Jain College, Jayanagar
II PUC Mock Paper I 2018
Mathematics

Duration: 3hr 15 min **Max.Marks: 100**

PART A

I. Answer all the questions: **1 × 10 = 10**

1. Let * be a binary operation defined on the set of rational numbers defined by a*b = 7ab + 7. Verify whether * is a binary operation.
2. Find the principal value of $\sec^{-1}\left(-\sqrt{2}\right)$
3. Define identity matrix.
4. Find K if the matrix $\begin{bmatrix} K & 4 \\ 3 & 2 \end{bmatrix}$ has no inverse.
5. Find the derivative of tan x° {x degree}
6. Find the anti derivative of $(2\cos^2 3x - 1)dx$
7. Find the value of λ for the vector $\vec{a} = 2\hat{i} - 3\lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ are perpendicular to each other.
8. What is the equation of the plane that cuts the co-ordinate axes at (a, 0, 0) (0, b, 0) and (0, 0, c).
9. Define objective function of LPP.
10. $If \quad P(A \cap B) = 0.32$ and P(A) = 0.8. Find P(B/A)

PART B

II. Answer any 10 questions: **10 × 2 = 20**

11. If $f : R \to R$ given by $f(x) = (3 - x^3)^{1/3}$. Then find (fof) (x).
12. Write the simple form of $\tan^{-1}\left[\sqrt{\frac{1+\cos 2x}{1-\cos 2x}}\right]$
13. If $\operatorname{cosec}\left[\tan^{-1}\left(\frac{1}{7}\right) + \cot^{-1} x\right] = 1,\ find\ x$
14. Find the area of the triangle whose vertices are (3, 8), (-4, 2) and (5, 1) using determinants.
15. If $x = \cos^{-1}\left(2t^2 - 1\right)$ and $y = \sin^{-1}\left(4t^3 - 3t\right)$ prove that $\frac{dy}{dx} = -\frac{3}{2}$
16. Prove that $f(x) = x^3 - 3x^2 + 4x, \quad x \in R$ is strictly increasing on R.
17. Find the points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to x-axis.
18. Evaluate $\int \frac{\cot(1+\log x)}{x} dx$
19. $\int x \sin^3\left(x^2\right) \cos x^2 dx$
20. Find the order and degree of $\left[\frac{d^3y}{dx^3} + y^2\right]^{3/2} = e^{y^{111}}$
21. Prove that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$
22. Find the shortest distance between $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}\ and\ \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$
23. If α, β, γ are the angles made by a vector with co-ordinate axes then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

24. Two dice are thrown. Find the probability of getting an odd number on the first die and a multiple of 3 on the other.

PART C

III. Answer any 10 questions: 10 × 3 = 30

25. Verify whether $f(x)=\frac{x-2}{x-3}$ is one-one and onto or not give reason.

26. Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

27. Express $A=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}$ as sum of symmetric and skew symmetric matrix.

28. If $\sqrt{1+x}+\sqrt{1+y}=0$ P.T $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$

29. If $x=a^{\sqrt{\tan^{-1}t}}$ $y=a^{\sqrt{\cot^{-1}t}}$ P.T $\frac{dy}{dx}=\frac{-y}{x}$

30. Verify mean value theorem for the function $f(x)=x^2-4x-3$ in the interval [1, 4]

31. Evaluate $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

32. Find $\int\frac{x}{(x+1)(x+2)}dx$

33. Find the area of the region bounded by $y^2=9x,\ x=2\ and\ x=4$.

34. Prove that $x^2\frac{dy}{dx}=x^2-2y^2+xy$ is a homogeneous differential equation.

35. Find a vector perpendicular to each of the vectors $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$ which has magnitude 10 units.

36. If $\vec{a}=-4\hat{i}-6\hat{j}-\lambda\hat{k}$

    $\vec{b}=-\hat{i}+4\hat{j}+3\hat{k}$

    $\vec{c}=-8\hat{i}-\hat{j}+3\hat{k}$ are coplanar. Find λ.

37. Find the distance between parallel lines $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+m(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+n(2\hat{i}+3\hat{j}+6\hat{k})$

38. Given that 2 numbers appearing on throwing two dice are different. Find the probability of the event "the sum of numbers on the dice is 4".

PART D

IV. Answer any 6 questions: 6 × 5 = 30

39. Let R^+ be the set of all non-negative real numbers. Prove that $f:R^+\to[4,\infty]$ defined by $f(x) = x^2 + 4$ is invertible. Also write the inverse of f.

40. If $A=\begin{bmatrix}1 & 1 & -1\\ 2 & 0 & 3\\ 3 & -1 & 2\end{bmatrix}$ $B=\begin{bmatrix}1 & 3\\ 0 & 2\\ -1 & 4\end{bmatrix}$ $C=\begin{bmatrix}1 & 2 & 3 & -4\\ 2 & 0 & -2 & 1\end{bmatrix}$ then prove that A(BC) = (AB) C

41. Using matrix method solve the following equations

    2x – 3y + 5z = 11

    3x + 2y – 4z = -5

    x + y – 2z = -3

42. If y = 5cos(logx) + 7sin(logx). Prove that $x^2y_2+xy_1=0$

43. A man of height 2 meters walks at a uniform speed of 5 km/hr away from a lamp post which is 6 meters height. Find the rate at which the length of his shadow increases.

44. Find the integral of $\frac{1}{\sqrt{x^2+a^2}}$ w.r.t x and hence evaluate $\int \frac{1}{\sqrt{x^2+2x+4}}dx$

45. Find the area of the region enclosed by the parabola $x^2 = 4y$ and the line x = 4y – 2 and x-axis.

46. Derive the equation of the line in space passing through a point and parallel to a vector both in the vector and Cartesian form.

47. Solve the differential equation $x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x$

48. If a fair coin is tossed 8 times. Find the probability of (i) at least five heads and (ii) at most five heads.

PART E

V. Answer any 1 question: 1 × 10 = 10

49. a) Prove $\int_0^{2a} f(x)dx=\int_0^{a} f(x)dx+\int_0^{a} f(2a-x)dx$ then prove

$$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^{a} f(x)dx & when \quad f(2a-x)=f(x) \\ 0 & when f(2a-x)=-f(x) \end{cases}$$

b) Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (x^3-1)^2$

50. a) Solve the following L.P.P.

Minimize and maximize z = 3x + 9y subject to the constraints

$x+3y \le 60$

$x+y \ge 10$

$x \le y$

$x \ge 0, y \ge 0$

b) Determine the value of k if $f(x)=\begin{cases} \frac{k\cos x}{\pi-2x} & if \quad x \ne \frac{\pi}{2} \\ 3 & if \quad x=\frac{\pi}{2} \end{cases}$

****