JAIN COLLEGE, Bangalore
Mock Paper - 1 December - 2017
II PUC – Mathematics (35)

Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all questions: **10 × 1 = 10**

1. Give an example to show that *: N X N→ N, given by (a ,b) = a – b is not a binary operation.
2. Find the value of $\cos^{-1}(\frac{1}{2}) + 2\sin^{-1}(\frac{1}{2})$.
3. If A= [3 1 -1] and B= $\begin{bmatrix}2\\4\\4\end{bmatrix}$, find AB.
4. If A is an invertible matrix of order 2x2 and $|A| = 15$ then find determinant (A^{-1}).
5. Differentiate log (cos e^x) with respect to x.
6. Integrate secx (secx+tanx) with respect to x.
7. Find the sum of the vectors $\vec{a} = i - 2j + k, \vec{b} = -2i + 4j + 5k$ and $\vec{c} = i - 6j - 7k$
8. Find the equation of the plane having intercept 3 on Y-axis and parallel to zox plane.
9. Define feasible region.
10. A fair die is rolled. Consider events E = {2,4,6} and F = {1,2}, find $P\left(\frac{E}{F}\right)$.

PART B

II. Answer any ten : **10 × 2 = 20**

11. A relation R is defined on the set A = {1,2,3,4,5,6} by R = {(x,y): y is divisible by x}. Verify whether R is symmetric and reflexive or not. (Give reason).
12. Prove that $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$, where $\frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$
13. Write the simplest form of $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right], x \neq 0$
14. Prove that, " if each element of a row of 3X3 determinant is multiplied by a constant k ,then the value of the determinant is multiplied by k".
15. If $ax + by^2 = \cos y$, find $\frac{dy}{dx}$.
16. Differentiate $\sqrt{e^{\sqrt{x}}}$ w.r.t x.
17. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at x=4.
18. Evaluate: $\int \sin 4x \sin 8x \, dx$.
19. Integrate : $\int_0^{\frac{\pi}{4}} \tan x \, dx$.
20. Find the order and degree of the differential equation $\left(\frac{ds}{dt}\right)^4 + 3s\left(\frac{d^2s}{dt^2}\right) = 0$.
21. Find $|\vec{a} + \vec{b}|$ if $\vec{a} = i - 7j - 7k$ and $\vec{b} = 3i - 2j + 2k$.
22. Find a vector of magnitude 8 units in the direction of the vector $\vec{a} = 5i - j + 2k$.

23. Find the angle between the pair of lines $\vec{r} = 3i+5j-k+\lambda(i+j+k)$, $\vec{r} = 7i+4k+\mu(2i+2j+2k)$

24. Probability distribution of X is

X	0	1	2	3	4
P(x)	0.1	K	2k	2k	K

PART C

III. Answer any ten: 10 × 3 = 30

25. Let $f : X \to Y \, and \, g : Y \to Z$ be two invertible functions. Then show that (gof) is invertible is also invertible with $(gof)^{-1} = f^{-1} o \, g^{-1}$.

26. Prove that $\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{63}{16}$.

27. If $x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$ find the values of x and y.

28. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ w.r.t x.

29. Verify Rolle's theorem for the function $f(x) = x + 2x - 8, x \in [-4,2]$.

30. Evaluate: $\int \frac{1}{1+\tan x} dx$.

31. Find the point on the curve $y = x^3 - 11x + 5 \, at \, which \, the \tan gent \, is \, y = x - 11$.

32. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x} dx$.

33. Find the region bounded by the line y=3x+2 and x-axis and the ordinates x=-1and x=1.

34. Find the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

35. Find the area of a rectangle having vertices A,B,C and D with position vectors $-i + \frac{1}{2}j + 4k \, , i + \frac{1}{2}j + 4k \, , i - \frac{1}{2}j + 4k \, and \, -i - \frac{1}{2}j + 4k.$ respectively.

36. If $\vec{a}, \vec{b} \, and \, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$?

37. Find the equation of the plane passing through the points R(2,5,-3), S(-2,-3,5) and T(5,3,-3).

38. Find the probability of getting 5 exactly twice in 7 throws of a die.

PART D

IV. Answer any six : 6 × 5 = 30

39. Consider $f : R^+ \to [-5, \infty)$ given f(x)=$9x^2 + 6x - 5$ show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6} - 1}{3}\right)$

40. If $A = \begin{bmatrix}1 & 2 & 3\\3 & -2 & 1\\4 & 2 & 1\end{bmatrix}$ Show That: $A^3 - 23A - 40I = 0$.

41. Solve the system of equations 3x-2y+3z=8 ,2x+y-z=1 , 4x-3y+2z=4 by matrix method.

42. If $e^y (x+1) = 1 \, show that \, \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

43. A ladder 5m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 cm/sec. How fast is its height on the wall decreases when the foot of the ladder is 4m away from the wall?

44. Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ w.r.t x, and hence evaluate $\int \frac{1}{\sqrt{x^2+2x+2}}dx$.

45. Find the area bounded by the triangle whose vertices are (1,0) (2,2) (3,1) using integration method.

46. Find the general solution of the differential equation $\cos^2 x\frac{dy}{dx}+y=\tan x$.

47. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in cartesian and vector form.

48. An urn contains 25 balls of which 10 balls bear a mark X and the remaining 15 bear a mark Y. A ball is drawn at random from the urn its mark is noted down and it is replaced. if six balls are drawn in this way find the probability that

(i) All will bear X mark

(ii) Not more than two will bear Y mark

(iii) Atleast one ball will bear Y mark.

PART E

V. Answer any one : **10 × 1 = 10**

49. (a) Prove that $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ and hence Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$.

(b) Find the values of a and b such that the function defined $f(x)=\begin{Bmatrix} 5 & if\ x\le 2 \\ ax+b & if\ 2<x<10 \\ 21 & if\ x\ge 10 \end{Bmatrix}$ is continuous function.

50. (a) A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nut. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs. 17.5 per packet of nuts and Rs. 7 per packet of bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates both the machines atmost 12 hours a day.

(b) Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}=(1-x^3)^2$ using properties of determinants .

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Mock Paper - 2 December - 2017

II PUC – Mathematics (35)

Time: 3 Hours 15 Minutes **Max. Marks: 100**

PART A

I. Answer all : **10 × 1 = 10**

1. Let * be a binary operation defined on set of rational numbers by $a*b=\frac{ab}{4}$ .Find the identity element.
2. Find the principal values of $\cos ec^{-1}(-\sqrt{2})$.
3. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}, A+A'=I$ then find the value of α .
4. Let A be a non-singular matrix of order 3X3 and $|A|=25$ then find $|adj\ A|$
5. Differentiate $\sin(x^2+5)$ w.r.t x .
6. Evaluate $\int_2^3 \frac{1}{x}dx.$
7. If $\vec{a}\ \ and\ \ \vec{b}$ are any two vectors such that $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$ .Find the angle between $\vec{a}\ \ and\ \ \vec{b}$.
8. Find the direction cosines of a line which makes equal angles with the co-ordinate axis.
9. Define optimal solution.
10. If $P(A)=\frac{3}{5}\ \ and\ \ P(B)=\frac{1}{5}$ .Find $P(A\cap B)$ where A and B are independent events.

PART B

II. Answer any 10 : **10 × 1 = 10**

11. Examine whether the binary operation * on the set 'R 'defined as $\vec{a}*\vec{b}=\frac{a+b}{2}\ \forall a,b\in R$ is associative.
12. Write the simplest form of $\tan^{-1}\left[\frac{\cos x-\sin x}{\cos x+\sin x}\right], 0<x<\frac{\pi}{2}.$
13. Prove that $2\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{24}{7}\right)$.
14. Without expansion prove that $\begin{vmatrix}1 & bc & a(b+c)\\ 1 & ca & b(c+a)\\ 1 & ab & c(a+b)\end{vmatrix}=0$.
15. Find two positive numbers x and y such that $x+y=60\ \ and\ \ xy^3$ is maximum.
16. Differentiate $y=(\log x)^{\cos x}$ wrt x.
17. Find $\frac{dy}{dx}$ if $x=4t\ \ and\ \ y=\frac{4}{t}$.
18. Evaluate $\int\frac{1}{x-\sqrt{x}}dx$.
19. Evaluate $\int\frac{1}{e^x-1}dx\ \ w.r.t\ \ x$.

20. Form the differential equations of the family of circles touching the x-axis at origin.
21. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ then show that the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ is perpendicular.
22. For the vectors $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$ .Find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.
23. Find the vector equation of the line passing through the points $(-1,0,2)$ and $(3,4,6)$.
24. Find the probability distribution of the number of heads in two tosses of a coin.

PART C

III. Answer any 10 : 10 × 3 = 30

25. Show that $f : N \to N$ given by $f(x) = \begin{cases} x+1 & if\ x\ is\ odd \\ x-1 & if\ x\ is\ even \end{cases}$ is a bijective function.
26. Solve for x: $\tan^{-1} 2x + \tan^{-1} 3x = \frac{x}{4}$.
27. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .show that $F(x).F(y) = F(x+y)$.
28. If $y = \sin^{-1}\left[\frac{2^x + 1}{1 + 4^x}\right]$ .Find $\frac{dy}{dx}$.
29. Using differentials find the approximate value of $26^{\frac{1}{3}}$.
30. A balloon which remains spherical has a variable radius. Find the rate at which its volume increasing with the radius, when the radius is 10cm.
31. Evaluate $\int \tan^4 x dx$.
32. Evaluate $\int \frac{1}{x(x^n + 1)} dx$.
33. Find the area bounded by the curve $x^2 = 4y$ and line $x = 4y - 2$.
34. Find the equation of the curve through the point $(-2,3)$, given that the slope of the tangent at any point (x,y) is $\frac{2x}{y^2}$.
35. If $\vec{a}$ is unit vector , $\vec{a}$ makes an angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and acute angle θ with $\hat{k}$ .Find θ and hence the components of $\vec{a}$
36. Find the area of triangle having the points $A(1,1,1), B(1,2,3)$ and $C(2,3,1)$ as its vertices using vector method.
37. Determine whether the given planes $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$ are parallel or perpendicular and in the case of neither both, find the angle between them .
38. Two dice are thrown simultaneously if X denotes the number sixes. Find the expectation of X and variance of X.

PART D

IV. Answer any six : 6 × 5 = 30

39. Let $f : R \to R$ be defined as $f(x) = 10x + 7$ .Find the function $g : R \to R$ such that $g \circ f = f \circ g = I_R$.

40. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix} and\ C = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$ .Find a matrix D such that $CD - AB = 0$.

41. Solve the system of equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2,$

42. If $y = (\tan^{-1})^2$ then Show that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2$.

43. A particle is moving along the curve $6y = x^3 + 2$ .Find the points on the curve at which the y-co ordinate is changing 8 times as fast as the x-co ordinate,.

44. Find the integration of $\sqrt{a^2 + x^2}$ w.r.t x and hence evaluate $\int \sqrt{x^2 + 25x + 5}dx$.

45. Find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a>b) by the method of integration and hence find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

46. Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$.

47. Derive the equation of the plane in normal form (both vector and Cartesian form).

48. A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize $\frac{1}{100}$ .what is the probability that he will win a prize at least once and exactly once.

PART –E

V. Answer any ONE question: 1 × 10 =10

49. a) Solve the following problem graphically

$$Minimize\ and\ Maximise$$
$$z = 3x + 9y$$
$$constraints\ \ x + 3y \geq 60$$
$$x + y \geq 10$$
$$x \leq y, x \geq 0, y \geq 0$$

b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

50. a) Prove that $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if\ \ f(x)\ is\ even \\ 0 & if\ \ f(x)\ is\ odd \end{cases}$ and hence evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$

b) Determine the value of k $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x} & if \;\; x\neq\dfrac{\pi}{2}\\ 3 & if \;\; x=\dfrac{\pi}{2}\end{cases}$ is continuous at $\dfrac{\pi}{2}$.