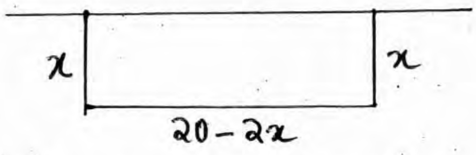
SECOND YEAR HIGHER SECONDARY EXAMINATION JUNE 2018

SUBJECT :MATHEMATICS (SCIENCE) CODE. NO: 2018

Qn No	Sub Qns	Answer Key/Value Points	Score	Total
1	a	$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$	$\frac{1}{2}+\frac{1}{2}$	
	b	$A^2 = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} 29 & 36 \\ 45 & 56 \end{bmatrix}$	1+1	3
		Remarks for any THREE correct entries in A^2 give full score for any A, Correct A^2 give 2 score		
2	a	$2x$	1	
	b	$\int xe^x dx = x\int e^x dx - \int e^x dx$	1	
		$= xe^x - e^x + C = e^x(x-1)+C$	1	3
		Remarks formula give ½ score		
3		$(x-a)^2 + y^2 = a^2 \quad -①$	1	
		$x^2+y^2 = 2ax$ $2x+2y\frac{dy}{dx} = 2a \Rightarrow a = x+y\frac{dy}{dx}$	1	
		$① \Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + y^2 = \left(x+y\frac{dy}{dx}\right)^2$	1	3
		$\Rightarrow \frac{dy}{dx} = \frac{y^2-x^2}{2xy}$		
		Remarks for $(x-a)^2+(y-b)^2=r^2$ give ½ score		
4		not reflexive , $(2,2)\notin R$	1	
		Symmetric $(x,y)\in R, \Rightarrow (y,x)\in R$ $xy = yx$	1	
		not transitive $(3,2)\in R$ and $(2,3)\in R$ but $(3,3)\notin R$	1	3
		Remarks def. for reflexive, symmetric, transitive give ½ score each		
5		$A = \int_0^{\frac{\pi}{2}} \cos x\, dx + \left\lvert \int_{\frac{\pi}{2}}^{\pi} \cos x\, dx \right\rvert$	1	
		$= \left[\sin x\right]_0^{\frac{\pi}{2}} + \left\lvert \left[\sin x\right]_0^{\frac{\pi}{2}} \right\rvert$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$= 1+1 = 2$ Remarks find the area using $\int_0^{\pi} \cos x \, dx$ give $1\frac{1}{2}$ score for diagram only give ONE score	1	3
6		 length $= 20-2x$, breadth $= x$ $A = x(20-2x) = 20x - 2x^2$ $A'(x) = 20 - 4x$ $A'(x) = 0 \Rightarrow x = 5$ $A''(x) = -4 < 0 \Rightarrow$ A is maximum at $x=5$ maximum area $= 5 \times 10 = 50$ Remarks Area $= l \times b$ give ½ score	1 ½ ½ ½ ½	3
7		Max $Z = 25x + 12y$ Subject to $2x+3y \leq 15$ $3x+y \leq 15$ $x, y \geq 0$ Remarks for $2x+3y \geq 15$, $3x+y \geq 15$ give 1 score	1 1 1	3
8	a	$\frac{dy}{dx} = 3x^2+8$ $3x^2+8 = 20$ $x = \pm 2$ Points $(2, 27), (-2, -21)$ Remarks Any one point give FULL score Slope of the tangent $= \frac{dy}{dx}$ give ½ score	1 1 ½ ½	4
	b	No $3x^2+8 \geq 0$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
9	a	iii) $f(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$	1	
	b	$\lim_{x \to 3^+} f(x) = 3a+b$; $3a+b=10$	1	
		$\lim_{x \to 4^-} f(x) = 4a+b$; $4a+b=20$	1	4
		Solving, $a=10$, $b=-20$	1	
		<u>Remarks</u> def. of continuity give ½ Score		
10	a	$2x-3y+z=k$	1	
		passing through $(1,1,3)$		
		$2-3+3=k \Rightarrow k=2$	1	
		$2x-3y+z=2$		
		<u>Remarks</u> for $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ give ½ Score		
	b	Distance between planes = Distance of the point $(1,1,3)$ from the plane	1	4
		$= \left\lvert \frac{2\times1-3\times1+3-5}{\sqrt{4+9+1}} \right\rvert = \frac{3}{\sqrt{14}}$	$\frac{1}{2}+\frac{1}{2}$	
		<u>Remarks</u> for formula give ½ Score Alternate method give fullscore		
11	a	Projection $\frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert}$	1	
		$\vec{a}\cdot\vec{b}=3$, $\lvert\vec{b}\rvert=\sqrt{18}$	$\frac{1}{2}+\frac{1}{2}$	
		projection $= \frac{3}{\sqrt{18}}$ or $\frac{1}{\sqrt{2}}$		4
	b	0	1	
	c	$\vec{d}=\vec{a}$, or any vector parallel to $\vec{a}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
12	a	$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD} = 4\hat{i} + 4\hat{j}$	$\frac{1}{2} + \frac{1}{2}$	
		$\overrightarrow{DB} = \overrightarrow{AB} - \overrightarrow{AD} = 2\hat{i} - 2\hat{j}$	$\frac{1}{2} + \frac{1}{2}$	
	b	Let $\vec{c} = \vec{a} + \vec{b}$ and $\vec{d} = \vec{a} - \vec{b}$	1	4
		$\vec{c} \times \vec{d} = (\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = -2(\vec{a}\times\vec{b})$	$\frac{1}{2}$	
		$\lvert\vec{c}\times\vec{d}\rvert = 2\lvert\vec{a}\times\vec{b}\rvert$	$\frac{1}{2}$	
		<u>Remarks</u> Verify the result using any vectors give FULL score		
13		$4x+7 = A(2x+4) + B$	1	
		$A = 2$, $B = -1$	1	
		$I = 2\int(2x+4)\sqrt{x^2+4x+13}\,dx - \int\sqrt{x^2+4x+13}\,dx$		
		$= 2I_1 - I_2$		
		$I_1 = \int\sqrt{t}\,dt = \frac{2}{3}t^{3/2} + C,\ \frac{2}{3}(x^2+4x+13)^{3/2} + C_1$	1	4
		$I_2 = \int\sqrt{x^2+4x+13}\,dx = \int\sqrt{(x+2)^2+9}\,dx$		
		$= \frac{x+2}{2}\sqrt{(x+2)^2+9} + \frac{9}{2}\log\lvert(x+2)+\sqrt{(x+2)^2+9}\rvert + C_2$	1	
		$I = 2I_1 - I_2$		
14	a	$IF = e^{\int\frac{1}{x}dx} = e^{\log x} = x$	$\frac{1}{2} + \frac{1}{2}$	
	b	$\frac{dy}{dx} = 2x$	1	
		$y = x^2 + C$	1	4
		$4 = 1 + C \Rightarrow C = 3$	1	
		equation $y = x^2 + 3$		

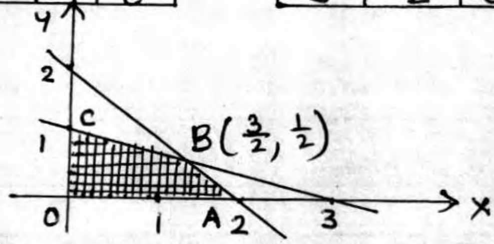
<table>
<tr><th>Qn. No</th><th>Sub Qns</th><th>Answer Key/Value Points</th><th>Score</th><th>Total</th></tr>
<tr><td>15</td><td></td><td>

$x+3y=3$

x	0	3
y	1	0

$x+y=2$

x	0	2
y	2	0



Corner points	$Z=3x+5y$
$O(0,0)$	0
$A(2,0)$	6
$B\left(\frac{3}{2},\frac{1}{2}\right)$	7
$C(0,1)$	5

Maximum 7 at $B\left(\frac{3}{2},\frac{1}{2}\right)$

<u>Remark</u> Correct graph and incorrect shaded region give $2\frac{1}{2}$ score
</td><td>1

2

1</td><td>4</td></tr>
<tr><td>16</td><td>a

b</td><td>

$x=\frac{5}{12}$

$\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{5}{12}\right)$

$=\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}\right)$

$=\tan^{-1}\left(\frac{56}{33}\right)$

<u>Remarks</u> FOR ATTEMPTING AND ANALYSING GIVE FULL SCORE
</td><td>1

3</td><td>4</td></tr>
<tr><td>17</td><td>a

b</td><td>

$\frac{ab}{4}=\frac{ab}{4}$, $*$ commutative, $a*b=b*a$

$\frac{abc}{16}=\frac{abc}{16}$, $*$ associative $(a*b)*c=a*(b*c)$

$a*e=a \Rightarrow \frac{ae}{4}=a$

$\Rightarrow e=4$
</td><td>$\frac{1}{2}+\frac{1}{2}$
$\frac{1}{2}+\frac{1}{2}$
$\frac{1}{2}$
$\frac{1}{2}$</td><td></td></tr>
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Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
	c	$5 * b = 4 \Rightarrow \frac{5b}{4} = 4$	$\frac{1}{2}$	4
		$\Rightarrow b = \frac{16}{5}$	$\frac{1}{2}$	
		<u>Remark</u> $b = \frac{16}{a}$ give full score		
18	a	$A = IA$		
		$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 4 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$	1	
		$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix} A \quad R_3 \to R_3 - 4R_2$	$\frac{1}{2}$	
		$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 8 & -2 \\ 0 & 9 & -2 \\ 0 & -4 & 1 \end{bmatrix} A \quad \begin{matrix} R_1 \to R_1 - 2R_3 \\ R_2 \to R_2 - 2R_3 \end{matrix}$	1	
		$A^{-1} = \begin{bmatrix} 1 & 8 & -2 \\ 0 & 9 & -2 \\ 0 & -4 & 1 \end{bmatrix}$	$\frac{1}{2}$	
		<u>Remark</u> Alternate method, correct A^{-1} give 2 score		6
	b	$AX = B$	$\frac{1}{2}$	
		$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$	$\frac{1}{2}$	
		$X = A^{-1}B = \begin{bmatrix} 1 & 8 & -2 \\ 0 & 9 & -2 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$	$\frac{1}{2} + 1$	
		$= \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$	$\frac{1}{2}$	
		$x = 4, \quad y = 3, \quad z = -1$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
19	a	$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$	1	
		$= (b-a)(c-a) \begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix}$	1	
		$= (b-a)(c-a)(c-b)$	1	6
		$= (a-b)(b-c)(c-a)$		
	b	$\lvert A \rvert = -14$	1	
		$\operatorname{adj} A = \begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$	1	
		$A(\operatorname{Adj} A) = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} = \lvert A \rvert I$	1	
20	a	$2\int_0^a f(x)\,dx$	1	
	b	$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = 2\int_0^{\frac{\pi}{2}} \cos x\,dx$	1	
		$= 2\left[\sin x\right]_0^{\frac{\pi}{2}}$	$\frac{1}{2}$	
		$= 2\left(\sin\frac{\pi}{2} - \sin 0\right) = 2$	$\frac{1}{2}$	
		<u>Remark</u> Alternate method give full score		
	c	$h = \frac{1}{n}$, $b - a = 1$		
		$\int_a^b f(x)\,dx = \lim_{n\to\infty} \frac{b-a}{n}\left[f(a) + f(a+h) + \cdots + f(a+(n-1)h)\right]$	1	
		$\int_0^1 (x^2+1)\,dx = \lim_{n\to\infty} \frac{1}{n}\left[1 + \left(\left(\frac{1}{n}\right)^2+1\right) + \left(\left(\frac{2}{n}\right)^2+1\right) + \cdots + \left(\left(\frac{n-1}{n}\right)^2+1\right)\right]$	1	
		$= \lim_{n\to\infty} \frac{1}{n}\left[(1+1+\cdots+1) + \frac{1}{n^2}\left(1^2+2^2+\cdots+(n-1)^2\right)\right]$	$\frac{1}{2}$	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		$= \lim_{n\to\infty} \frac{1}{n}\left(n + \frac{(n-1)n(2n-1)}{6n^2}\right)$		6
		$= \lim_{n\to\infty}\left(1 + \frac{(1-\frac{1}{n})(2-\frac{1}{n})}{6}\right)$	$\frac{1}{2}$	
		$= 1 + \frac{2}{6} = \frac{4}{3}$		
		Remark: for direct integration give 1 score		
21	a	f is continuous	$\frac{1}{2}$	
		$f'(x) = 2x-4$, f is differentiable	$\frac{1}{2}$	
		$f(4) = -3$, $f(1) = -6$	$\frac{1}{2}$	
		$f'(x) = \frac{f(b)-f(a)}{b-a} \Rightarrow 2x-4 = \frac{-3+6}{4-1} = 1$	1	
		$\Rightarrow x = \frac{5}{2}$	$\frac{1}{2}$	
		$c = \frac{5}{2} \in [1, 4]$		
		Mean Value theorem is verified		6
		Remarks Write the statement only give 1 score		
	b(i)	Put $x = \sin\theta$	$\frac{1}{2}$	
		$f(x) = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$	$\frac{1}{2}$	
		$= \sin^{-1}(\sin 2\theta)$		
		$= 2\theta$	$\frac{1}{2}$	
		$= 2\sin^{-1}x$		
	(ii)	$f'(x) = \frac{2}{\sqrt{1-x^2}}$	$\frac{1}{2}$ 1	
		Remarks Use $x = \cos\theta$, proceed (b) give 2 Score		
22	a	Points $(2,-1,3)$ and $(3,1,4)$	$\frac{1}{2}$	
		$\vec{a_2} - \vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$	$\frac{1}{2}$	
		$(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2}) = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix} = 0$	$\frac{1}{2}+\frac{1}{2}$	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total
		So the lines are Coplanar		
	b	$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{vmatrix} = 3\hat{i} - 0\hat{j} - 3\hat{k}$	1	
		dr's of normal $(3, 0, -3)$	½	
		equation of the plane		
		$3(x-2) + -3(z-3) = 0$	$\frac{1}{2}$	6
		$x - z + 1 = 0$		
		Remarks Alternate method give full score		
	c	$(3,1,4)$ is on the second line, any point	$\frac{1}{2}$	
		on the first line is $(\lambda+2, 2\lambda-1, \lambda+3)$	$\frac{1}{2}$	
		$(3,1,4) = (\lambda+2, 2\lambda-1, \lambda+3)$		
		$\Rightarrow \lambda+2 = 3, \quad \lambda = 1$	$\frac{1}{2}$	
		Put $\lambda=1$ in $(\lambda+2, 2\lambda-1, \lambda+3) \Rightarrow (3,1,4)$	$\frac{1}{2}$	
23	a	$X = \{0,1,2,3\}$ OR $P(X=x) = {}^nC_x\, p^x q^{n-x}$	1	
		X: 0, 1, 2, 3; P(X): $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{8}$, $\frac{1}{8}$	2	
		Remark for writing sample space only give 1 Score		
	b	E_1 : balls added are black		
		E_2 : balls added are white		6
		A : ball drawn is white	1	
		$P(E_1) = \frac{1}{2}, \; P(E_2) = \frac{1}{2}, \; P(A/E_1) = \frac{2}{5}, \; P(A/E_2) = \frac{2}{3}$	1	
		$P(E_1/A) = \dfrac{P(E_1) \cdot P(A/E_1)}{P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2)}$	$\frac{1}{2}$	
		$= \frac{3}{8}$	$\frac{1}{2}$	

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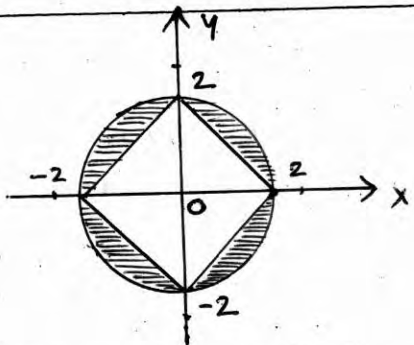
24				
		Equation of the circle : $x^2+y^2=2^2$	1	
		Area of the first quadrant $=\int_0^2 \sqrt{2^2-x^2}\,dx$	1	
		$=\left[\frac{x}{2}\sqrt{2^2-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$	1	6
		$=\frac{4}{2}\sin^{-1}1 = \pi$	1	
		Area of the triangle $=\frac{1}{2}\times 2\times 2 = 2$	$\frac{1}{2}$	
		Area of the shaded region in the first quadrant $=\pi-2$	$\frac{1}{2}$	
		Required area $=4(\pi-2) = 4\pi-8$ sq. units	1	
		<u>Remarks</u> for $\int\sqrt{a^2-x^2}\,dx$ give 1 score		

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