Rao IIT Academy

Symbol of Excellence and Perfection

JEE | MEDICAL-UG | BOARDS | KVPY | NTSE | OLYMPIADS

SSC - BOARD - 2018

Date: 10.03.2018   **MATHEMATICS - PAPER-1 - SOLUTIONS**

Q.1 **Attempt any FIVE of the following sub-questions :**   **[5]**

(i)   Find next two terms of an A.P.

4, 9, 14, ...

Ans. Given sequence

4, 9, 14, ...

Given sequence is an A.P. with $a = 4, d = 5$

$t_4 = a + 3d = 4 + 3(5) = 19$

$t_5 = a + 4d = 4 + 4(5) = 24$

Topic:Arithmetic Progression_ ; Sub-topic:__L-1__SSC Board Test_Mathematics

(ii) State whether the given equation is quadratic or not. Give reson.

$\dfrac{5}{4}m^2 - 7 = 0$

Ans. $\dfrac{5}{4}m^2 - 7 = 0$. Here maximum index of the variable is 2.

Here $a = \dfrac{5}{4}, \ b = 0, \ c = -7$ are real numbers and $a \neq 0$

So it is a quadratic equation in variable m.

Topic:Quadratic Equation_ ; Sub-topic:__L-1__SSC Board Test_Mathematics

(iii) If $D_x = 25, D = 5$ are the values of the determinants for certain simultaneous equations in x and y, find x.

Ans. By Cramer's Rule,

$x = \dfrac{D_x}{D} = \dfrac{25}{5} = 5$

Topic:Linear equation in two variables_ ; Sub-topic:__L-1__SSC Board Test_Mathematics

(iv)   If $S = \{2, 4, 6, 8, 10, 12\}$ and $A = \{4, 8, 12\}$, find A'.

Ans. $S = \{2, 4, 6, 8, 10, 12\}$ and $A = \{4, 8, 12\}$

$A' = \{2, 4, 6, 8, 10, 12\} - \{4, 8, 12\}$

$A' = \{2, 6, 10\}$

Topic:Probability_ ; Sub-topic:__L-1__SSC Board Test_Mathematics

(v)   Write any one solution of equation $x + 2y = 7$.

Ans. $x + 2y = 7$

Substituting $x = 1$ and $y = 3$
L.H.S $= 1 + 2(3) = 7 = $ R.H.S

$\therefore$ $x = 1$ and $y = 3$ is the solution of $x + 2y = 7$

Topic:Linear equation in two variables_; Sub-topic:__L-1__SSC Board Test_Mathematics

(vi)   If $S_5 = 15$ and $S_6 = 21$, find t_6.

Ans. $S_{n+1} - S_n = t_{n+1}$

$t_6 = S_6 - S_5 = 21 - 15 = 6$

Topic:Arithmetic Pregression_ ; Sub-topic:__L-2__SSC Board Test_Mathematics

Q.2   Attempt any FOUR of the following subquestions :   **[8]**

(i)   Find 'n' if the nth term of the following A.P. is 68 :

5, 8, 11, 14, ...

Ans.   Given that

$a = 5$, $d = 3$, $t_n = 68$

$t_n = a + (n - 1)d$

$68 = 5 + (n - 1)3$

$63 = (n - 1)3$

$n - 1 = 21 \Rightarrow n = 22$

Topic:Arithmetic Pregression_ ; Sub-topic:__L-1__SSC Board Test_Mathematics

(ii)   If one of the roots of the quadratic equation $x^2 - 11x + k = 0$ is 9, then find the value of k.

Ans. $x^2 - 11x + k = 0$
Given that
One root of given equation is 9

$\therefore$ $(9)^2 - 11(9) + k = 0$

$\therefore 81 - 99 + k = 0$

$\therefore k = 18$

Topic:Quadratic Equation_ ; Sub-topic:Formation of roots_L-1__SSC Board Test_Mathematics

(iii)   A box contains 20 cards marked with numbers 1 to 20. One card is drawn at random. Event A is the number of the card which is multiple of 5. Write S, $n(S)$, A and $n(A)$.

Ans. $S = \{1, 2, 3, 4, \ldots\ldots\ldots 20\}$

$n(S) = 20$

A = The number on the card is multiple of 5

$A = \{5, 10, 15, 20\}$

$n(A) = 4$

Topic:Probability_ ; Sub-topic:__L-1__SSC Board Test_Mathematics

(iv)   Find the value of $x - y$ if $4x + 3y = 25$, $3x + 4y = 24$.

Ans. $4x + 3y = 25$ ...(i)

$3x + 4y = 24$ ...(ii)

Equation (i) is multiply by 3 and (ii) by 4

$12x + 9y = 75$

$12x + 16y = 96$

$\overline{}$

$-7y = -21$

$\therefore\ y = 3$

$\therefore\ 4x + 3(3) = 25$

$4x = 16$

$x = 4$

$\therefore\ x - y = 4 - 3 = 1$

Topic:Linear equation in two variables_ ; Sub-topic:__L-1__SSC Board Test_Mathematics

(v)   Form the quadratic equation if its roots are -3 and 4.

Ans.   Given that

$\alpha = -3,\ \beta = 4$

$\therefore\ \alpha + \beta = (-3) + 4 = 1$

$\alpha \cdot \beta = -3 \times 4 = -12$

$\therefore$ The quadratic equation which roots are α and β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\therefore\ x^2 - x - 12 = 0$

Topic:Quadratic Equation_ ; Sub-topic:Formation of roots_L-1__SSC Board Test_Mathematics

(vi)   For a certain frequency distribution, the values of mean and median are 72 and 78 respectively. Find the value of mode.

Ans.   Mean = 72

Median = 78

Mean – Mode = 3(Mean – Median)

72 – Mode = 3(72 – 78)

Mode = 72 + 18 = 90

Topic:Statistics I_ ; Sub-topic:Mean, Median and Mode_L-1__SSC Board Test_Mathematics

Q.3 **Attempt any THREE of the following subquestions :** [9]

(i) For an A.P., find S_7 if $a = 5$ and $d = 4$.

Ans. $a = 5,\ d = 4$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_7 = \frac{7}{2}\left[2(5) + (7-1)(4)\right]$$

$$= \frac{7}{2}\left[10 + 24\right]$$

$$= \frac{7}{2} \times 34$$

$$S_7 = 119$$

Topic:Arithmetic Progression_ ; Sub-topic:__L-1__SSC Board Test_Mathematics

(ii) Solve the following quadratic equation by using formula method :

$2x^2 - 3x = 2$

Ans. Given quadratic equation

$2x^2 - 3x = 2$

$2x^2 - 3x - 2 = 0$

$a = 2,\ b = -3,\ c = -2$

By Formula method,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3 \pm \sqrt{9 - 4(2)(-2)}}{2(2)}$$

$$= \frac{3 \pm \sqrt{9 + 16}}{4}$$

$$= \frac{3 \pm 5}{4}$$

$$x = \frac{3+5}{4} \qquad \text{or} \qquad x = \frac{3-5}{2}$$

$$\therefore\ x = 2, -1$$

Topic:Quadratic Equation_ ; Sub-topic:Solution of QE_L-2__SSC Board Test_Mathematics

(iii) Solve the following simultaneous equations using Cramer's rule :

$3x - 2y = 3$;

$2x + y = 16$

Ans. $3x - 2y = 3$ ...(i)

$2x + y = 16$ ...(ii)

$$D = \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} = 3(1) - 2(-2) = 7$$

$$D_x = \begin{vmatrix} 3 & -2 \\ 16 & 1 \end{vmatrix} = 3(1) - 16(-2) = 3 + 32 = 35$$

$$D_y = \begin{vmatrix} 3 & 3 \\ 2 & 16 \end{vmatrix} = 3(16) - 3(2) = 48 - 6 = 42$$

Now,

$$x = \frac{D_x}{D} \qquad\qquad y = \frac{D_y}{D}$$

$$x = \frac{35}{7} \qquad\qquad y = \frac{42}{7}$$

$$x = 5 \qquad\qquad y = 6$$

Topic:Linear equation in two variables_; Sub-topic:Cramer's Rule_L-1__SSC Board Test_Mathematics

(iv) A die is thrown, find the probability of the event of getting a number less than 3.

Ans. Sample space when a die is thrown

$S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

Let A = Getting a number less than 3

$\therefore A = \{1, 2\}$

$\therefore n(A) = 2$

P(getting a number less than 3) $= P(A) = \dfrac{n(A)}{n(S)} = \dfrac{2}{6} = \dfrac{1}{3}$

*Topic:Probability_; Sub-topic:__L-1__SSC Board Test_Mathematics*

(v) The marks obtained by a student in an examination out of 100 are given below. The total marks obtained in various subjects are as follows :

Subject	Marks
Marathi	85
English	85
Science	90
Mathematics	100
Total	360

Represent the above data using pie diagram.

Ans. First of all, we compute the central angle for each subject as shown in following table.

Sr.No.	Subject	Marks	Measure of central angle
1	Marathi	85	$\dfrac{85}{360} \times 360° = 85°$
2	English	85	$\dfrac{85}{360} \times 360° = 85°$
3	Science	90	$\dfrac{90}{360} \times 360° = 90°$
4	Mathematics	100	$\dfrac{100}{360} \times 360° = 100°$
	Total	**360**	**360°**
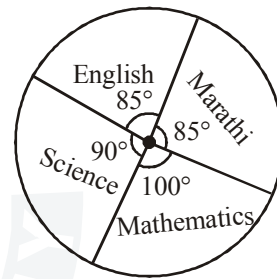
Topic:Statistics II_ ; Sub-topic:Pie Diagram_L-1__SSC Board Test_Mathematics

Q.4 Attempt any TWO of the following subquestions : [8]

(i) If $\alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 35,$ find the quadratic equation whose roots are α and β.

Ans. Here α and β are the roots of the quadratic equation, so required equations is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0 \qquad\qquad ...(1)$$

We have $\alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 35$

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

$$\therefore 35 = (5)^3 - 3\alpha\beta \times 5$$

$$\therefore 35 = 125 - 15\alpha\beta$$

$$\therefore 15\alpha\beta = 90$$

$$\therefore \alpha\beta = 6$$

So from (1) required quadratic equation is $x^2 - 5x + 6 = 0$

Topic:Quadratic Equation_ ; Sub-topic:Formation of QE_L-3__SSC Board Test_Mathematics

(ii) Two dice are thrown. Find the probability of getting :

(a) The sum of the numbers on their upper faces is at least 9.

(b) The sum of the numbers on their upper faces is 15.

(c) The number on the upper face of the second die is greater than the number on the upper face of the first die.

Ans.
$$S = \{(1,1)\,(1,2)\,(1,3)\,(1,4)\,(1,5)\,(1,6)$$
$$(2,1)\,(2,2)\,(2,3)\,(2,4)\,(2,5)\,(2,6)$$
$$(3,1)\,(3,2)\,(3,3)\,(3,4)\,(3,5)\,(3,6)$$
$$(4,1)\,(4,2)\,(4,3)\,(4,4)\,(4,5)\,(4,6)$$
$$(5,1)\,(5,2)\,(5,3)\,(5,4)\,(5,5)\,(5,6)$$
$$(6,1)\,(6,2)\,(6,3)\,(6,4)\,(6,5)\,(6,6)\}$$

$n(S) = 36$

Let A = sum of the numbers in their upper faces is at least 9.

$A = \{(3,6)\,(4,5)\,(4,6)\,(5,4)\,(5,5)\,(5,6)\,(6,3)\,(6,4)\,(6,5)\,(6,6)\}$

$n(A) = 10$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

Let B = sum of the number on their upper faces is 15.

$B = \{\ \}$ (Null set)

$n(B) = 0$

$$P(B) = \frac{n(B)}{n(S)} = \frac{0}{36} = 0$$

Let C = number on the upper face of second die is greater than the number on the upper face of first die.

$C = \{(1,2)\,(1,3)\,(1,4)\,(1,5)\,(1,6)\,(2,3)\,(2,4)\,(2,5)\,(2,6)\,(3,4)\,(3,5)\,(3,6)\,(4,5)\,(4,6)\,(5,6)\}$

$n(C) = 16$

$$P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

Topic:Probability_; Sub-topic:Probability_L-2__SSC Board Test_Mathematics

(iii) Frequency distribution of daily commission received by 100 salemen is given below :

Daily Commission (in Rs.)	No. of Salesmen
100-120	20
120-140	45
140-160	22
160-180	09
180-200	04

Find mean daily commission received by salemen, by assumed mean method.

Ans.

Daily commission	Classmark	$d_i = x_i - A$ $d_i = x_i - 150$	No. of salemen f_i	$f_i d_i$
$100-120$	110	-40	20	-800
$120-140$	130	-20	45	-900
$140-160$	$150 \rightarrow A$	0	22	0
$160-180$	170	20	09	180
$180-200$	190	40	04	160
			$\sum f_i = 100$	$\sum f_i x_i = -1360$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f} = -\frac{1360}{100} = -13.60$$

$$\therefore \bar{x} = A + \bar{d} = 150 + (-13.60) = 136.4$$

Topic:Statistics I_; Sub-topic: Mean__L-2__SSC Board Test_Mathematics

Q.5 Attempt any TWO of the following subquestions : [10]

(i) A boat takes 10 hours to travel 30 km upsteam and 44 km downstream, but it takes 13 hours to travel 40 km upstream and 55 km downstream. Find the speed of the boat in still water and the speed of the stream.

Ans. Let the speed of the boat in still water be x km/hr and the speed of the stream by y km/hr.

Therefore, the speed of the boat downstream $= (x + y) km / hr$ and the speed of the boat upstream

$= (x - y) km / hr$

Now, time $= \dfrac{\text{distance}}{\text{speed}}$

Therefore, time taken by the boat to cover 30 km upstream $= \dfrac{30}{x - y}$ *hours* and the time taken by the

boat to cover 24km down stream $= \dfrac{44}{x + y}$ *hours*

But the total time taken by the boat to cover 30 km upstream and 44 km downstream is 10 hours.

$$\therefore \frac{30}{x-y} + \frac{44}{x+y} = 10 \dots (i)$$

similarly by second condition,

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \dots (ii)$$

substituting $\dfrac{1}{x-y} = a$ and $\dfrac{1}{x+y} = b$ in

equation (i) and (ii)

$$\therefore 30a + 44b = 10 \dots (iii)$$

$$40a + 55b = 13 \dots (iv)$$

Equation (iii) x (iv) and eqation (iv) x (iii), we get

$$120a + 176b = 40 \dots (v)$$

$$120a + 165b = 39 \dots (vi)$$

equation (v) – equation (vi) , we get

$$11b = 1$$

$$b = \frac{1}{11}$$

substituting $b = \dfrac{1}{11}$ in equation (v), we get

$$120a + 176\left(\frac{1}{11}\right) = 40$$

$$120a = 40 - 16$$

$$120a = 24$$

$$a = \frac{24}{120}$$

$$a = \frac{1}{5}$$

Now, $\dfrac{1}{x-y} = \dfrac{1}{5}$ and $\dfrac{1}{x+y} = \dfrac{1}{11}$

$x - y = 5$ and $x + y = 11$

$$x + y = 11 \dots (vii)$$
$$x - y = 5 \dots (viii)$$

Adding equation (vii) and equation (viii), we get

$$2x = 16$$

$$x = 8$$

Subsidity $x = 8$ in equation (vii) we get $y = 3$

∴ speed of the boat in still water is 8 km/hr and speed of the stream is 3 km/hr

Topic:Linear equation in two variables_; Sub-topic:__L-3__SSC Board Test_Mathematics

(ii) If the 9th term of an A.P. is zero, then prove that 29th term is double of 19th term.

Ans. $t_n = a + (n-1)d$

9th term i.e. $n = 9$

$$\therefore t_9 = a + (9-1)d$$

$$= a + 8d$$

It is given that $t_9 = 0$

$$\therefore a + 8d = 0 \dots (i)$$

29th term i.e. t_{29} where $n = 29$

$$\therefore t_{29} = a + (29-1)d$$

$$t_{29} = a + 28d \dots (ii)$$

$$= (a + 8d) + 20d$$

$$= 0 + 20d \qquad \dots \text{by eq (i)}$$

$$\therefore t_{29} = 20d \dots (ii)$$

$$t_{19} = a + (19-1)d$$

$$t_{19} = a + 18d$$

$$= a + 8d + 10d$$

$$= 0 + 10d$$

$$t_{19} = 10d \dots (iii)$$

by equation (ii) & (iii)

$$t_{29} = 2t_{19}$$

Topic:Arithmetic Progression_; Sub-topic:__L-3__SSC Board Test_Mathematics

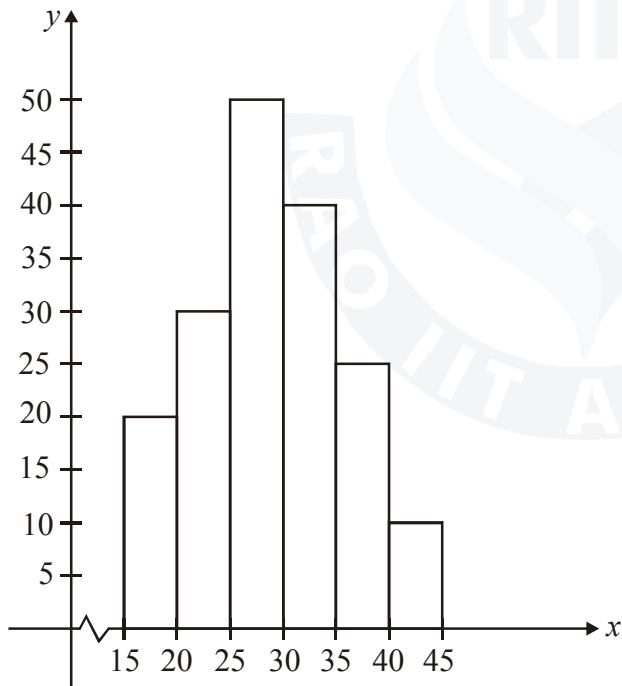(iii) Draw histrogram and frequency polygon on the same graph paper for the following frequency distribution :

Class	Frequency
15-20	20
20-25	30
25-30	50
30-35	40
35-40	25
40-45	10

Ans.

Class	15 − 20	20 − 25	25 − 30	30 − 35	35 − 40	40 − 45
Frequency	20	30	50	40	25	10
Classmark	17.5	22.5	27.5	32.5	37.5	42.5

Scale - on x axis : 1 cm = 5 units and y axis : 1 cm = 5 units

Histogram

Frequency Polygon curve
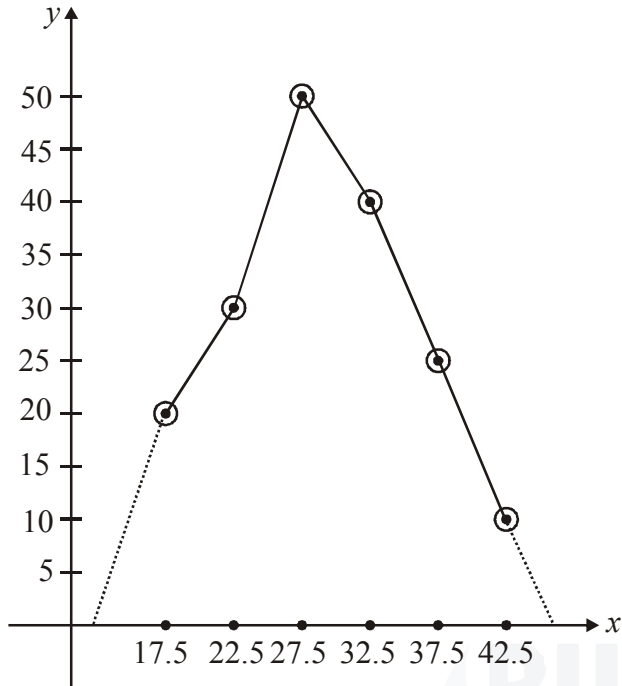
Topic:Statistics II_ ; Sub-topic:__L-1__SSC Board Test_Mathematics