



# JAIN COLLEGE

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Bangalore - 560 098

**Date: Dec 2017**

**SUBJECT: Mathematics**

## II PUC Mock II

**Timings Allowed: 3 Hrs.**

**Total Marks: 100**

Instructions :

- (i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.
- (ii) Use the graph sheet for the question on Linear programming in PART E.

### PART A

**I Answer all**

**1X10=10**

1. Give an example of a relation which is reflexive, symmetric but not transitive
2. Find the principal value of  $\cos^{-1}(-1/2)$
3. If A be  $\begin{bmatrix} 4 & 7 \\ 6 & 5 \end{bmatrix}$ , Find |3A|
4. What is the number of possible square matrices order 2 with each entry 0 or 1
5. Differentiate  $\tan(2x+5)$  w.r.t. 'x'
6. Evaluate  $\int_0^2 \log 5 dx$
7. Define collinear vectors
8. Find the direction ratio of the line passing through the points (-2,4,-5) and (1,2,3)
9. Define Optimal solution
10. If  $P(A)=6/11$ ,  $P(B)=5/11$  and  $p(A \cup B)=7/11$  then Find  $P(A/B)$

### PART -B

**II ANSWER ANY TEN**

**2X10=20**

11. Verify the operation \* on Q, defined by  $a*b = \frac{ab}{4} \forall a, b \in Q$  associative or not
12. Prove that  $\cos^{-1}(-x) = \pi - \cos^{-1}x$ ,  $x \in [-1, 1]$
13. Write the simplest form of  $\tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}$ ,  $0 < x < \pi$
14. Find the values of K, if the area of the triangle is 4 sq.units and vertices are (k,0), (4,0), (0,4)
15. Find the derivative of  $x^{\sin x}$ ,  $x > 0$  w.r.t 'x'
16. Differentiate  $\sin^2 x$  w.r.t.  $e^{\cos x}$
17. Find slope of normal to the curve  $x = a \cos^3 t$ ,  $y = a \sin^3 t$  at  $t = \pi/4$
18. Evaluate  $\int_0^{\pi/4} (2 \sec^2 x + x^3 + 4) dx$
19. Evaluate  $\int \tan^{-1} x dx$
20. Find the order and degree of the differential equation,

$$xy \frac{d^2 y}{dx^2} + x \left( \frac{dy}{dx} \right)^2 - y \frac{dy}{dx} = 0$$

21. Find K if the vectors  $\hat{i} + 3\hat{j} + \hat{k}$ ,  $2\hat{i} - \hat{j} - \hat{k}$  and  $k\hat{i} + 7\hat{j} + 3\hat{k}$  are coplanar
22. Find the area of the parallelogram whose adjacent sides are given by  $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$  and  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$
23. The Cartesian equation of a line is  $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$ , find the vector equation for the line
24. Determine P(E/F). A coin is tossed three times where, E: head on 3<sup>rd</sup> toss, F: heads on the first two toss

### PART -C

#### III ANSWER ANY TEN

3X10=30

25. Show that the relation R in the set Z of integers given by  $R = \{(a,b) : 5 \text{ divides } a-b\}$  is an equivalence relation
26. Prove that  $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$
27. By using elementary transformation, Find the inverse of the matrix  $A \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$
28. If  $y = \tan^{-1} \left[ \frac{\sin x}{1 + \cos x} \right]$ , Prove that  $\frac{dy}{dx} = \frac{1}{2}$
29. If a function is differentiable at  $x=c$ , prove that it is continuous at  $x=c$
30. A square piece of tin of side 18cm is to be made into a box without top by cutting a square from each corner and folding up flaps to form the box, What should be the side of the square to be cut off so that the volume of the box is maximum
31. Evaluate  $\int_0^2 e^{2x} dx$
32. Evaluate  $\int \sin(ax + b) \cos(ax + b) dx$
33. Find the smaller area enclosed by circle  $x^2 + y^2 = 2^2$  and the line  $x+y=2$
34. Find the general solution of differential equation  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$
35. Find the cosine of the angle between the vectors  $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$
36. Prove that  $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$
37. Find the shortest distance between the lines  $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{-1}$  and  $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$
38. Find the probability of getting 5 exactly twice in 7 throws of a die

### PART -D

#### IV ANSWER ANY SIX

5X6=30

39. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 4x^2 + 12x + 15$ . Show that  $f: \mathbb{N} \rightarrow \mathbb{S}$ , Where  $\mathbb{S}$  is the range of the function, is invertible. Also find the inverse of  $f$ .
40. If  $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ , Verify  $A^3 - 3A^2 - 10A + 24I = 0$
41. Solve the system of equations by Matrix method  $2x + 3y + 3z = 5, x - 2y + z = -4, 3x - y - 2z = 3$
42. If  $y = (\tan^{-1} x)^2$ , Show that  $(x^2 + 1)y_2 + 2x(x^2 + 1)y_1 = 2$
43. A particle moving along the curve  $6Y = X^3 + 2$ , Find the points on the curve at which the y coordinate is changing 8 times as the x-coordinate
44. Find the integral of  $\frac{1}{\sqrt{a^2 - x^2}}$  w.r.t., 'x' and hence Evaluate  $\int \frac{1}{\sqrt{2x - x^2}} dx$
45. Using method of integration, Find the area of the region bounded by the triangle whose vertices are (1,0), (2,2) and (3,1)
46. Find general solution of differential equation  $\frac{dy}{dx} - y = \cos x$

47. Derive the equation of a plane passing through three non collinear points both in vector and Cartesian form
48. If a fair coin is tossed 10 times , Find the probability of
- (i) exactly six heads
  - (ii) At least six heads
  - (iii) At most six heads

**PART –E**

**V ANSWER ANY TWO**

**10X2=20**

49(a) Prove that  $\int_{-a}^a f(x)dx = 2 \int_0^a f(x) dx$  if  $f(x)$  is even.

Hence evaluate  $\int_{-\pi}^{\pi} \sin^4 x dx$

(b) Define continuity of a function . Find all points of discontinuity of  $f$  defined by  $f(x) = |x| - |x + 1|$

50.(a) (a) A manufacturing company makes two products A and B . Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hour for finishing. For fabricating and finishing the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs.8000 on each piece of model A and Rs.12000 on each piece of model B. How many pieces of model A and model B should be manufactured per week to realize a maximum profit? What is the maximum profit?

$$(b) \begin{vmatrix} x + y + 2z & x & y \\ z & y + z + 2x & y \\ z & x & z + x + 2y \end{vmatrix} = 2(x+y+z)^2$$

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