Jain College, Jayanagar
II PUC Mock Paper 2
Subject - Mathematics

Duration: 3.15 minutes **Max.Marks: 100**

Section – A

I. Answer all the following:

1. Find whether * on Z^+ defined by $a*b=a^b, \forall a,b \in Z^+$ is a binary operation or not.
2. Find the principle value of $\text{cosec}^{-1}(2)$.
3. Construct a 2 × 2 matrix, $A=[aij]$ whose elements are $aij=\frac{(i+j)^2}{2}$.
4. Find the value of x if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$
5. Find $\frac{dy}{dx}$ if y = tan(2x + 3).
6. Find antiderivative of $(ax+b)^2$ w.r.t 'x'.
7. Define coplanar vectors.
8. Find direction cosines of y-axis.
9. Define feasible region.
10. If P(A) = 0.3, P(B) = 0.5, P(B/A) = 0.2 find $P(A \cap B)$.

Section – B

II. Answer any ten of the following:

11. Show that the relation R on the set A = {1,2,3,4,5,6} as R = {(x,y); y is divisible by x} is reflexive and transitive.
12. Prove that $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=2\cos^{-1}x, \ 1/\sqrt{2} \le x \le 1$.
13. Write $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), 0<x<\pi$ in simplest form.
14. Show that the points A(a, b+c), B(b, c+a), C(c, a+b) are collinear.
15. Find $\frac{dy}{dx}$ if $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0<x<\frac{1}{\sqrt{2}}$.
16. Differentiate $(x+3)^2\cdot(x+4)^3(x+5)^4$ with respect to x.
17. Find the intervals in which the function $f(x)=2x^2-3x$ is strictly increasing and strictly decreasing.
18. Integrate $\frac{1}{x(\log x)^m}, m\neq 1$, x > 0 w.r.t 'x'.
19. Evaluate $\int_0^{\pi/4}\left(2\sec^2 x+x^3+2\right)dx$.
20. Find order and degree of a differential equation $\left(\frac{ds}{dt}\right)^4+3s\left(\frac{d^2s}{dt^2}\right)=0$
21. Find $\left|\vec{a}\times\vec{b}\right|$ if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$.
22. If $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2$. Prove that $\vec{a}$ and $\vec{b}$ are perpendicular to each other, given $\vec{a}\neq 0, \vec{b}\neq 0$.
23. Find the angle between the lines whose direction ratios are a,b,c and b-c, c-a, a-b.
24. A dies thrown, E is an event 'the number. appearing is a multiple of 3' and F be the event 'the number. appearing is even' then find whether E and F are independent.

Section – C

III. Answer any ten of the following:

25. Let $f : x \to y$ and $g : y \to z$ be two invertible functions. Then prove that gof is also invertible with $(gof)^{-1} = f^{-1}og^{-1}$.

26. Simplify $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right]$, if $\frac{a}{b}\tan x > -1$.

27. Using elementary transformation find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

28. If $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.

29. Verify mean value theorem, if $f(x) = x^3 - 5x^2 - 3x$ in [a, b] where a = 1, b = 3.

30. Find two numbers whose sum is 24 and whose product is as large as possible.

31. Evaluate $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx$

32. Evaluate $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

33. Find the area of the region bounded by the curve $y^2 = 4x$ and the line y = 2x.

34. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

35. Prove that $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

36. Show that the four points A,B,C and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-(\hat{j}+\hat{k})$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ are respectively coplanar.

37. Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

38. Two dice are thrown simultaneously, if x denotes the number of sixes, find the expectation of x and variance of x.

Section – D

IV. Answer any two of the following:

39. Consider $f : R_+ \to (-5\infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible.

40. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ verify that (A+B)C = AC + BC.

41. Solve the system of equations by matrix method 3x – 2y + 3z =8; 2x + y – z =1 and 4x – 3y + 2z = 4.

42. If $y = \cos^{-1} x$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

43. Show that height of cylinder of maximum volume that can be inscribed in a sphere of radius 'a' is $\frac{2a}{\sqrt{3}}$.

44. Find the integral of $\sqrt{x^2 - a^2}$ with respect to x and hence evaluate $\int \sqrt{x^2 + 4x + 1}\,dx$.

45. Using integration method find the area of the region bounded by the triangle whose vertices are (1, 0) (2, 2) and (3, 1).

46. Find the general solution of the differential equation $(1+x^2)dy + 2xy\ dx = \cot x\,dx$.

47. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and cartesian form.

48. A fair coin is tossed 10 times, find the probability of (a) exactly six head (b) atleast six heads (c) atmost six heads.

Section – E

V. Answer any one of the following:

49. a) Prove that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ and hence evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

b) Find the relationship between 'a' and 'b' so that the function f defined by

$f(x) = \begin{cases} ax+1, & if \ \ x \le 3 \\ bx+3, & if \ \ x > 3 \end{cases}$ is continuous at x = 3

50. a) Solve the following problem graphically minimize and maximize z = 3x + qy Subject to constraints $x+3y \le 60; x+y \ge 10, x \le y, x \ge 0, y \ge 0$

b) If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ then show that 1 + xyz = 0

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