XII CBSE - BOARD - MARCH - 2016

CODE (65/ 3 /S) SET -3

Date: 14.03.2016

MATHEMATICS - SOLUTIONS

1. $\left|A \cdot A^T\right| = |A|^2 = (5)^2 = 25$ **[1 mark]**

Topic:Determinants; Sub-Topic: Determinant_L-_1_Target-2016_ XII-CBSE Board Exam_Mathematics

2. $P_1 \rightarrow \vec{r}\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) - 4 = 0$

$P_2 \rightarrow \vec{r}\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) + 10 = 0$

Distance between plane P_1 and P_2 is

$$d = \left|\frac{10 - (-4)}{\left|2\hat{i} - 3\hat{j} + 6\hat{k}\right|}\right|$$

$$d = \left|\frac{14}{\sqrt{4 + 9 + 36}}\right|$$

$$d = \left|\frac{14}{50}\right|$$

$$d = \frac{7}{25}$$ **[1 mark]**

Topic:3D; Sub-Topic: Plane_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

3. $\left|\vec{a} - 2\vec{b}\right|^2 = |\vec{a}|^2 + 4\left|\vec{b}\right|^2 - 4\vec{a} \cdot \vec{b}$

$\left|\vec{a} - 2\vec{b}\right|^2 = 1 + 4 - 4|\vec{a}| \cdot \left|\vec{b}\right| \cdot \cos\theta$

$1^2 = 5 - 4\cos\theta$

$-4 = -4\cos\theta$

$\cos\theta = 1$

$\cos\theta = \cos 0$

$\theta = 2n\pi$ where $n \varepsilon z$ **[1 mark]**

Topic:Vector; Sub-Topic: Dot product_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

4. $AB=\begin{bmatrix}1 & 2\\ 3 & -1\end{bmatrix}\begin{bmatrix}1 & -4\\ 3 & -2\end{bmatrix}$

$AB=\begin{bmatrix}1+6 & -4-4\\ 3-3 & -12+2\end{bmatrix}$

$AB=\begin{bmatrix}7 & -8\\ 0 & -10\end{bmatrix}$

$|AB|= 7\times(-10)-(-8)(0)$

$|AB|=-70$ **[1 mark]**

Topic:Determinant; Sub-Topic: Determinant_L-_1_Target-2016_ XII-CBSE Board Exam_Mathematics

5. $A=\begin{bmatrix}0 & 3\\ 2 & -5\end{bmatrix}$, $KA=\begin{bmatrix}0 & 4a\\ -8 & 5b\end{bmatrix}$

$K\begin{bmatrix}0 & 3\\ 2 & -5\end{bmatrix}=\begin{bmatrix}0 & 4a\\ -8 & 5b\end{bmatrix}$

$\begin{bmatrix}0 & 3K\\ 2K & -5K\end{bmatrix}=\begin{bmatrix}0 & 4a\\ -8 & 5b\end{bmatrix}$

Compairing both sides

$\rightarrow \quad 2K=-8$

$K=-4$ ....(1)

$3K=4a$ ....(2)

(3) put $K=-4$ in equation (2)

$a=-3$

$\rightarrow \quad -5K=5b$

$b=-K$

$\rightarrow \quad b=4$ **[1 mark]**

Topic:Matrices; Sub-Topic: Comparison of two matrices_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

6. $|\vec{a}|=\frac{1}{2}, |\vec{b}|=\frac{4}{\sqrt{3}},$

$|\vec{a}\times\vec{b}|=|\vec{a}|\cdot|\vec{b}|\cdot\sin\theta$

$\frac{1}{\sqrt{3}}=\frac{1}{2}\times\frac{4}{\sqrt{3}}\cdot\sin\theta$

$\sin\theta=\frac{1}{2}$

$\theta=\frac{\pi}{6}$

$\left|\vec{a}\cdot\vec{b}\right| = |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$

$= \frac{1}{2}\times\frac{4}{\sqrt{3}}\cos\frac{\pi}{6}$

$= \frac{1}{2}\times\frac{4}{\sqrt{3}}\times\frac{\sqrt{3}}{2}$

$\left[\left|\vec{a}\cdot\vec{b}\right| = 1\right]$ **[1 mark]**

Topic:Vectors_; Sub-Topic: Dot and cross products_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

7. at x = 0

LHL

$\lim_{x\to 0^-} \quad K\cdot\sin\frac{\pi}{2}(x+1)$ **[1 mark]**

$\lim_{h\to 0} \quad K\cdot\sin\frac{\pi}{2}(0-h+1)$

$\lim_{h\to 0} \quad \left(K\cdot\sin\left(\frac{\pi}{2}-\frac{\pi h}{2}\right)\right)$

$= \lim_{h\to 0} \quad K\cos\left(\frac{\pi h}{2}\right)$ **[1 mark]**

$= \quad K$

RHL

$= \lim_{x\to 0^+} \frac{\tan x - \sin x}{x^3}$

$= \lim_{h\to 0} \frac{\tan h - \sin h}{h^3}$

$= \lim_{h\to 0} \frac{\frac{\sin h}{\cos h} - \sin h}{h^3}$

$= \lim_{h\to 0} \frac{\sin h(1-\cos h)}{\cos h\cdot h^3}$ **[1 mark]**

$= \lim_{h\to 0} \frac{1-\cos h}{h^2}$

$= \lim_{h\to 0} \frac{2.\sin^2\frac{h}{2}}{4\frac{h^2}{4}} = \frac{1}{2}$

function is continuous at x = 0

$\therefore \quad LHL = RHL = f(0)$

$\left[K = \frac{1}{2}\right]$ **[1 mark]**

Topic:Continuity and differentiability Sub-Topic: Continuity_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

8. Let $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$ and $\upsilon = \cos^{-1}x^2$

putting $x^2 = \cos\theta$, we get **[1 Mark]**

$u = \tan^{-1}\left\{\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right\}$

$\Rightarrow u = \tan^{-1}\left\{\frac{\sqrt{2\cos^2\theta/2}-\sqrt{2\sin^2\theta/2}}{\sqrt{2\cos^2\theta/2}+\sqrt{2\sin^2\theta/2}}\right\}$ **[1 Mark]**

$\Rightarrow u = \tan^{-1}\left\{\frac{\cos\theta/2-\sin\theta/2}{\cos\theta/2+\sin\theta/2}\right\}$ **[1 Mark]**

$\Rightarrow u = \tan^{-1}\left\{\frac{1-\tan\theta/2}{1+\tan\theta/2}\right\}$ [Dividing numerator and denominator by $\cos\frac{\theta}{2}$]

$\Rightarrow u = \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right\}$

$\Rightarrow u = \frac{\pi}{4}-\frac{1}{2}\theta = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$

$\therefore \frac{du}{dx} = -\frac{1}{2}\times\frac{-2x}{\sqrt{1-x^4}} = \frac{x}{1-x^4}$

Now, $\upsilon = \cos^{-1}x^2 \Rightarrow \frac{d\upsilon}{dx} = \frac{-2x}{\sqrt{1-x^4}}$ **[1 Mark]**

$\therefore \frac{du}{d\upsilon} = \frac{du/dx}{d\upsilon/dx} = -\frac{1}{2}$

Topic:Continuity and differentiabiluty; Sub-Topic: Inverse functions_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

OR

Let

$y = (\sin 2x)^x + \sin^{-1}\sqrt{3x}$ **[1 mark]**

Let $u = (\sin 2x)^x$, $v = \sin^{-1}\sqrt{3x}$

Taking logarithm of both sides

$\log u = x \log \sin 2x$

differentiating both sides w.r.t.x

$$\frac{1}{u}\frac{du}{dx} = \frac{2x.\cos 2x}{\sin 2x} + \log \sin 2x$$

$$\frac{du}{dx} = u\left[2x \cdot \cot 2x + \log \sin 2x\right]$$ **[1 mark]**

$$\frac{du}{dx} = (\sin 2x)^x \left[2x \cdot \cot 2x + \log \cdot \sin 2x\right]$$

$v = \sin^{-1}\sqrt{3}x$

differentiating both sides w.r.t x

$$\frac{dv}{dx} = \frac{1}{1-\left(\sqrt{3}x\right)^2}\sqrt{3}$$ **[1 mark]**

$$\frac{dv}{dx} = \frac{\sqrt{3}}{1-3x^2}$$

We know that

$y = u + v$

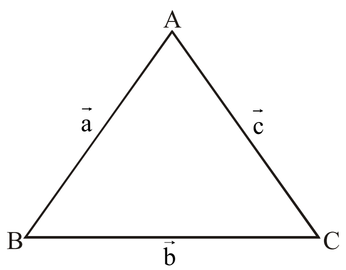
differentiating both sides w.r.t x

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{dy}{dx} = (\sin 2x)^x \left[2 \cdot x \cdot \cot 2x + \log \sin 2x\right] + \frac{\sqrt{3}}{1-3x^2}$$ **[1 mark]**

Topic:Continuity and differentiabiluty; Sub-Topic: Inverse functions_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

9.



Area of $\Delta ABC = \frac{1}{2}\left|\bar{b} \times \bar{c}\right|$

$$5\sqrt{6} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix}$$ **[1 Mark]**

$$10\sqrt{6} = \left|\hat{i}(-6-4) - \hat{j}(-2s-12) + \hat{k}(8-9)\right|$$

$10\sqrt{6}=\sqrt{100+(2s+12)^2+(8-9)^2}$

Square both sides

$600=100+4s^2+48s+144+8^2-18s+81$ **[1 Mark]**

$500=5s^2+30s+225$

$s^2+60s-275=0$

$s^2+60s-55=0$

$(s-5)(s+11)=0$

$s=5$ and $s=-11$

and given

$\vec{a}=\vec{b}+\vec{c}$ **[1 Mark]**

$p\hat{i}+9\hat{j}+r\hat{k}=(r+3)\hat{i}+4\hat{j}+2\hat{k}$

$p=r+3, q=4, r=2$

For r = 5, for r = – 11

p = 8, p = – 8 **[1 Mark]**

then (p, q, r, s) = (8 , 4 , 2 , 5) or (–8 , 4 , 2, – 11)

Topic:Vectors; Sub-Topic: Vector_L-_3_Target-2016_ XII-CBSE Board Exam_Mathematics

10. $P(E_1)=P(E_2)=\frac{1}{2}$

$A\rightarrow 2W\ \&1R$

$P(A/E_1)=\frac{{}^3C_2\times{}^1C_1}{{}^7C_3}=\frac{3\times1}{{}^7C_3}=\frac{3}{35}$ **[1 mark]**

$P(A/E_2)=\frac{{}^4C_2\times{}^3C_1}{{}^7C_3}=\frac{6\times3}{35}=\frac{18}{35}$ **[1 mark]**

By Baye's theorem

$P(E_2/A)=\frac{P(A/E_2)P(E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$ **[1 mark]**

$=\frac{\frac{18}{35}\times\frac{1}{2}}{\frac{1}{2}\times\frac{3}{35}+\frac{1}{2}\times\frac{18}{35}}=\frac{18}{3+18}$

$=\frac{18}{21}=\frac{6}{7}$ **[1 mark]**

Topic:Probability; Sub-Topic: Bayes theorem_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

11. $A = xy$

$(x-50)(y+50) = xy$

$xy + 50x - 50y - 2500 = xy$

$x - y = 50$ ......(1) **[1 mark]**

$(x-10)(y-20) = xy - 5300$

$xy - 20x - 10y + 200 = xy - 5300$

$-20x - 10y = -5300 - 200$

$20x + 10y = 5500$

$2x + y = 550$ ......(2) **[1 mark]**

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

A $\quad X = B$

$X = A^{-1}B$ ......(3) **[1 mark]**

$$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$\therefore$ from (3)

$$X = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$$\boxed{\begin{matrix} x = 200 \\ y = 150 \end{matrix}}$$

He wants to donate because he wants the upliftment of village students. **[1 mark]**

Topic:Matrices; Sub-Topic: Application of matrices_L-2_Target-2016_ XII-CBSE Board Exam_Mathematics

12. We have,

$$2y e^{x/y} dx + \left(y - 2x e^{x/y}\right) dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x e^{x/y} - y}{2y e^{x/y}}$$ **[1 Mark]**

Clearly, the given differential equation is a homogeneous differential equation. As the right hand side of (i) is expressible as a function of $\frac{x}{y}$. So, we put $x = \upsilon y$ and $\frac{dx}{dy} = \upsilon + y\frac{d\upsilon}{dy}$ to get

$$\upsilon + y\frac{d\upsilon}{dy} = \frac{2\upsilon e^{\upsilon} - 1}{2e^{\upsilon}}$$ **[1 Mark]**

$$\Rightarrow y\frac{d\upsilon}{dy} = \frac{2\upsilon e^{\upsilon} - 1}{2e\upsilon} - \upsilon$$

$$\Rightarrow y\frac{d\upsilon}{dy} = -\frac{1}{2e^{\upsilon}}$$

$$\Rightarrow 2ye^{\upsilon}d\upsilon = -dy$$

$$\Rightarrow 2e^{\upsilon}\,d\upsilon = -\frac{1}{y}dy$$

$$\Rightarrow 2\int e^{\upsilon}d\upsilon = -\int \frac{1}{y}dy$$ [On integrating]

$$\Rightarrow 2e^{\upsilon} = -\log\left|\frac{C}{y}\right|$$ **[1 mark]**

$$\Rightarrow e^{x/y} = \log\left|\frac{C}{y}\right|$$

Hence, $2e^{x/y} = \log\left|\frac{C}{y}\right|$ gives the general solution of the given differential equation. **[1 mark]**

Topic:Differential equation; Sub-Topic: Homogeneous equation_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

13. $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^3$

$$\frac{dy}{dx} + \left[\frac{-1}{x+1}\right]y = e^{3x}(x+1)^2$$ **[1 mark]**

$$\therefore P = \frac{-1}{x+1}, \qquad Q = e^{3x}(x+1)^2$$

$$IF = e^{\int \frac{-1}{x+1}dx} = e^{-\log(x+1)} = e^{\log\frac{1}{x+1}} = \frac{1}{x+1}$$

$$\therefore\ y\cdot(IF) = \int Q\cdot(IF)\,dx + C$$

$$y\cdot\frac{1}{x+1} = \int \frac{e^{3x}(x+1)^3}{(x+1)}\cdot dx + C$$ **[1 mark]**

$$\frac{y}{x+1} = \int e^{3x}(x+1)^2\,dx + C$$

$$\frac{y}{x+1} = \int (x+1)^2 e^{3x}dx + C$$

By parts method

$$\frac{y}{x+1}=(x+1)^2\frac{e^{3x}}{3}-\int 2(x+1)\frac{e^{3x}}{3}dx+C$$

$$\frac{y}{x+1}=\frac{(x+1)^2e^{3x}}{3}-\frac{2}{3}\int \underset{u}{(x+1)}\underset{v}{e^{3x}}\,dx+C$$ **[1 mark]**

$$\frac{y}{(x+1)}=\frac{(x+1)^2e^{3x}}{3}-\frac{2}{3}\left[\frac{(x+1)e^{3x}}{3}-\int\frac{1\cdot e^{3x}}{3}\cdot dx\right]+C$$

$$\frac{y}{x+1}=\frac{(x+1)^2e^{3x}}{3}-\frac{2}{3}\left[\frac{(x+1)e^{3x}}{3}-\frac{e^{3x}}{9}\right]+c$$

$$\frac{y}{x+1}=\frac{(x+1)^2e^{3x}}{3}-\frac{2}{9}(x+1)e^{3x}+\frac{2}{27}e^{3x}+c$$ **[1 mark]**

$$y=\frac{(x+1)^3e^{3x}}{3}-\frac{2}{9}\times(x+1)^2e^{3x}+\frac{2}{27}e^{3x}(x+1)+c(x+1)$$

Topic:Differential equation; Sub-Topic: Linear differential equation_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

14. $$\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$$

$\log_e x=t$

$\Rightarrow x=e^t \qquad \therefore dx=e^t dt$ **[1 mark]**

$$\therefore I=\int\left[\log(t)+\frac{1}{t^2}\right]e^t\cdot dt$$

$$=\int e^t\left[\log t+\frac{1}{t}-\frac{1}{t}+\frac{1}{t^2}\right]dt$$ **[1 mark]**

$$=\int e^t\left(\log t+\frac{1}{t}\right)dt+\int\left(\frac{-1}{t}+\frac{1}{t^2}\right)e^t dt \qquad \left\{\because\int e^x\left(f(x)+f'(x)\right)dx=e^x f(x)+c\right\}$$ **[1 mark]**

$$=e^t\log^t-e^t\frac{1}{t}+C$$

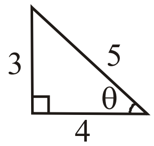
$$=\left[x\log(\log x)-\frac{x}{\log x}+C\right]$$ **[1 mark]**

Topic:Integrals; Sub-Topic: Substitution methods _L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

15. Prove $2\sin^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{17}{31}\right)=\frac{\pi}{4}$

LHS

Let $\sin^{-1}\left(\frac{3}{5}\right)=\theta$ **[1 mark]**



$\therefore \tan\theta=\frac{3}{4}$

$\theta=\tan^{-1}\left(\frac{3}{4}\right)$

$\therefore$ LHS : $2\tan^{-1}\left(\frac{3}{4}\right)-\tan^{-1}\left(\frac{17}{31}\right)$ **[1 mark]**

$$\tan^{-1}\left[\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right]-\tan^{-1}\left[\frac{17}{31}\right]$$

$$=\tan^{-1}\left[\frac{3/2}{7/16}\right]-\tan^{-1}\left[\frac{17}{31}\right]$$ **[1 mark]**

$$=\tan^{-1}\left[\frac{3\times8}{7}\right]-\tan^{-1}\left[\frac{17}{31}\right]$$

$$=\tan^{-1}\left[\frac{24}{7}\right]-\tan^{-1}\left[\frac{17}{31}\right]$$

$$=\tan^{-1}\left[\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right]$$

$$=\tan^{-1}\left[\frac{744-119}{217+408}\right]=\tan^{-1}\left[\frac{625}{625}\right]$$

$$=\tan^{-1}(1)$$

$$=\frac{\pi}{4}=\text{RHS}$$ **[1 mark]**

Topic:Inverse trigonometric functions_; Sub-Topic: Inverse trigonometric functions_L-_1_Target-2016_ XII-CBSE Board Exam_Mathematics

OR

Solve $\cos\left(\tan^{-1} x\right) = \sin\left(\cot^{-1}\frac{3}{4}\right)$ **[1 mark]**

$\cos\left[\tan^{-1} x\right] = \cos\left[\frac{\pi}{2} - \cot^{-1}\frac{3}{4}\right]$ $\left\{\therefore \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right\}$

$\tan^{-1} x = \frac{\pi}{2} - \cot^{-1}\frac{3}{4}$ **[1 mark]**

$\tan^{-1} x = \tan^{-1}\frac{3}{4}$ **[1 mark]**

$\Rightarrow \boxed{x = \frac{3}{4}}$ **[1 mark]**

Topic:Inverse trigonometric functions_; Sub-Topic: Inverse trigonometric functions_L-_1_Target-2016_ XII-CBSE Board Exam_Mathematics

16. Let $I = \int \frac{1-\sin x}{\sin x\left(1+\sin x\right)} dx$

$= \int \frac{1+\sin x - 2\sin x}{\sin x\left(1+\sin x\right)} dx$ **[1 mark]**

$= \int \frac{1}{\sin x} dx - 2\int \frac{1}{1+\sin x} dx$

$= \int \operatorname{cosec} x \, dx - 2\int \frac{1}{1+\sin x} dx$ **[1 mark]**

$= \int \operatorname{cosec} x \, dx - 2\int \frac{1-\sin x}{1-\sin^2 x} dx$

$= \int \operatorname{cosec} x \, dx - 2\int \frac{1-\sin x}{\cos^2 x} dx$ **[1 mark]**

$= \int \operatorname{cosec} x \, dx - 2\int \sec^2 x \, dx + 2\int \tan x \sec x \, dx$

$I = \boxed{\log\left|\operatorname{cosec} x - \cot x\right| - 2\tan x + 2\sec x + c}$ **[1 mark]**

Topic:Integral _; Sub-Topic: Integral_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

17. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$ ______(1)

$I = \int_0^{\pi/2} \frac{\sin^2\left(\pi/2 - x\right) dx}{\sin\left(\pi/2 - x\right) + \cos\left(\pi/2 - x\right)} dx$ **[1 mark]**

$= \int_0^{\pi/2} \frac{\cos^2 x \, dx}{\cos x + \sin x} dx$ ______(2)

(1) + (2)

$$2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$$ **[1 mark]**

$$2I = \int_0^{\pi/2} \frac{1}{\cos x + \sin x} dx$$

$$2I = \int_0^{\pi/2} \frac{1}{\frac{2\tan x/2}{1+\tan^2 x/2} + \frac{1-\tan^2 x/2}{1+\tan^2 x/2}}$$

$$= \int_0^{\pi/2} \frac{1+\tan^2 x/2}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int_0^{\pi/2} \frac{1+\tan^2 x/2}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int_0^{\pi/2} \frac{\sec^2 x/2}{2\tan x/2 + 1 - \tan^2 x/2} dx$$

Put $\tan x/2 = t \Rightarrow \sec^2\frac{x}{2} dx = 2dt$

$x = 0, \Rightarrow t = 0 \;\; \& \, x = \pi/2 \Rightarrow t = 1$

$$\therefore 2I = \int_0^1 \frac{2dt}{2t + 1 - t^2}$$ **[1 mark]**

$$= 2\int_0^1 \frac{dt}{\left(\sqrt{2}\right)^2 - (t-1)^2}$$

$$= 2 \times \frac{1}{2\sqrt{2}} \left[\log\left(\frac{\sqrt{2} + t - 1}{\sqrt{2} - t + 1} \right) \right]_0^1$$

$$2I = \frac{1}{\sqrt{2}} \left\{ 0 - \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1} \right) \right\}$$

$$= \frac{-1}{\sqrt{2}} \log\left[\frac{\left(\sqrt{2}-1\right)^2}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)} \right]$$

$$\therefore I = \frac{-1}{\sqrt{2}} \log\left(\sqrt{2}-1\right)$$ **[1 mark]**

Topic:Integral; Sub-Topic: Definite integral_L- 3 _Target-2016_ XII-CBSE Board Exam_Mathematics

OR

$$\int_0^1 \cot^{-1}(1-x+x^2)\,dx$$

$$=\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$$ **[1 mark]**

$$=\int_0^1 \tan^{-1}\left(\frac{1}{1-x(1-x)}\right)dx$$

$$=\int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx$$

$$=\int_0^1 \tan^{-1}x+\int_0^1 \tan^{-1}\left(1-(1-x)\right)dx$$ **[1 mark]**

$$=2\int_0^1 \tan^{-1}x = 2\int_0^1 \underset{u}{\tan^{-1}x}\cdot \underset{v}{1}dx$$

$$I = 2\left[x\tan^{-1}x\right]_0^1 - 2\int_0^1 \frac{x}{1+x^2}dx$$

$$=2\left[x\tan^{-1}x\right]_0^1 - \int_0^1 \frac{2x}{1+x^2}dx$$

$$=2\left[x\tan^{-1}x\right]_0^1 - \left[\log(1+x^2)\right]_0^1$$ **[1 mark]**

$$=2\left(\frac{\pi}{4}-0\right)-\left(\log 2-\log 1\right)$$

$$=\frac{\pi}{2}-\log 2$$ **[1 mark]**

Topic:Integral; Sub-Topic: Definite integral_L- 2 _Target-2016_ XII-CBSE Board Exam_Mathematics

18. Given Curve

$ay^2 = x^3$ ....(1)

and X - Co-ordinate

$x = am^2$ Substituting in (1)

$ay^2 = \left(am^2\right)^3$ **[1 mark]**

$ay^2 = a^3m^6$

$y^2 = a^2m^6$

$y = am^3$

So Point $(x_1, y_1) = (am^2, am^3)$

Now, $ay^2 = ax^3$

diff. w.r.t. x

$$a \cdot 2y\frac{dy}{dx} = 3a\,x^2$$

$$\frac{dy}{dx} = \frac{3ax^2}{2ay} = \frac{3}{2}\frac{x^2}{y}$$

So slope at point (am^2, am^3) **[1 mark]**

$$\text{Slope} = \left(\frac{dy}{dx}\right)_{(am^2, am^3)} = \frac{3}{2}\frac{a^2m^4}{am^3} = \frac{3}{2}am$$

Now slope of normal

$$= -\frac{1}{\frac{dy}{dx}} = \frac{-2}{3am} = \frac{-2}{3am}$$

So Equation of normal at point

$(x_1, y_1) = (am^2, am^3)$ and slope $(m) = \dfrac{-3}{3am}$ **[1 mark]**

$$y = y_1 = m(x - x_1)$$

$$y - am^3 = \frac{-2}{3am}(x - am^2)$$

$$3a\,my - 3a^2m^4 = -2x + 2a\,m^4$$

$$2x + 3a\,my = 2am^2 + 3a^2\,m^4$$

$$\left[2x + 3a\,my = am^2\left(2 + 3a\,m^2\right)\right]$$ **[1 mark]**

Topic:AOD_; Sub-Topic: Tangent nomral_L-_3_Target-2016_ XII-CBSE Board Exam_Mathematics

19. Plane passing through three points A(3,2,1), B(4,2,–2), C(6,5,–1)

then equation of line passing through three points

$$\begin{vmatrix} x - x & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 3 & y - 2 & z - 1 \\ 4 - 3 & 2 - 2 & -2 - 1 \\ 6 - 3 & 5 - 2 & -1 - 1 \end{vmatrix} = 0$$ **[1 mark]**

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$(x-3)(0+9)-(y-2)(-2+9)+(z-1)(3-0)=0$ **[1 mark]**

$9x-27-(y-2)(7)+(z-1)3=0$

$9x-27-7y+14+3z-3=0$

$\boxed{9x-7y+3z+16=0}$

and λ of value of if $A(3,2,1), B(4,2,-2), C(6,5,-1), D(\lambda,5,5)$ are coplanar

If $\begin{vmatrix} x_4-x_1 & y_1-x_1 & z_4-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

$$\begin{vmatrix} \lambda-3 & 5-2 & 5-1 \\ 4-3 & 2-2 & -2-1 \\ 6-3 & 5-2 & -1-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} \lambda-3 & 3 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$(\lambda-3)-3(-2+9)+4(3)=0$ **[1 mark]**

$9\lambda-27-21+12=0$

$9\lambda-36=0$

$9\lambda=36$

$\lambda=\frac{36}{9}=4$

$\lambda=4$ **[1 mark]**

Topic: 3 D; Sub-Topic: Line and plane_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

OR

Given

$\vec{n} = i + \hat{j} + 3\hat{k}$

and $d = \frac{4}{\sqrt{11}}$ **[1 mark]**

so equation of plane

$\vec{r}\cdot\hat{n} = d$

$(xi + y\hat{j} + z\hat{k})\cdot\frac{(\hat{i} + \hat{j} + 3\hat{k})}{\sqrt{1+1+9}} = \frac{4}{\sqrt{11}}$

$x + y + 3z = 4$ ...(1)

and line given,

So point

$\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ **[1 mark]**

$x\hat{i} + y\hat{j} + z\hat{k} = (-1 + 3\lambda)\hat{i} + (-2 + 4\lambda)\hat{j} + (-3 + 3\lambda)\hat{k}$

$\Rightarrow (x,y,z) = (-1+3\lambda, -2+4\lambda, -3+3\lambda)$

Point lie on plane (1)

$(-1 + 3\lambda) + (-2 + 4\lambda) + 3(-3 + 3\lambda) = 4$ **[1 mark]**

$-1 + 3\lambda - 2 + 4\lambda - 9 + 9\lambda = 4$

$16\lambda = 16$

$\lambda = 1$

So the co-ordinates of the point

$(x, y, z) = (-1 + 3\lambda, -2 + 4\lambda, -3 + 3\lambda)$

$= (-1 + 3, -2 + 4, -3 + 3) = (2, 2, 0)$

$(x, y, z) = (2, 2, 0)$ **[1 mark]**

Topic: 3 D; Sub-Topic: Line and plane_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

20. Fix positive Integer $= \{1,2,3,4,56,\}$

If we choose 3 numbers from 6 then we choose smallest number, 1, 2, 3, 4 then

$P(\text{smallest digit is } 1) = \frac{^5C_2}{^6C_3} = \frac{10}{20} = \frac{1}{2}$

$P(\text{smallest digit is } 2) = \frac{^4C_2}{^6C_3} = \frac{6}{20} = \frac{3}{10}$

$P(\text{smallest digit is } 3) = \frac{^3C_2}{^6C_3} = \frac{3}{20} = \frac{3}{20}$

$P(\text{smallest digit is } 4) = \frac{^2C_2}{^6C_3} = \frac{1}{20} = \frac{1}{20}$

$P(X = 5) = 0$

$P(X = 6) = 0$

So, probability distribution of X

X	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$	0	0

[2 marks]

Now for mean

$$E(x)=\sum P_i\cdot x_i=\frac{1}{2}\times 1+2\times\frac{3}{10}+3\times\frac{3}{20}+4\times\frac{1}{20}+=0+0$$

$$=\frac{1}{2}+\frac{6}{10}+\frac{9}{20}+\frac{4}{20}$$

$$=\frac{10+12+9+4}{20}=\frac{35}{20}=1.75$$

$\text{Mean}=E(x)=1.75$ **[2 marks]**

Now, $Var(X)=\sum_{i=1}^{x}P_i x_i^2-[E(x)]^2$

$$Var(X)=\left(\frac{1}{2}\times 1+\frac{3}{10}\times 4+\frac{3}{20}\times 9+\frac{1}{20}\times 16+0+0\right)-(1\cdot 75)^2$$

$$=\left(\frac{1}{2}+\frac{12}{10}+\frac{27}{20}+\frac{16}{20}\right)-(1\cdot 75)^2$$

$$=\left(\frac{10+24+27+16}{20}\right)-(1.75)^2$$

$$Var(X)=\frac{77}{20}-(1\cdot 75)^2$$

$$=\frac{7\cdot 7}{2}-(1\cdot 75)^2$$

$$=3.85-3.0625$$

$Var(X)=0.7875$ **[2 marks]**

Topic:Probability; Sub-Topic: Mean and variance_L- 2_Target-2016_ XII-CBSE Board Exam_Mathematics

21. Suppose the diet contains x units of food F_1 and y units of food F_2.

Foods	F_1	F_2	Minimum Required
Vitamin(A)	4	3	80
Minerals	3	6	100

Since one unit cost of food $F_1=5\,Rs.$

Since one unit cost of food $F_2=6\,Rs.$

Then total cost of one unit food

F_1 and F_2.

$Z = 5x + 6y$

F_1 food contain u units of vitamin of A and 3 units of minerals them minimum requirement of F_1 food

$4x + 3y \geq 80$

and similarly for F_2.

$3x + 6y \geq 100$

So formulation of LPP objective function

Minimum $Z = 5x + 6y$ **[2 marks]**

Subject to

$4x + 3y \geq 80$

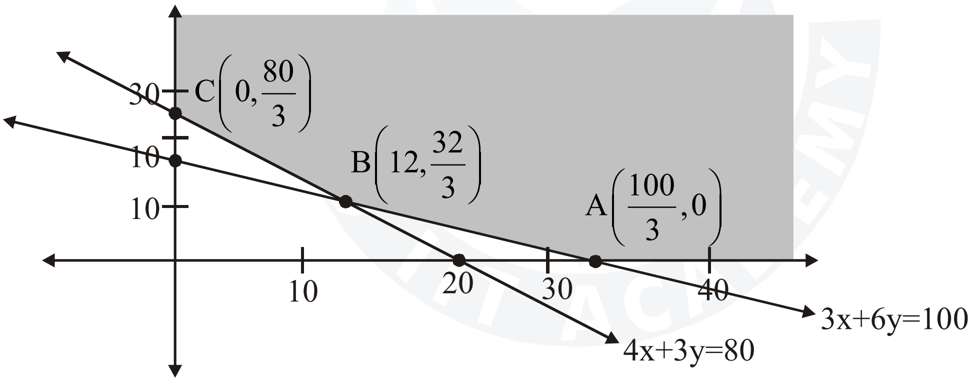
$3x + 6y \geq 100$

Now solution of L.P.P. by Graphical method

Convert Inequation to equation

$4x + 3y = 80$ ______ (1)

$3x + 6y = 100$ ______ (2)



[2 marks]

Solve (1) and (2) find B point

$(1)\times 2 - (2)$

$8x + 6y = 100$

$3x + 6y = 100$

$-\quad -\quad -$

$5x = 60$

$x = 12$

$48 + 3y = 80$

$3y = 32$

$y = \frac{32}{3}$

So three corner point

$A\left(\frac{100}{3},0\right), B\left(12,\frac{32}{3}\right), C\left(0,\frac{80}{3}\right)$

Points	Min z
$A\left(\frac{100}{3},0\right)$	166.6
$B\left(12,\frac{32}{3}\right)$	124
$C\left(0,\frac{80}{3}\right)$	16

So minimum cost of diet = 124 Rs. at point $B\left(12,\frac{32}{3}\right)$ **[2 marks]**

Topic:LPP; Sub-Topic: Solution of LPP_L-3_Target-2016_ XII-CBSE Board Exam_Mathematics

22. $C_1 \to C_1 - 2C_3 + C_2$

$$\begin{vmatrix} b^2+c^2+a^2 & a^2 & bc \\ c^2+a^2+b^2 & b^2 & ca \\ a^2+b^2+c^2 & c^2 & ab \end{vmatrix}$$

$$=\left(a^2+b^2+c^2\right)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$ **[2 marks]**

$R_1 \to R_1 \to R_2$ & $R_2 \to R_2 \to R_3$

$$=\left(a^2+b^2+c^2\right)\begin{vmatrix} 0 & a^2-b^2 & bc-ca \\ 0 & b^2-c^2 & ca-ab \\ 1 & c^2 & ab \end{vmatrix}$$

$$=\left(a^2+b^2+c^2\right)\begin{vmatrix} 0 & (a-b)(a+b) & -c(a+b) \\ 0 & (b-c)(b+c) & -a(b-c) \\ 1 & c^2 & ab \end{vmatrix}$$

$$=\left(a^2+b^2+c^2\right)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & b+c & -a \\ 1 & c^2 & ab \end{vmatrix}$$ **[2 marks]**

$R_1 \to R_1 - R_2$

$$=\left(a^2+b^2+c^2\right)(a-b)(b-c)\begin{vmatrix}0 & a-c & -c+a\\0 & b+c & -a\\1 & c^2 & ab\end{vmatrix}$$

$$=\left(a^2+b^2+c^2\right)(a-b)(b-c)(c-a)\begin{vmatrix}0 & -1 & -1\\0 & b+c & -a\\1 & c^2 & ab\end{vmatrix}$$

$$=\left(a^2+b^2+c^2\right)(a-b)(b-c)(c-a)(a+b+c)=\text{RHS}$$

$\text{LHS}=\text{RHS}$ **[2 marks]**

Topic:Determinants; Sub-Topic: Properties of determinants_L- 3_Target-2016_ XII-CBSE Board Exam_Mathematics

OR

$A.A^{-1}=I$

$$A=\begin{bmatrix}2 & -1 & 3\\-5 & 3 & 1\\-3 & 2 & 3\end{bmatrix}A^{-1}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$ **[2 marks]**

$R_1\to 2R_1+R_3$

$$\begin{bmatrix}1 & 0 & 9\\-5 & 3 & 1\\-3 & 2 & 3\end{bmatrix}A^{-1}=\begin{bmatrix}2 & 0 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$R_2\to R_2+5R_1,\ R_3\to R_3+3R_1$

$$\begin{bmatrix}1 & 0 & 9\\0 & 3 & 46\\0 & 2 & 30\end{bmatrix}A^{-1}=\begin{bmatrix}2 & 0 & 1\\10 & 1 & 5\\6 & 0 & 4\end{bmatrix}$$

$R_2\to R_2-R_3$

$$\begin{bmatrix}1 & 0 & 9\\0 & 1 & 16\\0 & 2 & 30\end{bmatrix}A^{-1}=\begin{bmatrix}2 & 0 & 1\\4 & 1 & 1\\6 & 0 & 4\end{bmatrix}$$

$R_3\to R_2-2R_2$

$$\begin{bmatrix}1 & 0 & 9\\0 & 1 & 16\\0 & 0 & -2\end{bmatrix}A^{-1}=\begin{bmatrix}2 & 0 & 1\\4 & 1 & 1\\-2 & -2 & 2\end{bmatrix}$$ **[2 marks]**

$R_3 \rightarrow -\frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & 16 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & 0 & 1 \\ 4 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 9R_3, R_2 \rightarrow R_2 - 16R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$$ **[2 marks]**

Topic:Matrices; Sub-Topic: Elementary transformation_L- 3_Target-2016_ XII-CBSE Board Exam_Mathematics

23. $\frac{x-1}{2} = \frac{y-(-1)}{-1} = \frac{z-0}{3}$

$\frac{x-0}{4} = \frac{y-2}{-2} = \frac{z-(-1)}{6}$

$\therefore a_2 - a_1 = 0-1, 2-(-1), -1-0$ **[2 marks]**

$= -1, 3, -1$

$\bar{b} = 2, -1, 3$

$$\therefore \left(\overline{a_2} - \overline{a_1}\right) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$$

$= \hat{i}(9-1) - \hat{j}(-3+2) + \hat{k}(1-6)$

$= 8\hat{i} + \hat{j} - 5\hat{k} = \bar{n}$

$\therefore \bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\bar{r}.\left(8\hat{i} + \hat{j} - 5\hat{k}\right) = \left(0\hat{i} - 2\hat{j} - \hat{k}\right)\left(8\hat{i} + \hat{j} - 5\hat{k}\right)$ **[2 marks]**

$8x + y - 5z = -2 + 5$

$\boxed{8x+y-5z-3=0}$

Line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{5}$

$\therefore 8(3)+1(1)-5(5)$

$=0$

$\Rightarrow$ line lies on plane **[2 marks]**

Topic:3D; Sub-Topic: Plane_L-_2_Target-2016_ XII-CBSE Board Exam_Mathematics

24. Let $R=\{(x,y): y^2 \le 6ax \text{ and } x^2+y^2 \le 16a^2\}$

$\Rightarrow \quad R=\{(x,y): y^2 \le 6ax\} \cap \{(x,y): x^2+y^2 \le 16a^2\}$

$=R_1 \cap R_2$

Point of intersection

$x^2+6ax=16a^2$

$x^2+6ax-16a^2=0$

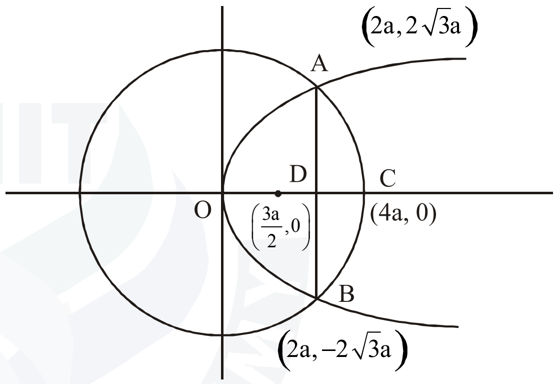
$(x+8a)(x-2a)=0$

$x=-8a \quad x=2a$

$x=2a$

$\therefore \quad y^2=12a^2$

$y=\pm\sqrt{12}a$



$\therefore$ Intersection point $A(2a, 2a\sqrt{3})$ $B(2a, -2a\sqrt{3})$ **[2 marks]**

Required area

$A=2[Area\ AODO + Area\ ADCA]$

So area of AODO $S=\int_0^{2a}\sqrt{6ax}\,dx$

and area of ADCA $=\int_{2a}^{4a}\sqrt{16a^2-x^2}\,dx$

$\therefore \quad A=2\int_0^{2a}\sqrt{6ax}\,dx+2\int_{2a}^{4a}\sqrt{16a^2-x^2}\,dx$ **[2 marks]**

$2\times\frac{2}{3}\times\frac{1}{6a}\left[(6ax)^{3/2}\right]_0^{2a}+2\left[\frac{1}{2}x\sqrt{16a^2-x^2}+\frac{1}{2}16a^2\sin^{-1}\left(\frac{x}{4a}\right)\right]_{2a}^{4a}$

$=16a^2\sqrt{3}+16a^2\sin^{-1}(1)-2a\times 2a\sqrt{3}-16a^2\sin^{-1}\left(\frac{1}{2}\right)$

$=4a^2\left(3\sqrt{3}+\frac{4\pi}{3}\right)$ sq.units **[2 marks]**

Topic:AOI; Sub-Topic: Area between two curves_L-_3_Target-2016_ XII-CBSE Board Exam_Mathematics

25 In order to prove that f is invertible, it is sufficient to show that $f : N \to Range(f)$ is a bijection.

f is one-one : For any $x, y \in N$, we find that

$f(x) = f(y)$

$\Rightarrow \quad 4x^2 + 12x + 15 = 4y^2 + 12y + 15$

$\Rightarrow \quad 4(x^2 - y^2) + 12(x - y) = 0$

$\Rightarrow \quad (x - y)(4x + 4y + 3) = 0$

$\Rightarrow \quad x - y = 0 \qquad [\because 4x + 4y + 3 \neq 0 \text{ for any } x, y \in N]$ **[2 marks]**

$\Rightarrow \quad x = y$

So, $f : N \to$ Range (f) is one-one.

Obviously, $f : N \to$ Range (f) is onto. Hence, $f : N \to$ Range (f) is invertible.

Let f^{-1} denote the inverse of f. Then,

$fof^{-1}(x) = x$ for all $x \in$ Range (f)

$\Rightarrow \quad f(f^{-1}(x)) = x$ for all $x \in$ Range (f)

$\Rightarrow \quad 4\{f^{-1}(x)\}^2 + 12f^{-1}(x) + 15 = x$ for all $x \in$ Range (f)

$\Rightarrow \quad 4\{f^{-1}(x)\}^2 + 12f^{-1}(x) + 15 - x = 0$

$$\Rightarrow \quad f^{-1}(x) = \frac{-12 \pm \sqrt{144 - 16(15 - x)}}{8}$$ **[2 marks]**

$$\Rightarrow \quad f^{-1}(x) = \frac{-12 \pm \sqrt{16x - 96}}{8} = \frac{-3 \pm \sqrt{x - 6}}{2}$$

$$\Rightarrow \quad f^{-1}(x) = \frac{-3 + \sqrt{x - 6}}{2} \qquad [\because f^{-1}(x) \in N \therefore f^{-1}(x) > 0]$$

$$f^{-1}(x) = \frac{-3 + \sqrt{x - 6}}{2}$$

$$f^{-1}(31) = \frac{-3 + \sqrt{31 - 6}}{2}$$

$$f^{-1}(31) = \frac{-3 + 5}{2} = 1$$

$$f^{-1}(87) = \frac{-3 + \sqrt{87 - 6}}{2} = \frac{-3 + 9}{2}$$

$f^{-1}(87) = 3$ **[2 marks]**

Topic:Relation and function; Sub-Topic: Inverse of a function_L- 2_Target-2016_ XII-CBSE Board Exam_Mathematics

26. $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

$f'(x) = 4x^3 - 24x^2 + 44x - 24$

$= 4(x^3 - 6x^2 + 11x - 6)$

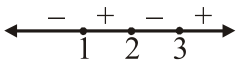
$= 4(x-1)(x^2 - 5x + 6)$ **[2 marks]**

For increasing

$f'(x) > 0$

$4(x-1)(x^2 - 5x + 6) > 0$

$(x-1)(x-2)(x-3) > 0$



[2 marks]

$x \varepsilon (1,2) \cup (3,\infty)$

For decreasing

$f'(x) < 0$

$(x-1)(x-2)(x-3) < 0$

$\Rightarrow x \varepsilon (2,3) \cup (-\infty,1)$ **[2 marks]**

Topic:AOD_; Sub-Topic: Increasing & decreasing_L-_3_Target-2016_ XII-CBSE Board Exam_Mathematics

OR

$f(x) = \sec x + \log \cos^2 x$

$f'(x) = \sec x \tan x + \frac{1}{\cos^2 x} \times 2\cos x(-\sin x)$

$= \sec x \tan x - 2\tan x$ **[2 marks]**

$= \tan x(\sec x - 2)$

For extreme values

$f'(x) = 0$

$\Rightarrow \tan x(\sec x - 2) = 0$

$\Rightarrow \tan x = 0 \quad \text{or } \sec x = 2$

$x = \pi \quad \text{or} \quad x = \frac{\pi}{3}$

$f''(x) = \tan x(\sec x \tan x) + (\sec x - 2)\sec^2 x$

$f''(\pi) = 0 + 1(1-2) = -2 < 0$

$\Rightarrow \text{Maxima at } x = \pi$

$\therefore f(\pi) = \sec \pi + \log \cos^2 \pi$ **[2 marks]**

$= -1 + \log 1 = [-1]$

$f''\left(\frac{\pi}{3}\right) = \sqrt{3}\left(2\times\sqrt{3}\right) + (2-2)4$

$\Rightarrow$ minima at $x = \frac{\pi}{3}$

$\therefore f\left(\frac{\pi}{3}\right) = \sec\frac{\pi}{3} + \log\cos^2\frac{\pi}{3}$

$= 2 + \log\left(\frac{1}{4}\right)$

$= 2 + \log 1 - \log 4$

$= 2 - \log 4 = 2 - 2\log 2$

$= [2(1-\log 2)]$ **[2 marks]**

Topic:AOD; Sub-Topic:Maxima & minima_L-_3_Target-2016_ XII-CBSE Board Exam_Mathematics