

**Instructions:**

- (i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.
(ii) Use the graph sheet for the question on linear programming in PART E.

PART – A**I Answer all the question****10 x 1 = 10**

1. Give an example of a relation which is symmetric and transitive but not reflexive.
2. Write the domain of $f(x) = \cos^{-1} x$
3. Find the value of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$
4. Define an identity matrix
5. If $y = \sec(2x + 3)$ find $\frac{dy}{dx}$
6. Write the integral of $\frac{-1}{x\sqrt{x^2 - 1}}$ where $x \geq 1$
7. Write the vector joining the points A(2,3,1) and B (-1, -2, -4)
8. Define collinear vectors
9. Define feasible solution
10. A pair of die is rolled. Consider events $E = \{2, 4, 6\}$ and $F = \{1, 2\}$. Find $P(E | F)$.

PART – B**II Answer any TEN questions****10 x 2 = 20**

11. Define binary operation on a set. Verify whether the operation $*$ defined on z by $a * b = a^b$ is binary or not.
12. Find the simplest form of $\tan^{-1} \sqrt{3} - \sec^{-1}(\sqrt{2})$
13. Evaluate $\sin \left[\frac{\pi}{3} - \sin^{-1} \left(-\frac{1}{\sqrt{2}} \right) \right]$
14. Find the area of the triangle whose vertices are (3,8) (-4,2) and (5,1) using determinants.
15. If $y = \sin(\log_e x)$ prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{x}$
16. Find the derivative of $x^{\sin x}$ with respect to x
17. Find a point on the line $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.
18. Find $\int e^x \sec x (\sin x + \sec x) dx$
19. Evaluate $\int x^3 e^x dx$
20. prove that the differential equation $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ is a homogeneous differential equation of degree 0.
21. If the position vectors of the points A and B respectively are $\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{j} - \hat{k}$. Find the direction cosines of \vec{AB}

22. Find the unit vector in the direction of the vectors $\vec{a} = \hat{i} + 3\hat{j} + 4\hat{k}$
23. Find the distance of the point (2, 3, -5) from the plane $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 2\hat{k}) = 9$
24. Two cards drawn at random and without replacement from a pack of 52 playing cards are black. Find the probability that both the cards are black.

PART – C**III Answer any TEN questions****10 x 3 = 30**

25. Show that the relation R in the set of all integers, Z defined by $R = \{(a, b) : 2 \text{ divides } a - b\}$ is an equivalence relation.
26. If $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$ find x
27. Using elementary transformation find the inverse of $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$.
28. Verify the mean value theorem of $f(x) = x^2 - 4x + 3$ in the interval [a, b] where a = 1 and b = 4.
29. If $y = \cot^{-1} \left(\frac{2x}{1-x^2} \right)$ find $\frac{dy}{dx}$
30. Find the intervals in which the function is given by $f(x) = x^2 + 2x - 5$ is
(i) strictly increasing (ii) strictly decreasing.
31. Find the antiderivative of $f(x)$ given by $f(x) = 4x^3 - \frac{3}{x^4}$ such that $f'(2) = 0$.
32. Evaluate $\int \frac{dx}{(x+1)(x+2)}$
33. Find the area of the region bounded by the line $y^2 = 9x$ and the lines $x = 0$, $x = 4$ and the x – axis in the first quadrant.
34. Form the differential equation of the family of circles touching the x – axis at origin.
35. For any three vectors \vec{a}, \vec{b} and \vec{c} prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$
36. Find the area of a triangle having the points A(1,1,1), B(1,2,3) and C(2,3,1) as the vertices.
37. Find the equation of the line which passes through the point (1,2,3) and is parallel to the vector $\hat{i} + 2\hat{j} + 2\hat{k}$ both in vector form and Cartesian form.
38. A bag contains 4 red and 4 black balls another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

PART – D**IV Answer any SIX questions****6 x 5 = 30**

39. Verify whether the function $f : R_+ \rightarrow [4, \infty)$ defined by $f(x) = x^2 + 4$ is invertible or not, write the inverse of f(x) if exists.
40. If $A = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ $B = [2 \quad 4 \quad -6]$ verify that $(AB)^T = B^T A^T$
41. Solve the following system of equation by matrix method: $x + y + z = 6$; $y + 3z = 11$ and $x - 2y + z = 0$.
42. If $y = Ae^{mx} + Be^{-nx}$ prove that $\frac{d^2y}{dx^2} - (m-n)\frac{dy}{dx} - mny = 0$
43. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of an edge is 10 centimeters

- 44. Evaluate $\int_2^3 x^2 dx$ as a limit of a sum.
- 45. Solve the differential equation $ydx - (x + 3y^2) dy = 0$
- 46. Find the area of the circle $4x^2 + 4y^2 = 9$. Which is interior to the parabola $x^2 = 4y$
- 47. Derive the condition for the coplanarity of two lines in space both in the vector form and Cartesian form.
- 48. Find the probability of getting at most two sixes in six throws of a single die.

PART – E

V Answer any ONE questions

1 x 10 = 10

- 49. (a) Minimize and maximize $Z = 3x + 9y$ subject to the constraints $x + 3y \leq 60$; $x + y \geq 10$; $x \leq y$; $x \geq 0$, $y \geq 0$ by the graphical method

b) Prove that
$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + ab + bc + ca$$

- 50. a) Prove that
$$\int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$
 and evaluate $\int_0^{2\pi} \sin^3 x dx$

(b) Find all points of discontinuity of $f(x) = \begin{cases} 2x+3, & \text{if } x \leq 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$
