SRI BHAGAWAN MAHAVEER JAIN COLLEGE	Course:	II PUC
Vishweshwarapuram, Bangalore.	Subject:	**Mathematics**
	Max. Marks:	100
Mock Exam 1 - Feb.2016	Duration:	3:15 Hrs.

Instructions :

(i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.

(ii) Use the graph sheet for the question on linear programming in PART E.

PART – A

I Answer ALL the questions: 10 x1=10

1. Show that *: $R \to R$ defined by $(a,b) = a + 4b^2$ is a binary operation.
2. Find the principal value o f $y = \tan^{-1}(-1)$
3. Define a skew-symmetric matrix.
4. If A is a matrix of order 3 x 3 than find |adjA| where |A| = 2
5. If $y = \sin^3 x + \cos^6 x$ find $\frac{dy}{dx}$.
6. Evaluate $\int \sec^2(7-4x)\,dx$
7. Define the term "corner points" of LPP.
8. If $\vec{a}$ and $\vec{b}$ are any two vector such that $\vec{a}.\vec{b} = \left|\vec{a} \, x \, \vec{b}\right|$ Find the angle between $\vec{a}$ and $\vec{b}$.
9. Show that the planes $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are parallel.
10. If P(A) = 0.8. P (B) = 0.5 and P(B/A) = 0.4 find P $(A \cap B)$

PART – B

II Answer any TEN questions: 10 x 2 = 20

11. If f;R → R given by $f(x) = (3 - x^3)^{1/3}$. Find $fof(x)$
12. Prove that $\cos^{-1}(-x) = \pi - \cos^{-1}(x) \quad x \in [-1,1]$
13. Solve: $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$
14. For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ find the values of a and b such that $A^2 + aA + bI = 0$.
15. If $y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ Find $\frac{dy}{dx}$ where $\left(0 < x < \frac{1}{\sqrt{2}}\right)$
16. Find the derivative of $\cos\left[\log x + e^x\right]$ where $x > 0$.
17. Find the approximate change in surface of a cube of side x meters caused by increasing the side by 1%.
18. Evaluate $\int_0^{2/3} \frac{dx}{4 + 9x^2}$
19. Integrate $\frac{\cos(\sqrt{x})}{\sqrt{x}}$ with respect to x.
20. Form the differential equation of the family of parabolas having vertex at origin and axis along positive direction of X- axis.
21. Evaluate $\hat{i}.\left(\hat{j} \times \hat{k}\right) + \hat{j}.\left(\hat{i} \times \hat{k}\right) + \hat{k}.\left(\hat{j} \times \hat{i}\right)$.
22. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} \quad \vec{c} = \hat{i} - 2\hat{j} + \hat{k}$. Find the unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

23. Find the distance of the point (2,5,-3) from the plane $\vec{r}.\left(6\hat{i}-3\hat{j}+2\hat{k}\right)=4$

24. Given two independent event A and B such that $P(A)=0.3$ $P(B)=0.6$. Find (i) P(A and not B) (ii) P(neither A nor B).

PART- C

III Answer any TEN questions: 10 x 3 =30

25. Find fog and gof if $f:R\rightarrow R$ and g: $R\rightarrow R$ defined by $f\left(x\right)=8x^3$and $g\left(x\right)=x^{1/3}$ show that $fog \ \# \ gof$

26. Solve:- $\tan^{-1}\left(2x\right)+\tan^{-1}3x=\frac{\pi}{4}$

27. Using elementary transformation find the inverse of the matrix A = $\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}$

28. If $x=a\left[\cos t+log\tan\left(\frac{t}{2}\right)\right]$ $y=a\sin t$.find $\frac{dy}{dx}$.

29. Verify Rolle's theorem for the function $f\left(x\right)=x^2+2x-8$ $x\in\left[-4,2\right]$

30. Find the equation of the tangent to the curve $y=x^2-2x+7$ which is parallel to the line $2x-y+9=0$

31. Evaluate $\int\frac{4x+1}{\sqrt{2x^2+x-3}}\,dx.$

32. Evaluate $\int_4^9\frac{\sqrt{x}}{\left(30-x^{3/2}\right)^2}\,dx$

33. Find the area of the region bounded by two parabolas $y=x^2$ and $x^2=y$ using method of integration.

34. Find the general solution of the differential equation $\frac{dy}{dx}=e^{x+y}$

35. Find the cosine of the angle between the vectors $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, and $\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$.

36. The two adjacent sides of a parallelogram are $\vec{a}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}-4\hat{j}+5\hat{k}$. Find the unit vector parallel to its diagonals. Also find its area.

37. Find the co-ordinates of the point where the line passes through (5,1,6) and (3,4,1) crosses the YZ - plane.

38. One card is drawn at random from a well shuffled deck of 52 cards in which E: The card draws is a king or queen F: The card drawn is a queen or jack. Are E and Fare independent.

PART-D

IV Answer any SIX questions: 6 x 5 = 30

39. Consider $f:R^+\rightarrow[-5,\infty)$ given by $f\left(x\right)=9x^2+6x-5$ show that f is invertible with $f^{-1}\left(y\right)=\frac{\sqrt{y+6}-1}{3}$

40. If A = $\begin{bmatrix}1\\-4\\3\end{bmatrix}$ and B = [-1 2 1] verify $(AB)^{|}=B^{|}A^{|}$

41. Solve by matrix method $x-y+2z=7;\ 3x+4y-5z=5;\ 2x-y+3z=12$.

42. If $e^y\left(x+1\right)=1$ show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

43. A ladder 5 m long leaning against a smooth vertical wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 cms/ sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 mts away from the wall?

44. Find the integral of $\frac{1}{x^2+a^2}$ with respect to x and hence Evaluate $\int \frac{3x^2}{x^6+1}\,dx$

45. Find the area of the region in the first quadrant enclosed by $x-axis,$ the line $x=\sqrt{3}y$ and the circle $x^2+y^2=4$

46. Find the general solution of the differential equation $x\frac{dy}{dx}+2y=x^2\log x$.

47. Derive the formula to find the shortest distances between two skew-lines $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ in vector form.

48. A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize at least once exactly once.

PART – E

V Answer any ONE question: 1 x 10 = 10

49. (a) Minimize and Maximize $z=x+2y$ subject to the constraints $x+2y\geq 100$, $2x-y\leq 0$ $2x+y\leq 200$, $x,y\geq 0$ **[6]**

b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ **[4]**

50. a) Prove that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ and hence Evaluate $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$ **[6]**

b) Find all points of discontinuity of f where f is defined by $f(x)=\begin{cases} \frac{x}{1x1} & \text{if } x<0 \\ -1 & \text{if } x\geq 0 \end{cases}$ **[4]**

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