Reg. No. : ....

Name : ....

Code No. 2018

SECOND YEAR
SAY/IMPROVEMENT
JUNE 2018

Time : 2½ Hours
Cool-off time : 15 Minutes

Part – III

MATHEMATICS (SCIENCE)

Maximum : 80 Scores

General Instructions to Candidates :

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.

വിദ്യാർത്ഥികൾക്കുള്ള പൊതുനിർദ്ദേശങ്ങൾ :

- നിർദ്ദിഷ്ട സമയത്തിന് പുറമെ 15 മിനിറ്റ് 'കൂൾ ഓഫ് ടൈം' ഉണ്ടായിരിക്കും.
- 'കൂൾ ഓഫ് ടൈം' ചോദ്യങ്ങൾ പരിചയപ്പെടാനും ഉത്തരങ്ങൾ ആസൂത്രണം ചെയ്യാനും ഉപയോഗിക്കുക.
- ഉത്തരങ്ങൾ എഴുതുന്നതിന് മുമ്പ് ചോദ്യങ്ങൾ ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- നിർദ്ദേശങ്ങൾ മുഴുവനും ശ്രദ്ധാപൂർവ്വം വായിക്കണം.
- കണക്ക് കൂട്ടലുകൾ, ചിത്രങ്ങൾ, ഗ്രാഫുകൾ, എന്നിവ ഉത്തരപേപ്പറിൽ തന്നെ ഉണ്ടായിരിക്കണം.
- ചോദ്യങ്ങൾ മലയാളത്തിലും നല്കിയിട്ടുണ്ട്.
- ആവശ്യമുള്ള സ്ഥലത്ത് സമവാക്യങ്ങൾ കൊടുക്കണം.
- പ്രോഗ്രാമുകൾ ചെയ്യാനാകാത്ത കാൽക്കുലേറ്ററുകൾ ഒഴികെയുള്ള ഒരു ഇലക്ട്രോണിക് ഉപകരണവും പരീക്ഷാഹാളിൽ ഉപയോഗിക്കുവാൻ പാടില്ല.

Questions from 1 to 7 carry 3 scores each. Answer any 6. **(6 × 3 = 18)**

1. (a) Construct a 2 × 2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = 2i + j$. **(1)**

   (b) Find A^2. **(2)**

2. (a) If $\int \frac{f(x)}{x^2+1}\,dx = \log|x^2+1| + C$ then $f(x) =$ ______. **(1)**

   (b) Find $\int x e^x \, dx$. **(2)**

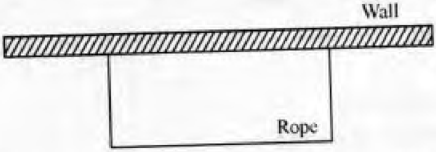
3. Form the differential equation of the family of all circles touching the y axis at the origin.

4. Consider the relation R in the set N of natural numbers defined as

   $R = \{(a, b) : ab \text{ is a factor of } 6\}$

   Determine whether the relation is reflexive, symmetric or transitive.

5. Find the area bounded by the curve $y = \cos x$ and the x axis between $x = 0$ and $x = \pi$.

6. 

   A rectangular plot is to be fenced using a rope of length 20 metres with one of its sides is a wall as shown in the figure. Find the maximum area of such a rectangle.

7. A manufacturer produces nuts and bolts. The time required to produce one packet of nuts and one packet of bolts on machines A and B is given in the following table :

	Machine A	Machine B
Nuts (1 packet)	2 hours	3 hours
Bolts (1 packet)	3 hours	1 hour

He earns a profit of ₹ 25 per packet of nuts and ₹ 12 per packet of bolts. He operates his machines for almost 15 hours a day. Formulate a linear programming problem to maximize his profit.

Questions from 8 to 17 carry 4 scores each. Answer any 8. **(8 × 4 = 32)**

8. Consider the curve $y = x^3 + 8x + 3$.

(a) Find the point on the curve at which the slope of tangent is 20. **(3)**

(b) Does there exist a tangent to the above curve with negative slope ? Justify your answer. **(1)**

9. (a) Which of the following functions is not continuous at zero ? **(1)**

(i) $f(x) = \sin x$

(ii) $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$

(iii) $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

(iv) $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

(b) Find the values of a and b such that the function defined by **(3)**

$$f(x) = \begin{cases} 10 & \text{if } x \le 3 \\ ax + b & \text{if } 3 < x < 4 \\ 20 & \text{if } x \ge 4 \end{cases}$$

is a continuous function.

10. Consider the plane $2x - 3y + z = 5$.

(a) Find the equation of the plane passing through the point (1, 1, 3) and parallel to the above plane. **(2)**

(b) Find the distance between above parallel planes. **(2)**

11. Consider the vectors

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and

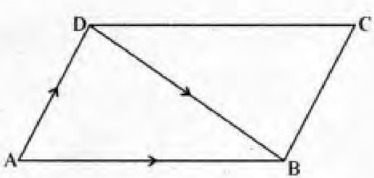
$\vec{b} = \hat{i} + 4\hat{j} - \hat{k}$

(a) Find the projection of $\vec{a}$ on $\vec{b}$ **(2)**

(b) If $\vec{a}$ is perpendicular to a vector $\vec{c}$ then projection of $\vec{a}$ on $\vec{c}$ = _______. **(1)**

(c) Write a vector $\vec{d}$ such that the projection of $\vec{a}$ on $\vec{d} = |\vec{a}|$. **(1)**

12. (a) **(2)**



In the figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{AD} = \hat{i} + \hat{j} + 2\hat{k}$ find $\overrightarrow{AC}$ and $\overrightarrow{DB}$.

(b) If $\vec{a}$ and $\vec{b}$ are adjacent sides of any parallelogram and $\vec{c}$ and $\vec{d}$ are its diagonals, show that $|\vec{c} \times \vec{d}| = 2\,|\vec{a} \times \vec{b}|$ **(2)**

13. Find $\int (4x+7)\sqrt{x^2+4x+13}\,dx$

14. (a) Write the integrating factor of the linear differential equation **(1)**

$$\frac{dy}{dx}+\frac{y}{x}=\sin x$$

(b) Slope of the tangent to a curve at any point is twice the x coordinate of the point.

If the curve passes through the point (1, 4), find its equation. **(3)**

15. Solve the linear programming problem graphically

Maximize $Z = 3x + 5y$

Subject to the constrains

$x + 3y \leq 3$

$x + y \leq 2$

$x, y \geq 0$

16. (a) If $\cos^{-1}\frac{12}{13} = \tan^{-1}x$ then find x. **(1)**

(b) Show that $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \tan^{-1}\frac{14}{33}$ **(3)**

17. Consider the binary operation $*$ on the set R of real numbers, defined by

$$a * b = \frac{ab}{4}$$

(a) Show that $*$ is commutative and associative. (2)

(b) Find the identity element for $*$ on R. (1)

(c) Find the inverse of 5. (1)

Questions from 18 to 24 carry 6 scores each. Answer any 5. **(5 × 6 = 30)**

18. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 4 & 9 \end{bmatrix}$$

(a) Find A^{-1} using elementary row operations. (3)

(b) Find the solution of the system of equations given below : (3)

(A^{-1} obtained above may be used)

$x + 2z = 2$

$y + 2z = 1$

$4y + 9z = 3$

19. (a) Show that (3)

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

(b) If $A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ (3)

verify that $A \text{ adj } A = |A| I$.

20. (a) If f is a function such that $f(-x) = f(x)$, then **(1)**

$$\int_{-a}^{a} f(x)\,dx = \_\_\_\_\_\_$$

(b) Evaluate $\int_{-\pi/2}^{\pi/2} \cos x \; dx$ **(2)**

(c) Evaluate $\int_{0}^{1} (x^2 + 1)\,dx$ as the limit of a sum **(3)**

21. (a) Verify mean value theorem for the function $f(x) = x^2 - 4x - 3$ in the interval $[1, 4]$. **(3)**

(b) Consider the function

$$f(x) = \sin^{-1} 2x\sqrt{1-x^2}, \; \frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$$

(i) Show that $f(x) = 2\sin^{-1} x$. **(2)**

(ii) Find $f'(x)$. **(1)**

22. (a) Show that the lines **(2)**

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{1} \text{ and}$$

$$\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-4}{2}$$

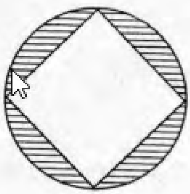
are coplanar.

(b) Find the equation of the plane that contains above lines. **(2)**

(c) Show that the above lines intersect at the point (3, 1, 4). **(2)**

23. (a) A coin is tossed 3 times. Find the probability distribution of the number of heads. **(3)**

(b) A bag contains 5 Black and 6 white balls. 4 balls of the same colour (Black or white) are added to the bag, shuffled well, and one ball is drawn. If the ball obtained is white, what is the probability that the balls added are black ? **(3)**

24. 

In a circle of radius 2, a square is inscribed as shown in the figure.

Using integration, find the area of the shaded region. (Area of a square or a triangle may be calculated using any convenient method)
