XII HSC - BOARD - FEB - 2016

Date: 26.02.2016

MATHEMATICS (40) - SOLUTIONS

The following scheme of marking is only for guidelines to help to evaluate answer papers. Any alternative, but logically correct approach, should be acceptable and must be given full credit. Part marking should be made strictly according to the number of correct steps.

SECTION - I

Q. 1 (A)

(i) (d)

$\sim\left[p \wedge (q \rightarrow r)\right]$

$= \sim p \vee \sim (q \rightarrow r)$

$= \sim p \vee (q \wedge \sim r)$ **[2 Marks]**

Topic:Logic; Sub-topic:Rules for negation L-1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii) (c)

Putting the value of x, equation get satisfied. **[2 Marks]**

Topic:Trigonometry function_; Sub-topic:ITF_ L-1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) (a)

equation of line parallel to $x-axis$ $y = k$

it is passing through (2, 3)

$\therefore\ y = 3$

$y - 3 = 0$

equation of line parallel to y-axis X = k

$x = 2$

$x - 2 = 0$

combined equation of line (2) and (3)

$(y-3)(x-2) = 0$

$xy - 3x - 2y + 6 = 0$ **[2 Marks]**

Topic:Pair of lines_; Sub-topic:Comb. equation_L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(B)

(i) $AB = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -2 \end{bmatrix}$

$AB = \begin{bmatrix} 1+2+3 & -1+4-6 \\ 1-2-3 & -1-4+6 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ -4 & 1 \end{bmatrix}$ **[1 Mark]**

Let $AB = P = \begin{bmatrix} 6 & -3 \\ -4 & 1 \end{bmatrix}$

$|P| = 6 - 12 = -6$

$|P| \neq 0$

$\therefore\ P^{-1}$ exists

Co-factors

$C_{11} = 1$

$C_{12} = 4$

$C_{21} = 3$

$C_{22} = 6$

$Adj\ P = \begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$

$p^{-1} = \dfrac{Adj\,P}{|P|} = \dfrac{\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}}{-6} = \begin{bmatrix} \frac{-1}{6} & \frac{-1}{2} \\ \frac{-2}{3} & -1 \end{bmatrix}$ **[1 Mark]**

Topic:Matrices ; Sub-topic:Inverse L-1__Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii) equation of plane passing through a point and $\perp$ to a vector in

$\overline{r} \cdot \overline{n} = \overline{a} \cdot \overline{n}$ **[1 Mark]**

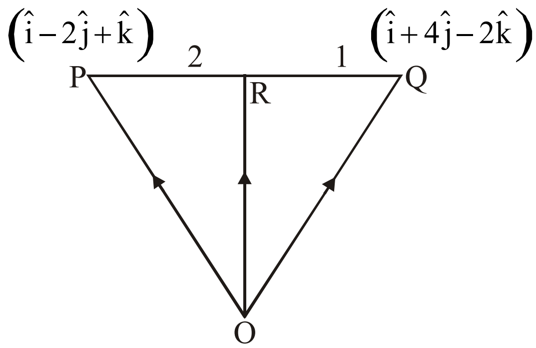
$given\ \overline{a} = 3\hat{i} - 2\hat{j} + \hat{k}\ ;\ \overline{n} = 4\hat{i} + 3\hat{j} + 2\hat{k}$

$\overline{r} \cdot \left(4\hat{i} + 3\hat{j} + 2\hat{k}\right) = \left(3\hat{i} - 2\hat{j} + \hat{k}\right) \cdot \left(4\hat{i} + 3\hat{j} + 2\hat{k}\right)$

$\overline{r} \cdot \left(4\hat{i} + 3\hat{j} + 2\hat{k}\right) = 8$ **[1 Mark]**

Topic:Plane_; Sub-topic:Plane_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii)



[1 Mark]

Position vector of point R in

$$\overrightarrow{OR} = \frac{\overrightarrow{OQ} \times 2 + 1 \times \overrightarrow{OP}}{2+1}$$

$$\overrightarrow{OR} = \frac{2\left(\hat{i} + 4\hat{j} - 2\hat{k}\right) + 1\left(\hat{i} - 2\hat{j} + \hat{k}\right)}{2+1}$$

$$\overrightarrow{OR} = \frac{2\hat{i} + 8\hat{j} - 4\hat{k} + \hat{i} - 2\hat{j} + \hat{k}}{3}$$

$$\overrightarrow{OR} = \frac{3\hat{i} + 6\hat{j} - 3\hat{k}}{3}$$

$\overrightarrow{OR} = \hat{i} + 2\hat{j} - \hat{k}$ **[1 Mark]**

Topic:Vector_; Sub-topic:Section formula_ L-1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iv) Auxilliary equation of

$6x^2 + kxy + y^2 = 0$ is $m^2 + km + 6 = 0$ .....(1)

one of the equation of pair of line (1) is $2x + y = 0$

slope of $2x + y = 0$ is $m = -2$ **[1 Mark]**

$\therefore\ m = -2$ is satisfies the equation (1)

$(-2)^2 + k(-2) + 6 = 0$

$4 - 2k + 6 = 0$

$2k = 10$

$k = 5$ **[1 Mark]**

Topic:Pair of straight line_; Sub-topic:Nature of roots_ L-2__Target-2016_XII-HSC Baord (40) Test_Mathematics

(v) Direction ratios of $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ are

$a_1=-3,\ b_1=2k,\ c_1=2$

$a_2=3k,\ b_2=1,\ c_2=-5$ **[1 Mark]**

Given lines are perpendicular

$\therefore\ a_1a_2+b_1b_2+c_1c_2=0$

$-3\cdot 3k+2k\cdot 1+2\cdot -5=0$

$-9k+2k-10=0$

$-7k-10=0$

$k=\frac{-10}{7}$ **[1 Mark]**

Topic:_Line_; Sub-topic:Application_ L-1__Target-2016_XII-HSC Baord (40) Test_Mathematics

Q.2 (A)

(i) Given : $[(p\to q)\wedge q]\to p$

No.of rows = $2^n=2^2=4$

No. of column = $m+n=3+2=5$ **[1 Mark]**

Truth table

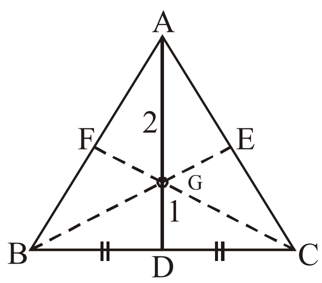
p	q	$p\to q$	$(p\to q)\wedge q$	$[(p\to q)\wedge q]\to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

[1 Mark]

From last column given statement pattern is contingency. **[1 Mark]**

Topic:Logic_; Sub-topic:_____ L- ___XII-HSC Baord (40) Test_Mathematics

(ii) Let $\bar{a},\bar{b},\bar{c},\bar{d},\bar{e}$ the position vectors of the vertices A, B, C of ΔABC and $\bar{d},\bar{e},\bar{f}$ be the position vectors of the midpoints D, E, F of the sides BC, CA and AB respectively



[1 Mark]

Then by the midpoint formula,

$\bar{d}=\frac{\bar{b}+\bar{c}}{2},\ \bar{e}=\frac{\bar{c}+\bar{a}}{2},\ \bar{f}=\frac{\bar{a}+\bar{b}}{2}$

$\therefore\ 2\bar{d}=\bar{b}+\bar{c}\ ;\ 2\bar{e}=\bar{c}+\bar{a}\ ;2\bar{f}=\bar{a}+\bar{b}$

$\therefore\ 2\bar{d}+\bar{a}=\bar{a}+\bar{b}+\bar{c}$

$2\bar{e} + \bar{b} = \bar{a} + \bar{b} + \bar{c}$

$2\bar{f} + \bar{c} = \bar{a} + \bar{b} + \bar{c}$ **[1 Mark]**

$$\therefore \frac{2\bar{d} + \bar{a}}{2+1} = \frac{2\bar{e} + \bar{b}}{2+1} = \frac{2\bar{f} + \bar{c}}{2+1} = \frac{\bar{a} + \bar{b} + \bar{c}}{3} = \bar{g} \text{ (let)}$$

lies on the three medians AD, BE and CF dividing each of them internally in the ratio 2 : 1.
Hence, the medians are concurrent at point G . **[1 Mark]**

Topic:Vector ; Sub-topic:Theorem L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) Equation of lines are.,

$\bar{r} = \left(4\bar{i} - \bar{j}\right) + \lambda\left(\bar{i} + 2\bar{j} - 3\bar{k}\right)$ &

$\bar{r} = \left(\bar{i} - \bar{j} + 2\bar{k}\right) + \mu\left(\bar{i} + 4\bar{j} - 5\bar{k}\right)$

$\therefore$ above lines passes through

$\overline{a_1} = \left(4\bar{i} - \bar{j}\right)$ and $\overline{a_2} = \left(\bar{i} - \bar{j} + 2\bar{k}\right)$

and parallel to

$\overline{b_1} = \bar{i} + 2\bar{j} - 3\bar{k}$ & $\overline{b_2} = \bar{i} + 4\bar{j} - 5\bar{k}$ **[1 Mark]**

Shortest distance $= \left|\dfrac{\left(\overline{a_2} - \overline{a_1}\right).\left(\overline{b_1} \times \overline{b_2}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$ **[1 Mark]**

$\Rightarrow \overline{a_2} - \overline{a_1} = -3\bar{j} + 2\bar{k}$

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix} = 2\bar{i} + 2\bar{j} + 2\bar{k}$$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = 2\sqrt{3}$

Shortest distance $= \left|\dfrac{\left(-3\bar{i} + 2\bar{k}\right).\left(2\bar{i} + 2\bar{j} + 2\bar{k}\right)}{2\sqrt{3}}\right|$

$= \left|\dfrac{-6+4}{2\sqrt{3}}\right|$

$= \left|\dfrac{-2}{2\sqrt{3}}\right|$

$d = \dfrac{1}{\sqrt{3}}$ units **[1 Mark]**

Topic:Line ; Sub-topic:Shortest distance_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

Q.2 (B)

(i) Taking LHS

$$=(a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2}$$

$$=(a^2+b^2-2ab)\cos^2\frac{C}{2}+(a^2+b^2+2ab)\cdot\sin^2\frac{C}{2}$$ **[1 Mark]**

$$=(a^2+b^2)\cdot\cos^2\frac{C}{2}-2ab\cos^2\frac{C}{2}+(a^2+b^2).\sin^2\frac{C}{2}+2ab\sin^2\frac{C}{2}$$

$$=(a^2+b^2)\left(\cos^2\frac{C}{2}+\sin^2\frac{C}{2}\right)-2ab\left(\cos^2\frac{C}{2}-\sin^2\frac{C}{2}\right)$$ **[1 Mark]**

$$=(a^2+b^2)-2ab\cos C$$ {By cosine Rule} **[1 Mark]**

$$=c^2$$ **[1 Mark]**

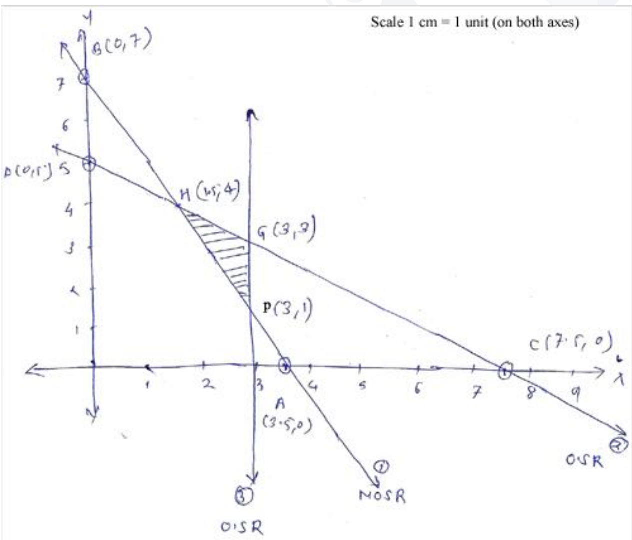
Topic:Trigo function; Sub-topic:SOT_ L- 2_Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii)

Line	Inequation	Point on X axis	Point on Y axis	Feasible region
AB	$2x+y\ge 7$	$A(3.5,0)$	$B(0,7)$	Non origin
CD	$2x+3y\le 15$	$C(7.5,0)$	$D(0,5)$	Origin side
EF	$x\le 3$	$E(3,0)$	$---$	Origin side

[1 Mark]

<u>Graph</u>



[1 Mark]

Extreme points	Minimum $Z=4x+5y$
$P(3,1)$	$Z(P)=4(3)+5(1)=17$
$G(3,3)$	$Z(G)=4(3)+5(3)=27$
$H(1.5,4)$	$Z(H)=4(1.5)+5(5)=26$

[1 Mark]

Minimum value of $Z=17$ at P (3,1) **[1 Mark]**

Topic:LPP_; Sub-topic:Graphical solution_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) Let the cost of 1 dozen of pencils, pen, and erasers respectively by given conditions we get,

$4x + 3y + 2z = 60$

$2x + 4y + 6z = 90 \Rightarrow x + 2y + 3z = 45$

$6x + 2y + 3z = 70$

$\therefore$ In matrix form we set,

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$ **[1 Mark]**

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 60 \\ 70 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1, R_3 \to R_3 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -10 & -15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ -120 \\ -200 \end{bmatrix}$$

$R_2 \to \frac{R_2}{-5}, R_3 \to \frac{R_3}{-5}$

$$-2\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 24 \\ 40 \end{bmatrix}$$ **[1 Mark]**

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 24 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x + 2y + 3z \\ y + 2z \\ -z \end{bmatrix} = \begin{bmatrix} 45 \\ 29 \\ -8 \end{bmatrix}$$ **[1 Mark]**

comparing,

(1) $-z = -8 \Rightarrow \boxed{z = 8}$

(2) $y + 2z = 24 \Rightarrow y + 16 = 24$

$\Rightarrow \boxed{y = 8}$

(3) $x + 2y + 3z = 45 \Rightarrow x + 16 + 24 = 45$

$\Rightarrow x + 40 = 45$

$\Rightarrow x = 5$

$\therefore$ cos t of 1 dozen pencils = 5/-

1 dozen pen = 8/-

1 dozen erasers = 8/- **[1 Mark]**

Topic:Matrix_; Sub-topic:Application _ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

Q.3 (A)

(i) Consider coterminus edges of tetrahedran

are $\bar{a} = 7\bar{i} + \bar{k}$

$\bar{b} = 2\bar{i} + 5\bar{j} - 3\bar{k}$

$\bar{c} = 4\bar{i} + 3\bar{j} + \bar{k}$

Volume of tetrahedran is

$\therefore V = \frac{1}{6}\left[\bar{a}\ \bar{b}\ \bar{c}\right]$ **[1 Mark]**

$V = \frac{1}{6}\begin{bmatrix} 7 & 0 & 1 \\ 2 & 5 & -3 \\ 4 & 3 & 1 \end{bmatrix}$ **[1 Mark]**

$= \frac{1}{6}\left[7(14) + 0 + 1(-14)\right]$

$= \frac{1}{6}[98 - 14]$

$V = 14 \text{ unit}^3$ **[1 Mark]**

Topic:Vector_; Sub-topic:Application_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii) LHS = $\sim(p \vee q) \vee (\sim p \wedge q)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ (by Demorgan's law) **[1 Mark]**

$= \sim p \wedge (\sim q \vee q)$ (by Distributive law)

$= \sim p \wedge T$ (by Complement law) **[1 Mark]**

$= \sim p = R.H.S$ (by Identity law) **[1 Mark]**

Topic:Logic_; Sub-topic:Algebra_ L- 2_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) Consider a homogenous equation of degree two in x and y

$ax^2 + 2hxy + by^2 = 0$ .....(i)

In this equation at least one of the coefficients a ,b or h is non zero.

We consider two cases

Case I : If $b = 0$, then the equation of lines $x = 0$ and $(ax + 2hy) = 0$ **[1 Mark]**

These lines passes through the origin.

Case II : $b \neq 0$,

Multiplying both the sides of equation (i) by b, we get **[1 Mark]**

$abx^2 + 2hbxy + b^2y^2 = 0$

$b^2y^2 + 2hbxy = -abx^2$

To make L.H.S a complete square, we add h^2x^2 on both the sides.

$b^2y^2 + 2hbxy + h^2x^2 = -abx^2 + h^2x^2$

$(by + hx)^2 = (h^2 - ab)x^2$

$$(by+hx)^2=\left[\left(\sqrt{h^2-ab}\right)x\right]^2$$

$$(by+hx)^2-\left[\left(\sqrt{h^2-ab}\right)x\right]^2=0$$

$$\left[(by+hx)+\left(\sqrt{h^2-ab}\right)x\right]\left[(by+hx)-\left(\sqrt{h^2-ab}\right)x\right]=0$$

It is the joint equation of two lines

$$(by+hx)+\left(\sqrt{h^2-ab}\right)x=0 \text{ and } (by+hx)-\left(\sqrt{h^2-ab}\right)x=0$$

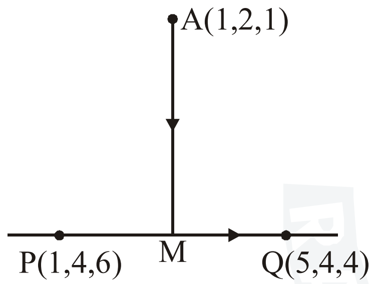
i.e. $\left(h+\sqrt{h^2-ab}\right)x+by=0$ and $\left(h-\sqrt{h^2-ab}\right)x+by=0$

These lines passes through the origin. **[1 Mark]s**

Topic: Pair of st. line_; Sub-topic:Theorem_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

Q.3 (B)

(i)



Equation of line passing through points P and Q is

$\bar{r}=\left(i+4\hat{j}+6\hat{k}\right)+\lambda\left(4\hat{i}-2\hat{k}\right)$ **[1 Mark]**

Let any point M on line PQ

$\therefore$ Co-ordinate of M **[1 Mark]**

$M(4\lambda+1,+4,-2\lambda+6)$

$\therefore \overrightarrow{AM}=(4\lambda)\hat{i}+2\hat{j}+(-2\lambda+5)\hat{k}$

$\therefore \overrightarrow{AM}$ is perpendicular to $\overline{PQ}$

$\therefore \overrightarrow{AM}.\overrightarrow{PQ}=0$ **[1 Mark]**

$\left[4\lambda\hat{i}+2\hat{j}+(-2\lambda+5)\hat{k}\right].\left(4\hat{i}-2\hat{k}\right)=0$

$16\lambda+0+4\lambda-10=0$

$20\lambda=10$

$\lambda=\frac{1}{2}$

$\therefore M\equiv(3,4,5)$

Foot of perpendicular is (3,4,5) **[1 Mark]**

Note : This question also can be solve by 3-D method

Topic:3– D_; Sub-topic:Foot of perp._ L- 2_Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii) The vector equation of the plane passing through the points $A(\bar{a}), B(\bar{b})$ and $C(\bar{c})$ is

$\bar{r}.(\overline{AB}\times\overline{AC}) = \bar{a}\cdot(\overline{AB}\times\overline{AC})$ **[1 Mark]**

Here, $\bar{a} = \hat{i}+\hat{j}-2\hat{k}$, $\bar{b} = \hat{i}+2\hat{j}+\hat{k}$, $\bar{c} = 2\hat{i}-\hat{j}+\hat{k}$

$\therefore \overline{AB} = \bar{b}-\bar{a} = (\hat{i}+2\hat{j}+\hat{k})-(\hat{i}+\hat{j}-2\hat{k}) = \hat{j}+3\hat{k}$

and $\therefore \overline{AC} = \bar{c}-\bar{a} = (2\hat{i}-\hat{j}+\hat{k})-(\hat{i}+\hat{j}-2\hat{k}) = \hat{i}-2\hat{j}+3\hat{k}$

$$\therefore \overline{AB}\times\overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 3 \\ 1 & -2 & 3 \end{vmatrix}$$ **[1 Mark]**

$= (3+6)\hat{i}-(0-3)\hat{j}+(0-1)\hat{k}$

$= 9\hat{i}+3\hat{j}-\hat{k}$

and $\bar{a}\cdot(\overline{AB}\times\overline{AC}) = (\hat{i}+\hat{j}-2\hat{k}).(9\hat{i}+3\hat{j}-\hat{k})$

$= 1(9)+1(3)+(-2)(-1)$

$= 9 + 3 + 2 = 14$

$\therefore$ from (1), the vector equation of the required plane is $\bar{r}.(9\hat{i}+3\hat{j}-\hat{k}) = 14$ **[1 Mark]**

Let $\bar{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore (x\hat{i}+y\hat{j}+z\hat{k}).(9\hat{i}+3\hat{j}-\hat{k}) = 14$

$\therefore$ Cartesian equation of required plane is

$9x+3y-z = 14$ **[1 Mark]**

Topic:Plane_; Sub-topic:Plane_ L- 3_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) As given

$\sin x + \sin 3x + \sin 5x = 0$

$\therefore \quad (\sin x + \sin 5x) + \sin 3x = 0$

$\therefore \quad 2\sin\left(\frac{x+5x}{2}\right)\cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0$ **[1 Mark]**

$\therefore \quad 2\sin 3x \cos 2x + \sin 3x = 0$

$\therefore \quad \sin 3x\,(2\cos 2x + 1) = 0$

$\therefore \quad \sin 3x = 0 \quad$ or $\quad 2\cos 2x + 1 = 0$

$\therefore \quad \sin 3x = 0$ ...(i) or $\cos 2x = -\frac{1}{2}$ ...(ii) **[1 Mark]**

For (ii) $\cos 2x = -\cos\frac{\pi}{3}$

$\therefore \quad \cos 2x = \cos\left(\pi - \frac{\pi}{3}\right)$ ...(by allied angles)

$\cos 2x = \cos\frac{2\pi}{3}$

$\therefore$ from (i) and (ii) we get

$\therefore \quad \sin 3x = 0$ or $\cos 2x = \cos\frac{2\pi}{3}$ **[1 Mark]**

$\therefore \ 3x = n\pi, \ n \in Z$ or $2x = 2m\pi \pm \frac{2\pi}{3}$, where $m \in Z$.

Hence the required solution is

$\therefore \ x = \frac{n\pi}{3}$ or $x = m\pi \pm \frac{\pi}{3}$, where $n, m \in Z$. **[1 Mark]**

Topic:Trigo. function_; Sub-topic:Gen. sol._ L- 3_Target-2016_XII-HSC Baord (40) Test_Mathematics

SECTION - II

Q.4 (A)

(i) (c)

If the function continuous $x = 1$

$\therefore$ LHL = RHL = $f(1)$

$\lim_{x \to 1^-} (K + x) = \lim_{x \to 1^+} 4x + 3$

$\lim_{h \to 0} (K + (1-h)) = \lim_{h \to 0} 4(1-h) + 3$

$K + 1 = 4 + 3$

$K = 6$ **[2 Marks]**

Topic:Continuity_; Sub-topic:At a point_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii) (a)

$y = x^2 + 4x + 1$

Slope of tangent $\frac{dy}{dx} = 2x + 4$

$\frac{dy}{dx}$ (at $x = -1$) $= -2 + 4 = 2$

Equation of tangent at $(-1, -2)$ having slope 2 is

$y - (-2) = 2(x - (-1))$

$y + 2 = 2(x + 1)$

$y + 2 = 2x + 2$

$2x - y = 0$ **[2 Marks]**

Topic:AOD_; Sub-topic:Tangent_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) (c)

n = 10, p = ?

E(x) = 8

We know that E(x) = nP = 8

Here n = 10

$$P = \frac{8}{10}$$

$$P = \frac{4}{5}$$

P = 0.8 **[2 Marks]**

Topic:Binomial dist._; Sub-topic:Mean_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

Q. 4 (B)

(i) $y = x^x$

Taking of logarithms of bothsides

$\log y = \log x^x$

$\log y = x \log x$ **[1 Mark]**

differentiating both sides w.r.t. x

$$\frac{1}{y}\frac{dy}{dx} = (1 + \log x)$$

$$\frac{dy}{dx} = y(1 + \log x)$$

$$\frac{dy}{dx} = x^x(1 + \log x)$$ **[1 Mark]**

Topic:Derivative_; Sub-topic:Log._ L- 1_XII-HSC Baord (40) Test_Mathematics

(ii) $s = 5 + 20t - 2t^2$

$$v = \frac{ds}{dt} = 20 - 4t$$ **[1 Mark]**

given, $v = 0$

$20 - 4t = 0$

$t = 5$

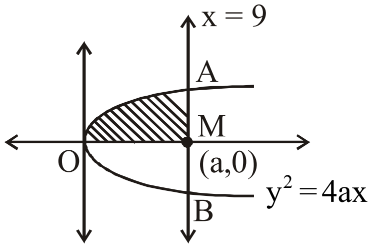
acceleration

$$a = \frac{dv}{dt} = -4 \text{ at } (t = 5)$$

acceleration $(a) = -4$ unit / sec^2 **[1 Mark]**

Topic:AOD_; Sub-topic:Rate measure_ L- 1_XII-HSC Baord (40) Test_Mathematics

(iii)



Area of OAM **[1 Mark]**

$$= \int_0^a \sqrt{4ax}$$

$$= 2\sqrt{a}\int_0^a \sqrt{x}$$

$$= 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$$

$$= 2\sqrt{a}\left[\frac{a^{\frac{3}{2}}}{\frac{3}{2}} - 0\right]$$

$$= 2\sqrt{a} \times \frac{2}{3}\left[a^{\frac{3}{2}}\right]$$

Area of $OAM = \frac{4}{3}a^2\, unit^2$ **[1 Mark]**

Topic:Application of Definite integration_; Sub-topic:Area under curve_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iv) Given X is discrete r . v .

$\therefore$ P(x) is pm f.

p.m.f. $= \sum pi = 1$ **[1 Mark]**

$= k + 2k + 3k + 4k + 5k = 1$

$= 15k = 1$

$k = \frac{1}{15}$

$p(x \le 4) = 4k + 3k + 2k + k$

$= 10k$

$= 10 \times \frac{1}{15} = \frac{2}{3}$ **[1 Mark]**

Topic:Prob. dist._; Sub-topic:PMF_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(v) $I=\int \frac{\sin x}{\sqrt{36-\cos^2 x}}\,dx$

Let $\cos x = t$

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$ **[1 Mark]**

$I = \int \frac{-dt}{\sqrt{36-t^2}}$

$I = -\int \frac{dt}{\sqrt{6^2-t^2}}$

$I = -\sin^{-1}\frac{t}{6} + c$

$I = -\sin^{-1}\left(\frac{\cos x}{6}\right) + c$ **[1 Mark]**

Topic:Integration_; Sub-topic:Substitution method_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

Q.5 (A)

(i) Let δx be a small increment (change) in x. Let δy and δu be the corresponding increments in y and u respectively. As $\delta x \to 0,\ \delta u \to 0,\ \delta y \to 0$

As u is differentiable function, it is continuous

Consider the incrementary ratio $\frac{\delta y}{\delta x}$

We have, $\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u} \times \frac{\delta u}{\delta x}$

Taking limit as $\delta x \to 0$, on both sides.

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\left(\frac{\delta y}{\delta u}\times\frac{\delta u}{\delta x}\right) = \lim_{\delta x\to 0}\frac{\delta y}{\delta u} \times \lim_{\delta x\to 0}\frac{\delta u}{\delta x}$$ **[1 Mark]**

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \lim_{\delta x\to 0}\frac{\delta y}{\delta u} \times \lim_{\delta x\to 0}\frac{\delta u}{\delta x} \qquad ...(1)$$

Since y is a differentiable function of u , $\lim_{\delta u\to 0}\left(\frac{\delta y}{\delta u}\right)$ exists and $\lim_{\delta u\to 0}\left(\frac{\delta u}{\delta x}\right)$ exists as u is a differentiable function of x.

R.H.S of (1) exists. Now

$$\lim_{\delta u\to 0}\frac{\delta y}{\delta u} = \frac{dy}{du} \quad \text{and} \quad \lim_{\delta x\to 0}\frac{\delta u}{\delta x} = \frac{du}{dx}$$

$$\therefore \lim_{\delta x\to 0}\frac{\delta y}{\delta x} = \frac{dy}{du}\times\frac{du}{dx}$$ **[1 Mark]**

Since R.H.S. exists $\therefore$ L.H.S. also exists and

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$ This is called chain rule **[1 Mark]**

Topic:Differentiation; Sub-Topic:Theorem_L-1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii) Since there are six part patients n = 6, if a patients recovers, we call the outcomes a success.

Thus, $P = P\text{ (success)} = 0.5,\ q = 1 - p = 0.5$

Let X : Number of patients, who recovered out of 6

Then $X \sim B\ (n = 6, P = 0.5)$ **[1 Mark]**

The p.m.f of X is given as

$$\therefore P(X = x) = {}^{n}C_x\, p^x q^{n-x}, x = 0, 1, 2, ..., 6$$

(a) Probability (None will recover) $= P(X = 0)\ {}^{6}C_0\,(0.5)^0\,(0.5)^6$

$= (0.5)^6$

$= 0.015625$ **[1 Mark]**

(b) Probability (half of them will recover) $= {}^{6}C_3\,(0.5)^3\,(0.5)^3$

$$= \frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times (0.5)^3 \times (0.5)^3$$

$= 0.3125$ **[1 Mark]**

Topic:Binomial dist._; Sub-topic:Binomial dist._ L- 2_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) $$\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x}\,dx$$

Let $$I = \int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x}\,dx \quad ......(1)$$

We use the property, $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Here, $a = \pi$. Hence changing x by $\pi - x$, we get

$$I = \int_0^{\pi} \frac{\pi - x}{a^2\left[\cos(\pi - x)\right]^2 + b^2\left[\sin(\pi - x)\right]^2}\,dx$$

$$= \int_0^{\pi} \frac{\pi - x}{a^2\cos^2 x + b^2\sin^2 x}\,dx \quad .....(2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2\sin^2 x}\,dx + \int_0^{\pi} \frac{\pi - x}{a^2\cos^2 x + b^2\sin^2 x}\,dx$$ **[1 Mark]**

$$= \int_0^{\pi} \frac{x + \pi - x}{a^2\cos^2 x + b^2\sin^2 x}\,dx$$

$$= \pi \int_0^{\pi} \frac{1}{a^2 \cos^2 x + b^2 \sin^2 x} dx$$

Dividing numerator and denominator by $\cos^2 x$, we get

$$2I = \pi \int_0^{\pi} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$$

We use the property, $\int_0^{2a} f(x) dx = \int_0^{a} [f(x) + f(2a - x)] dx$

$$\therefore 2I = \pi \int_0^{\pi/2} \left\{ \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} + \frac{[\sec(\pi - x)]^2}{a^2 + b^2 [\tan(\pi - x)]^2} \right\} dx$$ **[1 Mark]**

$$= \pi \int_0^{\pi/2} \left[\frac{\sec^2 x}{a^2 + b^2 \tan^2 x} + \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} \right] dx$$

$$\therefore 2I = 2\pi \int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$$

$$\therefore I = \pi \int_0^{\pi/2} \frac{\sec x}{a^2 + b^2 \tan^2 x} dx$$

Put $\tan x = t$ $\qquad \therefore \sec^2 x \, dx = dt$

When $x = 0, t = \tan 0 = 0$

When $x = \frac{\pi}{2}, t = \tan\frac{\pi}{2} = \infty$

$$\therefore I = \pi \int_0^{\infty} \frac{1}{a^2 + b^2 t^2} dt$$

$$= \frac{\pi}{b^2} \int_0^{\infty} \frac{1}{\frac{a^2}{b^2} + t^2} dt$$

$$= \frac{\pi}{b^2} \times \frac{1}{(a/b)} \left[\tan^{-1}\left(\frac{t}{a/b}\right) \right]_0^{\infty}$$

$$= \frac{\pi}{ab} \left[\tan^{-1} \infty - \tan^{-1} 0 \right]$$

$$= \frac{\pi}{ab} \left[\frac{\pi}{2} - 0 \right] = \frac{\pi^2}{2ab}$$ **[1 Mark]**

Topic:Definite integration_; Sub-topic:Properties of definite integration_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

Q.5 (B)

(i) $f(0)=\log\left(\frac{2}{3}\right)$ **[1 Mark]**

...Given .....(1)

$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{4^x-e^x}{6^x-1}$ **[1 Mark]**

$=\lim_{x\to 0}\frac{(4^x-1)-(e^x-1)}{6^x-1}$

$=\lim_{x\to 0}\frac{\left(\frac{4^x-1}{x}\right)-\left(\frac{e^x-1}{x}\right)}{\left(\frac{6^x-1}{x}\right)}$ ....$[x\to 0, x\neq 0]$

$=\frac{\lim_{x\to 0}\frac{4^x-1}{x}-\lim_{x\to 0}\frac{e^x-1}{x}}{\lim_{x\to 0}\frac{6^x-1}{x}}$

$=\frac{(\log 4)-1}{\log 6}$ ...$\left[\because \lim_{x\to 0}\frac{a^x-1}{x}=\log a\right]$

$\therefore \lim_{x\to 0} f(x)=\frac{(\log 4)-1}{\log 6}$ .....(2)

From (1) and (2), $\lim_{x\to 0} f(x)\neq f(0)$

$\therefore f$ is **discontinuous** at $x=0$ **[1 Mark]**

Here $\lim_{x\to 0} f(x)$ exists but not equal to $f(0)$. Hence, the discontinuity at $x=0$ is removable and can be removed by redefining the function as follows :

$f(x)=\frac{4^x-e^x}{6^x-1}$, $\quad for\ x\neq 0$

$=\frac{(\log 4)-1}{\log 6}$, $\quad for\ x=0$ **[1 Mark]**

Topic:Continuity_; Sub-topic:At a point_ L- 2_Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii) Let $I = \int \sqrt{a^2 - x^2}\,dx$

$= \int \sqrt{a^2 - x^2}.1\,dx$

$= \sqrt{a^2 - x^2}.\int 1dx - \int \left[\frac{d}{dx}(\sqrt{a^2 - x^2}).\int 1dx\right]dx$ (By integration by parts) **[1 Mark]**

$= \sqrt{a^2 - x^2}.x - \int \frac{1}{2\sqrt{a^2 - x^2}}(0 - 2x).x\,dx$

$= \sqrt{a^2 - x^2}\,.x - \int \frac{-x^2}{\sqrt{a^2 - x^2}}.xdx$ **[1 Mark]**

$= x\sqrt{a^2 - x^2} - \int \frac{a^2 - x^2 - a^2}{\sqrt{a^2 - x^2}}dx$

$= x\sqrt{a^2 - x^2} - \int \sqrt{a^2 - x^2}\,dx + a^2 \int \frac{dx}{\sqrt{a^2 - x^2}}$

$= x\sqrt{a^2 - x^2} - I + a^2 \int \frac{dx}{\sqrt{a^2 - x^2}}$ **[1 Mark]**

$\therefore\ 2I = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right) + c$

$\therefore\ I = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$ **[1 Mark]**

Topic:Integration_; Sub-topic:Theorem_ L- 2_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) Let $\theta^0 C$ he the temperature of the body at time t. The temperature of the air is 10^0C , i.e. $\theta_0 = 10$

According to the Newton's Law of cooling

$\frac{d\theta}{dt} \propto \theta - \theta_0$

$\therefore \frac{d\theta}{dt} = -K(\theta - \theta_0)$; where $K > 0$

$\therefore \frac{d\theta}{dt} = -K(\theta - 10)$ **[1 Mark]**

$\therefore \frac{1}{\theta - 10}d\theta = -kdt$

Intergrating both sides, we get

$\int \frac{1}{\theta - 10}d\theta = -K\int dt$

$\therefore \log(\theta - 10) = -Kt + C$ **[1 Mark]** when t = 0 , $\theta = 110^0 C$

$\therefore \log(110-10) = -K \times 0 + C$ $\qquad\qquad \therefore C = \log 100$

$\therefore \log(\theta - 10) = -Kt + \log 100$

$\log\left(\frac{\theta - 10}{100}\right) = -Kt$

Also $\theta = 60^0 C$; when t = 1

$\therefore \log\left(\frac{60-10}{100}\right) = -K \times 1$

$\therefore K = -\log\left(\frac{1}{2}\right)$

$\therefore \log\left(\frac{\theta - 10}{100}\right) = t \log\left(\frac{1}{2}\right)$ **[1 Mark]**

when $\theta = 35^0 C$, then

$\log\left(\frac{35-10}{100}\right) = t \log\left(\frac{1}{2}\right)$

$\therefore \log\left(\frac{25}{100}\right) = t \log\left(\frac{1}{2}\right)$

$\log\left(\frac{1}{4}\right) = t \log\left(\frac{1}{2}\right)$

$-\log 4 = -t \log 2$

$t = \frac{\log 4}{\log 2} = 2$

The additional time required for body to cool to 35^0C = $(2-1) = 1\, hour$ **[1 Mark]**

Topic:Diff. equation_; Sub-topic:Application_ L- 2_Target-2016_XII-HSC Baord (40) Test_Mathematics

Q.6 (A)

(i) Since 'a' lies between 0 and 2a,

we have,

$$\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_a^{2a} f(x)dx \quad (0 < a < 2a)$$

(by $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$) **[1 Mark]**

$= I_1 + I_2$ .....(Say)

$I_2 = \int_a^{2a} f(x)\,dx$

Put $x = 2a - t$

$\therefore dx = -dt$

When $x = a, 2a - t = a \quad \therefore t = a$

When $x = 2a, 2a - t = 2a \quad \therefore t = 0$ **[1 Mark]**

$\therefore I_2 = \int_0^{2a} f(x)dx = \int_a^0 f(2a-t)(-dt)$

$= -\int_a^0 f(2a-t)\,dt = \int_0^a f(2a-t)\,dt$ [By $\int_a^b f(x)dx = -\int_b^a f(x)\,dx$]

$= \int_0^a f(2a-x)\,dx$ [By $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$]

$\therefore \int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

$= \int_0^a [f(x) + f(2a-x)]\,dx.$ **[1 Mark]**

Topic:Definite integration _; Sub-topic:Property_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii) Let $I = \int \frac{1 + \log x}{x(2 + \log x)(3 + \log x)}$

$= \int \frac{1 + \log x}{(2 + \log x)(3 + \log x)} \cdot \frac{1}{x} dx$

Put $\log x = t \quad \therefore \frac{1}{x} dx = dt$ **[1 Mark]**

$\therefore I = \int \frac{1 + t}{(2 + t)(3 + t)}\, dt$

Let $\frac{1 + t}{(2 + t)(3 + t)} = \frac{A}{2 + t} + \frac{B}{3 + t}$

$\therefore 1 + t = A(3 + t) + B(2 + t)$

Put $2 + t = 0$

$1 - 2 = A(1) + B(0) \quad \therefore A = -1$

Put $3 + t = 0$, i.e. $t = -3$, we get

$1 - 3 = A(0) + B(-1) \quad \therefore B = 2$

$\therefore \frac{1+t}{(2+t)(3+t)} = \frac{-1}{2+t} + \frac{2}{3+t}$ **[1 Mark]**

$\therefore I = \int\left(\frac{-1}{2+t} + \frac{2}{3+t}\right) dt$

$= -\int \frac{1}{2+t}\, dt + 2\int \frac{1}{3+t}\, dt$

$= -\log|2+t| + 2\log|3+t| + c$

$= -\log|2+\log x| + 2\log|3+\log x| + c$

$= -\log|2+\log x| + \log\left|(3+\log x)^2\right| + c$

$= \log\left|\frac{(3+\log x)^2}{2+\log x}\right| + c$ **[1 Mark]**

Topic:Integration_; Sub-topic:Partial fraction_ L- 2_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) $y = \cos^{-1}(2x\sqrt{1-x^2})$

Let $x = \sin\theta \Rightarrow \theta = \sin^{-1} x$ **[1 Mark]**

$y = \cos^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \cos^{-1}(\sin 2\theta)$

$= \cos^{-1}\left(\cos\left(\frac{\pi}{2} - 2\theta\right)\right)$

$y = \frac{\pi}{2} - 2\theta$

$\therefore y = \frac{\pi}{2} - 2\sin^{-1} x$ **[1 Mark]**

Diff. w.r.t. x

$\frac{dy}{dx} = 0 - \frac{2}{\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$ **[1 Mark]**

Topic:Derivative_; Sub-topic:Inverse function_ L- 1_Target-2016_XII-HSC Baord (40) Test_Mathematics

Q.6 (B)

(i) $\cos(x+y)\,dy = dx$

$\therefore \frac{dy}{dx} = \frac{1}{\cos(x+y)}$

Let $x + y = t$

$\therefore 1 + \frac{dy}{dx} = \frac{dt}{dx}$ **[1 Mark]**

$$\therefore \frac{dt}{dx}-1=\frac{1}{\cos t}$$

$$\frac{dt}{dx}=\frac{1}{\cos t}+1$$

$$\frac{dt}{dx}=\frac{1+\cos t}{\cos t}$$

$$\therefore \frac{\cos t}{1+\cos t}\,dt=dx$$

Integrating both side.

$$\therefore \int \frac{\cos t}{1+\cos t}\,dt=\int dx$$ **[1 Mark]**

$$\therefore \int \frac{\cos t\left(1-\cos t\right)}{\sin^2 t}dt=x+c$$

$$\therefore \int \left(\operatorname{cosec} t.\cot t-\cot^2 t\right)dt=x+c$$

$$\therefore \int \left(\operatorname{cosec} t.\cot t-\operatorname{cosec}^2 t+1\right)dt=x+c$$

$$-\operatorname{cosec} t+\cot t+t=x+c$$

$$\frac{\cos t}{\sin t}-\frac{1}{\sin t}+t=x+c$$

$$-\tan\left(\frac{x+y}{2}\right)+x+y=x+c$$

$$\therefore\ -\tan\left(\frac{x+y}{2}\right)+y=c$$ **[1 Mark]**

puting $x=0,\ y=0$

$$\therefore\ -\tan\left(\frac{0+0}{2}\right)+0=c$$

$$\therefore c=0$$

$$\therefore\ y=\tan\left(\frac{x+y}{2}\right)$$ **[1 Mark]**

Topic:Diff. equation_; Sub-topic:Method of substitution_ L- 3_Target-2016_XII-HSC Baord (40) Test_Mathematics

(ii) Let r be the radius of the circle and x be the length of the side of the square.

Then, Circumference of the circle + perimeter of the square = total length of the wire

$\therefore 2\pi r + 4x = l \qquad \therefore 2\pi r = l-4x$

$$\therefore r=\frac{l-4x}{2\pi} \qquad ......(1)$$

Let A = Area of the circle + Area of the square

$= \pi r^2 + x^2$

$$= \pi\left(\frac{l-4x}{2\pi}\right)^2 + x^2$$ **[1 Mark]**

$$= \frac{l^2-8lx+16x^2}{4\pi} + x^2$$

$$= \frac{4\pi x^2+16x^2-8lx+l^2}{4\pi}$$

$$= \frac{4(\pi+4)x^2-8lx+l^2}{4\pi}$$

$$\therefore \frac{dA}{dx} = \frac{8(\pi+4)x-8l}{4\pi}$$

and $\dfrac{d^2A}{dx^2} = \dfrac{8(\pi+4)}{4\pi} = \dfrac{2(\pi+4)}{\pi}$ **[1 Mark]**

Now. A is minimum, if $\dfrac{dA}{dx} = 0$

i.e. if $\dfrac{8(\pi+4)x-8l}{4\pi} = 0$

i.e. if $8(\pi+4)x-8l=0$

i.e. if $8(\pi+4)x = 8l$ **[1 Mark]**

i.e. if $x = \dfrac{8l}{8(\pi+4)} = \dfrac{1}{\pi+4}$

Also, $\left(\dfrac{d^2A}{dx^2}\right)_{x=\frac{1}{\pi+4}} = \dfrac{2(\pi+4)}{\pi} > 0$

By the second derivative test, A is minimum when $x = \dfrac{l}{\pi+4}$

From (1), $r = \dfrac{l-4\left(\dfrac{l}{\pi+4}\right)}{2\pi} = \dfrac{\pi l+4l-4l}{2\pi}$

$$= \frac{\pi l}{2\pi} = \frac{l}{2}$$

$\Rightarrow$ Then, the sum of the areas of the circle and the square is the least, when radius of the circle is half of side of the square. **[1 Mark]**

Topic:Applications of Derivatives; Sub-Topic:Maxima-Minima_L-2_Target-2016_XII-HSC Baord (40) Test_Mathematics

(iii) Given p.d.f

$f(x) = \frac{x}{32} \qquad 0 < x < 8$

$c.d.f = F(X) = \int_0^X f(x)\,dx$ **[1 Mark]**

$= \int_0^X \frac{x}{32}\,dx$

$= \left[\frac{x^2}{64}\right]_0^X$

$= \frac{X^2}{64}$ **[1 Mark]**

at $x = 0.5$

$F(X) = F(0.5) = \frac{(0.5)^2}{64} = \frac{0.25}{64} = 0.0039$ **[1 Mark]**

For any value of x greater than 8

$F(X) = 1$

$\therefore F(9) = 1$ **[1 Mark]**

Topic:Probability distribution_; Sub-topic:P.d.f_ L- 3_Target-2016_XII-HSC Baord (40) Test_Mathematics