Jain College, Jayanagar
II PUC Mock Paper - I
Subject: Mathematics (35)

Duration: 3 hrs 15 minutes **Max. Marks: 100**

I Answer all the following questions: **10×1=10**

1. Find whether * on Q defined by a*b=ab+1 is commutative and associative or not.
2. For what values of x, 2 $\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ holds.
3. Construct a 3 ×4 matrix whose elements are a_{ij} = 2i-j.
4. If $|A| = 6$ then find the value of $|A\ A^1|$.
5. Find $\frac{dy}{dx}$ if y = $e^{\log}(\log x^2)$.
6. Find the anti derivative of sin2x – $4e^{3x}$ w.r.t 'x'.
7. Define collinear vector.
8. Find the equation of the line parallel to x-axis and passing through origin.
9. Define objective function.
10. If P(A) = 3/5 and P(B) = 1/5 find P(A∩B) where A and B are independent events.

PART - B

II Answer any Ten of the following **10× 2 = 20**

11. Show that if f:A→B and g: B→c are one – one then gof : A→C is also one-one.
12. Find the value of Cos(Sec^{-1}x + Cosec^{-1}x), $|x| \geq 1$.
13. Prove that $\tan^{-1}(2/11) + \tan^{-1}(7/24) = \tan^{-1}(1/2)$.
14. Find the value of k if area of a triangle is 4 sq. Units and vertices are (k,0), (4,0) (0,2)
15. Differentiate $\cos(x^3)\sin^2(x^5)$.
16. Differentiate $(\log x)^{\cos x}$.
17. Find the approximate change in volume of a cube of side x meters caused by increasing the side by 1%.
18. Integrate $\frac{1-\cos x}{1+\cos x}$ w.r.t 'x'.
19. Evaluate $\int \frac{xe^x}{(1+x)^2}dx$.
20. Find order and degree of a differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + Cos\left(\frac{dy}{dx}\right) = 0$.
21. Show that the points A (1, -2, -8), B(5,0,-2) and C(11,3,7) are collinear.
22. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ then show that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular.
23. Find the angle between the pair lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.
24. A random variable x has the following probability distribution

X:	0	1	2	3	4
P(x) :	0.1	k	2k	2k	k, then find the value of k and p(x≥2)

PART - C

III. Answer any Ten of the following: 10×3=30

25. Show that the relation R in the set Z of integers given by R = {(a,b): 2 divides a-b} is an equivalence relation.
26. Prove that $Sin^{-1}(5/13)+Cos^{-1}(3/5)=tan^{-1}(63/16)$.
27. If A and B are invertible matrices of he same order then prove that $(AB)^{-1} = B^{-1}, A^{-1}$.
28. If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1$0<x<1, prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.
29. Verify Rolle's theorem for the function $f(x) = x^2+2x-8$, xϵ[-4,2].
30. Find the equation of tangent to the curve given by x=a sin^3t and y=bcos^3t at a point t = $\frac{\pi}{2}$.
31. Evaluate $\int \frac{dx}{1+\tan n}$.
32. Evaluate the definite integral $\int_0^2 e^x$ dx as the limit of a sum.
33. Find the area of the circle $x^2+y^2=a^2$ by integration method.
34. Form the differential equation of the family of circles touching y – axis at origion.
35. Find the cosine angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ *and* $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$.
36. Prove that $\left[\vec{a}, \vec{b}, \quad \vec{c} + \vec{d}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right] + \left[\vec{a}\vec{b}\vec{d}\right]$
37. Find the equation of the plane that contains the point (1,-1,2) and is perpendicular to each of the planes 2x+3y-2Z=5 and x+2y-3Z=8.
38. In a factory which manufactures bolts, machines A, B and c manufactures respectively 25%, 35%, and 40% of the bolts. Of their outputs 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and found to be defective. What is the probability that it is manufactured by the machine B?

PART-D

IV. Answer any six of the following: 6× 5 = 30

39. Let f: N →R be a function defined as $f(x) = 4x^2 +12x+15$. Show that f; N→S, where S is the range of f is invertible. Find the inverse of f.
40. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ then show that $A^3-23A-40I=0$ hence find A^{-1}.
41. Solve the system of equations by matrix method: X+Y+Z =6; Y + 3Z = 11 and x-2y+Z=0.
42. If Y = $e^{a\cos^{-1}x}$ $-1 \le x \le 1$ show then$(1-x^2)$ $\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$.
43. A man of height 2 meters walks at a uniform speed of 5km/hr away a lamp post which is 6 meters high. Find the rate at which the length of his shadow increases.
44. Find the integral of $\frac{1}{\sqrt{a^2 - x^2}}$ w.r.t s and hence evaluate $\int \frac{dx}{\sqrt{7-6x-x^2}}$.
45. Using integration, find the area of a triangular region whose sides have the equations y = 2x+1, y = 3x+1 and x=4.

46. Find the particular solution of the differential equation $\frac{dy}{dx} - 3y\cot x = Sin2x; y = 2\, when\, x = \frac{\pi}{2}$.

47. Derive the equation of a plane in normal form (both in vector and Cartesian form)

48. Find the probability of getting at most two sixes in six throws of a single die.

PART-E

V. Answer any One of the following: 1×10=10

49. a)Prove that $\int_a^b f(x)dx = \begin{cases} 2\int_0^a f(x)dx \;\; if \;\; f(x)\, in\, an\, even\, function \\ 0, if \;\; f(x)\, is\, an\, odd\;\, function\;\, and\;\, hence\;\, calculate \int_{-1}^{1} Sin^5 x\cos^4 x\; dx \end{cases}$

b) Determine the value of k, if f(x) = $\begin{cases} \frac{K\cos x}{\pi - 2x}, \; if \;\; x \neq \frac{\pi}{2} \\ 3 \quad if\,(x) = \frac{\pi}{2} \end{cases}$, is continuous at x=$\frac{\pi}{2}$

50. a) There are two types of fertilizers $F_1$and F_2. F_1 consists of 10% Nitrogen and 6% phosphoric acid. And F_2 consists 5% Nitrogen and 10% Phosphoric acid. After testing the soil condition former finds that he needs atleast 14 kg Nitrogen and 14 kg of phosphoric acid, for his crop. If F1 costs Rs 6kg and F2 cost Rs 5per kg. Determine how much of each type of fertilizer should be used so that nutrient requirements are met at min cost. What is the minimum cost?

b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$.

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