 & -4-8+12\\ -7+1+6 & 7-2+3 & -7-2+9\\ 5-3-2 & -5+6-1 & 5+6-3\end{bmatrix}$$

$$=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix} \qquad =8I$$

$$If\ \ A=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$$

$$\text{then } A^{-1}=\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$$

$$\therefore\ \ X=A^{-1}B=\frac{1}{8}\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}\begin{bmatrix}4\\ 9\\ 1\end{bmatrix}$$

$$X=\frac{1}{8}\begin{bmatrix}-16+36+4\\ -28+9+3\\ 20-27-1\end{bmatrix}=\frac{1}{8}\begin{bmatrix}24\\ -16\\ -8\end{bmatrix}=\begin{bmatrix}3\\ -2\\ -1\end{bmatrix}$$

$$\Rightarrow x=3, y=-2, z=-1$$

***Topic:Matrices_ Sub-Topic:_Applications_L-2*_XII-CBSE Board Exam-2017_Mathematics.**

25. $f(x)=\dfrac{4x+3}{3x+4}$

Assume its not one – one

$\therefore x_1\ \&\ x_2$ belonging to domain such that $f(x_1)=f(x_2)$

$$\Rightarrow \frac{4x_1+3}{3x_1+4}=\frac{4x_2+3}{3x_2+4}$$

$$12x_1x_2+16x_1+9x_2+12=12x_1x_2+9x_1+16x_2+12$$

$$7x_1=7x_2$$

$$\Rightarrow x_1=x_2$$

$\Rightarrow$ Assumption is wrong.

$\therefore\ \ f(x)$ is one – one

Now, $y=\dfrac{4x+3}{3x+4}$

$$3xy+4y=4x+3$$

$$x(3y-4)=3-4y$$

$$x=\frac{3-4y}{3y-4}$$

for every 'y' there is a x.

$\therefore$ it is onto

$f^{-1}(x)=\frac{3-4y}{3y-4}$

Now, $f^{-1}(0)=\frac{3-0}{0-4}=-\frac{3}{4}$

Now, $f^{-1}(x)=2 \qquad \Rightarrow \frac{3-4y}{3y-4}=2$

$\Rightarrow 3-4y=6y-8$

$3+8=6y+4y$

$11=10y$

$y=\frac{11}{10}$

***Topic:Relation & Function_ Sub-Topic:Inverse of function_L-2_*XII-CBSE Board Exam-2017_Mathematics.**

(OR)

25. (a, b) * (c, d) = (ac, b + ad)
(c, d) * (a, b) = (ca, d + cf)
Not commutative
(a, b) * [(c, d) * (e, f)]
= (a, b) * [ce, d + cf]
= [ace, b + ad + acf]
Now,
[(a, b) * (c, d)] * (e, f)
= [ac, b + ad] * (e, f)
= [ace, b + ad + acf]

$\therefore$ Associative

$\because$ $(a, b) * [(c, d) * (e, f)] = [(a, b)] * [(c, d) * (e, f)]$

(i) (a, b) * e = (a, b)

$\Rightarrow$ a = ac

$\Rightarrow$ c = 1 & b = b + ad $\Rightarrow$ ad = 0

$\Rightarrow$ d = 0

$\therefore$ $(a, b) * (1, 0) = (a, b + a \times 0) = (a, b)$

$\Rightarrow$ $(1, 0)$ is identify.

(ii) (a, b) * (c, d) = e = (1, 0)

$\Rightarrow$ ac = 1 & b + ad = 0

$\Rightarrow d = \frac{-b}{a}$

$\therefore$ Inverse of element

$\therefore$ Inverse of element of (a, b) is $\left(\frac{1}{a}, \frac{-b}{a}\right)$

***Topic:Relation & Function_ Sub-Topic:Binary Operation_L-3_*XII-CBSE Board Exam-2017_Mathematics.**

26. Let V be the fixed volume of a closed cuboid length x, breadth x and height y.
Let S be the surface area of the cuboid. Then,

$V = x^2y$ .....(i)

and, $S = 2(x^2 + xy + xy) = 2x^2 + 4xy$ .....(ii)

Now, $S = 2x^2 + 4xy$

$$\Rightarrow S = 2x^2 + 4x\left(\frac{V}{x^2}\right) \qquad \left[\because V = x^2y \therefore y = \frac{V}{x^2}\right]$$

$$\Rightarrow S = 2x^2 + \frac{4V}{x}$$

$$\Rightarrow \frac{dS}{dx} = 4x - \frac{4V}{x^2} \qquad .....(iii)$$

The critical numbers of S are given by $\frac{dS}{dx} = 0$.

$$\therefore \frac{dS}{dx} = 0$$

$$\Rightarrow 4x - \frac{4V}{x^2} = 0$$

$$\Rightarrow V = x^3$$

$$\Rightarrow x^2y = x^3 \qquad \left[\because V = x^2y\right]$$

$$\Rightarrow x = y$$

Differentiating (iii) with respect to x, we get

$$\frac{d^2S}{dx^2} = 4 + \frac{8V}{x^3} = 4 + \frac{8x^2y}{x^3} = 4 + \frac{8y}{x}$$

$$\Rightarrow \left(\frac{d^2S}{dx^2}\right)_{y=x} = 12 > 0.$$

Hence, S is minimum when length = x, breadth = x and height = x i.e., when it is a cube.

Topic:AOD; Sub-Topic:Maxima & Minima_L-3_XII-CBSE Board Exam-2017_Mathematics.

27. Eq. of AB

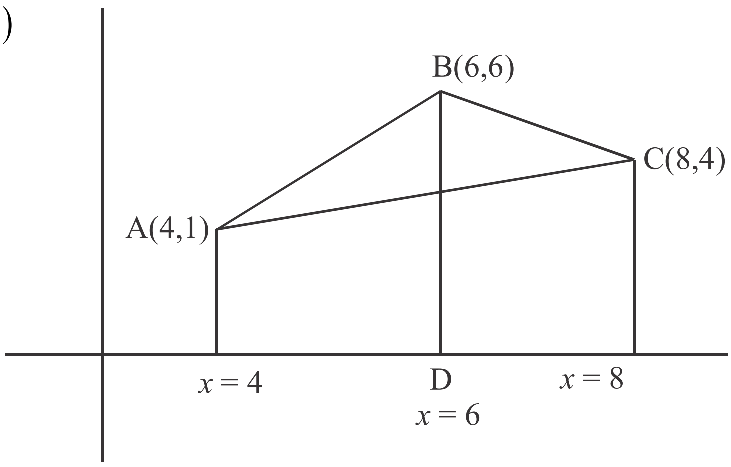
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{6-1}{6-4}(x - 4)$$

$$y - 1 = \frac{5}{2}(x - 4)$$

$$2y - 2 = 5x - 20$$

$$y = \frac{5x}{2} - 9$$



Eq. of BC

$y-6=\frac{4-6}{8-6}(x-6)$

$y-6=\frac{-2}{+2}(x-6)$

$y-6=-x+6$

$y=-x+12$

Eq. of AC

$y-1=\frac{4-1}{8-4}(x-4)$

$y-1=\frac{3}{4}(x-4)$

$4y-4=3x-12$

$y=\frac{3x}{4}-2$

$\therefore$ Area of $\Delta ABC=\int_4^6\left(\frac{5x}{2}-9\right)dx+\int_6^8(-x+12)dx$

$-\int_4^8\left(\frac{3}{4}x-2\right)dx$

$=\left[\frac{5x^2}{4}-9x\right]_4^6+\left[-\frac{x^2}{2}+12x\right]_6^8-\left[\frac{3x^2}{8}-2x\right]_4^8$

$=\left(\frac{5(36)}{4}-54\right)-\left(\frac{5(16)}{4}-36\right)+\left(\frac{64}{7}+96\right)-\left(-\frac{36}{2}+72\right)-\left(-\frac{24}{8}-16\right)+\left(\frac{34}{8}-8\right)$

$=(-9)+16+64-54+8-2$

$=7+10-10$

$=7 sq.units$

Topic:AOI_ Sub-Topic:AUC_L-2_XII-CBSE Board Exam-2017_Mathematics.

(OR)

27. The given quations are

$y=\frac{3x^2}{4}$ ....(1)

and, $3x-2y+12=0$ ....(2)

Solving equation no. (1) & (2)

$y=\frac{3x+12}{2}$

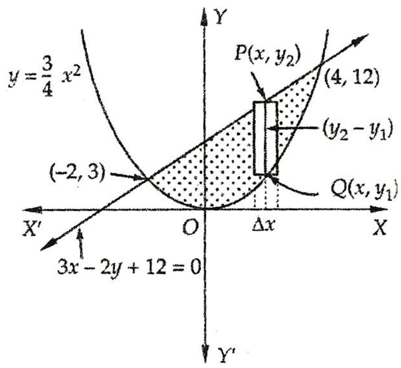
$\therefore \quad \frac{3x^2}{4} = \frac{3x+12}{2}$

$\therefore$ x = –2, 4

$\therefore$ y = 3, 12

$\therefore$ intersection point are $(-2,\ 3)$ & $(4,\ 12)$

$\therefore$ Require area is



$$A = \int_{-2}^{4} (y_2 - y_1)\, dx$$

$$A = \int_{-2}^{4} \left(\frac{3x+12}{2} - \frac{3}{4}x^2 \right) dx$$

$$\Rightarrow A = \left[\frac{3}{4}x^2 + 6x - \frac{x^3}{4} \right]_{-2}^{4}$$

$\Rightarrow A = 27$ sq. units

Topic:AOI_ Sub-Topic:AUC_L-2_XII-CBSE Board Exam-2017_Mathematics.

28. $\frac{dy}{dx} = \frac{x+2y}{x-y}$

Putting y = Vx

$$\frac{dy}{dx} = V + x\frac{dV}{dx}$$

$$\therefore \quad V + x\frac{dV}{dx} = \frac{x+2Vx}{x-Vx} = \frac{1+2V}{1-V}$$

$$\therefore x\frac{dV}{dx} = \frac{1+2V}{1-V} - V = \frac{1+2V-V+V^2}{1-V}$$

$$\therefore \int \frac{1-V}{V^2+V+1}dV = \int \frac{1}{x}dx$$

$$\therefore \int \frac{2-2V}{V^2+V+1}dV = 2\log|x| + c$$

$$\therefore \int \frac{3-(2V+1)}{V^2+V+1}dV = 2\log|x| + c$$

$$\therefore \int \frac{3}{V^2+V+1}dV - \int \frac{2V+1}{V^2+V+1}dV = \log|x|^2 + c$$

$$\therefore 3\int \frac{1}{\left(V+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dV - \log|V^2+V+1| = \log\left|x^2\right| + c$$

$$\therefore\ 3\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left(\frac{V+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)=\log|x^2\left(V^2+V+1\right)|+c$$

$$2\sqrt{3}\tan^{-1}\left(\frac{2V+1}{\sqrt{3}}\right)=\log|x^2\left(V^2+V+1\right)|+c$$

Putting $V=\frac{y}{x}$

$$2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)=\log\left|x^2\frac{\left(y^2+yx+x^2\right)}{x^2}\right|+c$$

Putting $y=0\quad x=1$

$$2\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\log|1|+c$$

$$C=2\sqrt{3}\cdot\frac{\pi}{6}=\frac{\pi}{\sqrt{3}}$$

$$\therefore\ 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)=\log\left|y^2+xy+x^2\right|+\frac{\pi}{\sqrt{3}}$$

$$6\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right)=\sqrt{3}\log\left(x^2+xy+y^2\right)+\pi$$

Topic:Differential equation; Sub-Topic:homogenous_L-3_XII-CBSE Board Exam-2017_Mathematics.

29. Equation of the line passing through $(3,-4,-5)$ and $(2,-3,1)$ is

$$\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=\lambda \qquad ...(1)$$

Let $x=-\lambda+3,\ y=\lambda-4,\ z=6\lambda-5$

Now equation of the plane passing through given three points $(1,2,3)$, $(4,2,-3)$ and $(0,4,3)$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix}=0$$

$\Rightarrow (x-1)(12)+(y-2)6+(z-3)6=0$

$12x-12+6y-12+6z-18=0$

$12x+6y+6z-42=0$

$2x+y+z-7=0$

$2(-\lambda+3)+1(\lambda-4)+(6\lambda-5)-7=0$

$-2\lambda+6+\lambda-4+6\lambda-5-7=0$

$5\lambda=10\Rightarrow\lambda=2$

$\therefore\ x=-2+3,\ \ y=2-4,\ \ z=12-5$

$\therefore\ x=1,\ \ y=-2,\ z=7$

$\therefore$ Intersection point is $(1,-2,7)$

Topic:3D; Sub-Topic:Plane_L-3_XII-CBSE Board Exam-2017_Mathematics.

(OR)

29. Let the equation of the plane be

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

where, a, b, c are variables.
This meets X, Y and Z axes at A(a, 0, 0), B(0, b, 0) and C(0, 0, c).

Let (α,β,γ) be the coordinates of the centroid of triangle ABC. Then,

$$\alpha=\frac{a+0+0}{3}=\frac{a}{3},\ \beta=\frac{0+b+0}{3}=\frac{b}{3},\ \gamma=\frac{0+0+c}{3}=\frac{c}{3}\quad ....(ii)$$

The plane (i) is at a distance 3p from the origin.

$\therefore\ 3p$ = Length of perpendicular from (0, 0, 0) to the plane (i)

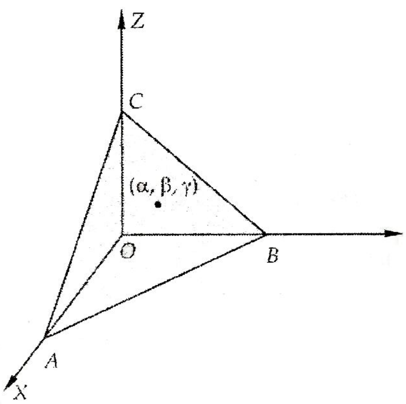
$$\Rightarrow\ 3p=\frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow\ 3p=\frac{1}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}\Rightarrow\frac{1}{9p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

From (ii), we have

$a=3\alpha, b=3\beta$ and $c=3\gamma$

Substituting the values of a, b, c in (iii), we obta



$$\frac{1}{9p^2}=\frac{1}{9\alpha^2}+\frac{1}{9\beta^2}+\frac{1}{9\gamma^2}$$

$$\Rightarrow\ \frac{1}{p^2}=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$$

So, the locus of (α,β,γ) is

$$\frac{1}{p^2}=\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}$$

Topic:3D; Sub-Topic:Plane_L-2_XII-CBSE Board Exam-2017_Mathematics.