Unit -I : Number System

Chapter - 1 : Real Numbers

TOPIC-1
Euclid's Division Lemma and Fundamental Theorem of Arithmetic

Quick Review

- **Algorithm :** An **algorithm** is a series of well defined steps which gives a procedure for solving a type of problems.
- **Lemma :** A lemma is a proven statement used for proving another statement.
- **Euclid's Division Lemma :** For given positive integers a and b, there exist unique integers q and r, satisfying $a = bq + r, 0 \leq r < b$. Here a = Dividend, b = Divisor, q = Quotient, r = Remainder *i.e.,*
  Dividend = (Divisor × Quotient) + Remainder
  For example,

$$\begin{array}{rrl} & & \underline{1 \rightarrow \text{Quotient}} \\ \text{Divisor} & 3) & 5 \\ & & \underline{-3} \\ & & 2 \rightarrow \ \text{Remainder} \end{array}$$

  According to the above formula, $5 = (3 \times 1) + 2$
- We state Euclid's division algorithm for positive integers only but it can be extended for all integers except zero, $b \neq 0$.
- When 'a' and 'b' are two positive integers such that $a = bq + r, 0 \leq r < b$, then $HCF\ (a, b) = HCF\ (b, r)$.
- **The steps to find the *HCF* of two positive integers by Euclid's division algorithm are given below :**
  (i) Let two integers be a and b such that $a > b$.
  (ii) Take greater number a as dividend and the number b as divisor.
  (iii) Now find whole numbers 'q' and 'r' as quotient and remainder respectively.
  $\therefore \quad a = bq + r, 0 \leq r < b.$
  (iv) If $r = 0$, b is the *HCF* of a and b. If $r \neq 0$, then take r as divisor and b as dividend.
  (v) Repeat step (iii), till the remainder is zero, the divisor thus obtained at last stage is the required *HCF*.
- **The Fundamental Theorem of Arithmetic**
  Every composite number can be expressed as the product of powers of primes and this factorisation is unique, apart from the order in which the prime factors occur. Fundamental theorem of arithmetic is also called a **Unique Factorisation Theorem.**

  Composite number = Product of prime numbers

  Or

  Any integer greater than 1 can either be a prime number or can be written as a unique product of prime numbers.
  e.g.,
  (i) $2 \times 11 = 22$ is the same as $11 \times 2 = 22$. [ignoring the order]
  (ii) 6 can be written as 2×3 or 3×2, where 2 and 3 are prime numbers.
  (iii) 15 can be written as 3×5 or 5×3, where 3 and 5 are prime numbers.
  The prime factorisation of a natural number is unique, except to the order of its factors.
  e.g., 12 made by multiplying the prime numbers 2, 2 and 3 together,
  $12 = 2 \times 2 \times 3$
  We would probably write it as $12 = 2^2 \times 3$
  It is still a unique combination of (2, 2 and 3).
- By using Fundamental Theorem of Arithmetic, we shall find the *HCF* and *LCM* of given numbers (two or more).

This method is also called **Prime Factorization Method.**

- **Prime Factorization Method to find *HCF* and *LCM* :**
  - **(i)** Find all the prime factors of given numbers.
  - **(ii)** *HCF* of two or more numbers = Product of the smallest power of each common prime factor involved in the numbers.
  - **(iii)** *LCM* of two or more numbers = Product of the greatest power of each prime factor involved in the numbers.
  - **(iv)** For two positive integers a and b, we have

$$HCF\,(a, b) \times LCM\,(a, b) = a \times b$$

or,
$$HCF\,(a, b) = \frac{a \times b}{LCM(a, b)}$$

and
$$LCM\,(a, b) = \frac{a \times b}{HCF(a, b)}$$

How it is done on GREENBOARD?

Q. Show that 6^n can never end with digit 0 for any natural number n.

Sol. : Step I : Any number which ends in zero must have at least 2 and 5 as its prime factors.

Step II : or, $6 = 2 \times 3$

or, $6^n = (2 \times 3)^n = 2^n \times 3^n$

Hence prime factor of 6 are 2 and 3.

Step III : Since 6^n does not contain 5 as a prime factor. Hence 6^n can never end in zero.

TOPIC-2
Irrational Numbers, Terminating and Non-Terminating Recurring Decimals

Quick Review

- **Rational Numbers :** The number in the form $\frac{p}{q}$, where p and q are coprime number and $q \neq 0$, is known as rational number.

  For Example : $2, -3, \frac{3}{7}, -\frac{2}{5}$ etc rational numbers.

- **Irrational Numbers :** A number is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. For example $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$ are irrational numbers.
- Let p be a prime number. If p divides a^2, then p divides a where a is a positive integer.
- **Terminating Decimals :** If decimal expansion of rational number $\frac{p}{q}$ comes to an end, then the decimal obtained from $\frac{p}{q}$ is called terminating decimal.
- **Non-terminating Repeating Recurring Decimals :** The decimal expansion obtained from $\frac{p}{q}$ repeats periodically, then it is called non-terminating repeating or recurring decimal.
- Just divide the numerator by the denominator of a fraction. If you end up with a remainder of O, you have a terminating decimal otherwise repeating or recurring decimal.
- The sum or difference of a rational and irrational number is irrational.
- The product and quotient of a non-zero rational and irrational number is irrational.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form 2^m5^n, where n and m are

non-negative integers. Then x has a decimal expansion which terminates after k places of decimals, where k is the largest of m and n.

- Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$, where p and q are co-prime and the prime factorisation of q is of the form 2^m5^n, where m and n are non-negative integers.
- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form 2^m5^n, where n and m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating.

Q. Show that $2\sqrt{3} + 5$ is an irrational number.

Sol. : Step I : Let $2\sqrt{3} + 5$ be a rational number.

A rational number can be expressed as $\frac{a}{b}$, where $b \neq 0$

Step II : or, $2\sqrt{3} + 5 = \frac{a}{b}$

or, $2\sqrt{3} = \frac{a}{b} - 5$

or, $\sqrt{3} = \frac{1}{2}\left(\frac{a}{b} - 5\right)$

Here R.H.S $= \frac{1}{2}\left(\frac{a}{b} - 5\right)$ is rational while L.H.S, $\sqrt{3}$ is irrational which is not possible.

Step III : Hence our assumption that $2\sqrt{3} + 5$ is a rational number is wrong. Hence $2\sqrt{3} + 5$ is an irrational number.

Unit -II : Algebra

Chapter - 2 : Polynomials

TOPIC-1
Zeroes of a Polynomial and Relationship between Zeroes and Coefficients of Quadratic Polynomials

Quick Review

- **Polynomial :** An algebraic expression of the form $a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots + a_2x^2 + a_1x + a_0$, (where n is a whole number and $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers) is called a polynomial in the variable x.
- **Value of a Polynomial at a given point :** If $p(x)$ is a polynomial in x and 'α' is a real number, then the value obtained by putting $x = \alpha$ in $p(x)$ is called the value of $p(x)$ at $x = \alpha$.
- **Zero of a Polynomial :** A real number k is said to be a zero of the polynomial $p(x)$, if $p(k) = 0$.

  Geometrically, the zeroes of a polynomial $p(x)$ are precisely the x-co-ordinates of the points, where the graph of $y = p(x)$ intersects the x-axis.

  (i) A linear polynomial has at most one zero.

  (ii) A quadratic polynomial has at most two zeroes.

  (iii) A cubic polynomial has at most three zeroes.

  (iv) In general, a polynomial of degree n has at most n zeroes.
- **Graphs of Different type of Polynomials :**
- **Linear Polynomial :** The graph of a linear polynomial $p(x) = ax + b$ is a straight line that intersects x-axis at one point only.
- **Quadratic Polynomial :** (i) Graph of a quadratic Polynomial $p(x) = ax^2 + bx + c$ is a parabola open upward like U. If a > 0 and intersects x-axis at maximum two distinct points.

(ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downward like $\cap$ if $a < 0$ and intersects X axis at maximum two distinct points.

Graph of a cubic polynomial : Graph of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ intersects x-axis at maximum three distinct points.

➢ **Relationship between the zeroes and the coefficients of Polynomials :**

(i) Zero of a linear polynomial $= \dfrac{-\text{Constant term}}{\text{Coefficient of } x}$

If $ax + b$ is a given linear polynomial, then zero of linear polynomial $ax + b$ is $\dfrac{-b}{a}$

(ii) In a quadratic polynomial,

$$\text{Sum of zeroes of a quadratic polynomial} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes of a quadratic polynomial} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$\therefore$ If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

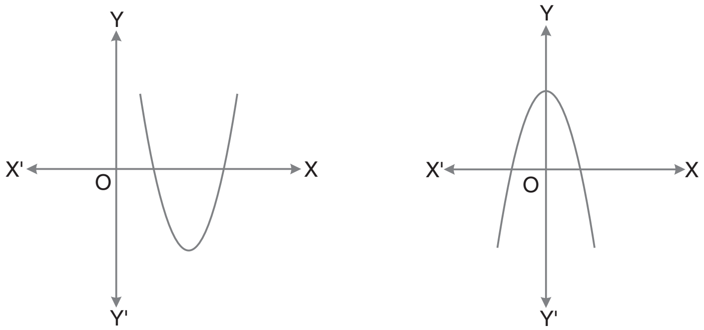
$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

(iii) If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = -\frac{b}{a},\ \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \text{ and } \alpha\beta\gamma = -\frac{d}{a}$$

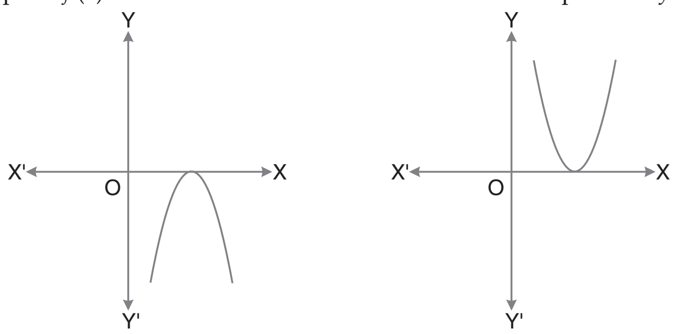
➢ **Discriminant of a Quadratic Polynomial :** For $f(x) = ax^2 + bx + c$, $a \neq 0$, $b^2 - 4ac$ is called its discriminant D. The discriminant D determines the nature of roots/zeroes of a quadratic polynomial.

Case I : If $D > 0$, graph of $f(x) = ax^2 + bx + c$ will intersect the X-axis at two distinct points, x-co-ordinates of points of intersection with X-axis is known as 'zeroes' of $f(x)$.



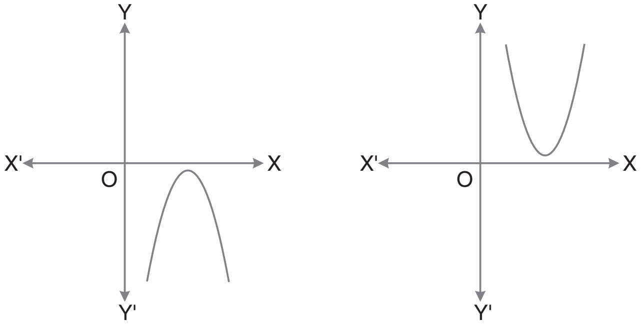
$\therefore f(x)$ will have two zeroes and we can say that roots/zeroes of the to given polynomials are real and unequal.

Case II : If $D = 0$, graph of $f(x) = ax^2 + bx + c$ will touch the X-axis at one point only.



$\therefore f(x)$ will have only one 'zero' and we can say that roots/zeroes of the given polynomial are real and equal.

Case III : If $D < 0$, graph of $f(x) = ax^2 + bx + c$ will neither touch nor intersect the X-axis.



$\therefore$ $f(x)$ will not have any real zeroes.

Relationship between the zeroes and the coefficients of a Polynomial :

S. No.	Type of polynomial	General form	Maximum Number of zeroes	Relationship between zeroes and coefficients
1.	Linear	$ax + b; a \neq 0$	1	$k = -\dfrac{b}{a}$, *i.e.*, $k = -\dfrac{\text{Constant term}}{(\text{Coefficient of } x)}$
2.	Quadratic	$ax^2 + bx + c; a \neq 0$	2	Sum of zeroes $(\alpha + \beta)$ $= -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\dfrac{b}{a}$ Product of zeroes $(\alpha\beta)$ $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{c}{a}$
3.	Cubic	$ax^3 + bx^2 + cx + d; a \neq 0$	3	Sum of zeroes $(\alpha + \beta + \gamma)$ $= -\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} = -\dfrac{b}{a}$ Product of sum of zeroes taken two at a time $(\alpha\beta + \beta\gamma + \gamma\alpha)$ $= +\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3} = \dfrac{c}{a}$ Product of zeroes $(\alpha\beta\gamma)$ $= -\dfrac{\text{Constant term}}{\text{Coefficient of } x^3} = -\dfrac{d}{a}$

How it is done on GREENBOARD ?

Q. If α and β are the roots of equation $x^2 - 5x + 6 = 0$, then find the value of $\alpha - \beta$.

Sol. : Step I : Compare $x^2 - 5x + 6 = 0$ with $ax^2 + bx - c = 0$ to get

$a = 1$, $b = -5$ and $c = 6$.

Step II : Sum of roots $\alpha + \beta = \dfrac{-b}{a} = \dfrac{-(-5)}{1}$

or, $\alpha + \beta = 5$ ...(i)

Step III : Product of roots

$\alpha\beta = \dfrac{c}{a} = \dfrac{6}{1}$

or, $\alpha\beta = 6$ ...(ii)

Step IV : $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

or, $\alpha - \beta = \sqrt{(5)^2 - 4 \times 6}$

$= \sqrt{25 - 24}$

$= \sqrt{1} = \pm 1$

$= 1$ or -1

TOPIC-2
Problems on Polynomials

Quick Review

- **Degree of a Polynomial :** The exponent of the highest degree term in a polynomial is known as its degree. In other words, the highest power of x in a polynomial $f(x)$ is called the degree of the polynomial $f(x)$. *e.g.,*

  (i) $f(x) = 5x + \frac{1}{3}$ is a polynomial in variable x of degree 1.

  (ii) $g(y) = 3y^2 - \frac{5}{2}y + 7$ is a polynomial in variable y of degree 2.

- **Constant Polynomial :** A polynomial of degree zero is called a constant polynomial.

  e.g., $f(x) = 8, \quad g(y) = \frac{-5}{2}$

- **Linear Polynomial :** A polynomial of degree 1 is called a linear polynomial.

  e.g., $f(x) = 3x + 5, g(y) = 7y - 1, h(z) = 5z - 3.$

  In general a linear polynomial in variable x with a real coefficient is of the form $f(x) = ax + b$, where a and b are real numbers and $a \neq 0$.

- **Quadratic Polynomial :** A polynomial of degree 2 is called a quadratic polynomial.

  e.g., $f(x) = 5x^2, g(y) = 7y^2 - 5y,$

  and $h(z) = 8z^2 + 6z + 5$

  In general, a quadratic polynomial in variable x with real coefficients is of the form $f(x) = ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.

- **Division Algorithm for Polynomials :** If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$, such that

  $$p(x) = g(x) \times q(x) + r(x),$$

  where degree of $r(x)$ < degree of $g(x)$. and $r(x)$ is denoted for remainder.

  > **Note :**
  > (i) If $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.
  > (ii) Dividend = Divisor × Quotient + Remainder.

  To Divide Quadratic Polynomial by Linear Polynomial :

  Let, $p(x) = ax^2 + bx + c$ and $g(x) = mx + n$

  $$\frac{a}{m}x + \frac{1}{m}\left(b - \frac{an}{m}\right) \longrightarrow \text{Quotient}$$

  $$\text{Divisor} \longrightarrow mx + n \overline{)\, ax^2 + bx + c} \longrightarrow \text{Dividend}$$

  $$+ax^2 + \frac{an}{m}x$$

  $$- \quad -$$

  $$\left(b - \frac{an}{m}\right)x + c$$

  $$\left(b - \frac{an}{m}\right)x + \frac{n}{m}\left(b - \frac{an}{m}\right)$$

  $$- \quad -$$

  $$c - \frac{n}{m}\left(b - \frac{an}{m}\right)$$

  Remainder = Constant term

  Step I : To obtain the first term of the quotient, divide the highest degree term of the dividend (*i.e.,* ax^2) by the highest degree term of the divisor (*i.e.,* mx). *i.e.,* $\frac{a}{m}x$. Then, carry out the division process.

  What remains is $\left(b - \frac{an}{m}\right)x + c$.

Step II : Now, to obtain the second term of the quotient, divide the highest degree term of the new dividend $\left\{ i.e., \left(b - \frac{an}{m}x \right) \right\}$ by the highest degree term of the divisor (*i.e., mx*). *i.e.,* $\frac{1}{m}\left(b - \frac{an}{m} \right)$. Then carry on the division process.

How it is done on GREENBOARD ?

Q. What should be added to $x^2 - 8x + 10$ to make it divisible by $x - 3$?

Sol. : Step I : Let k be added to $x^2 - 8x + 10$ to make it divisible by $x - 3$

$$\Rightarrow \qquad p(x) = x^2 - 8x + 10 + k$$

Step II : Applying Long division

$$
\begin{array}{r}
x - 3\,\overline{)x^2 - 8x + 10 + k(}\; x - 5 \\
\underline{\pm x^2 \mp 3x \qquad\quad} \\
-5x + 10 + k \\
\underline{\mp 5x \pm 15 \quad} \\
-5 + k
\end{array}
$$

Step III : Here remainder is $-5 + k$ which must be zero.

or, $\qquad -5 + k = 0$

$\therefore \qquad k = 5$

Chapter - 3 : Pair of Linear Equations In Two Variables

TOPIC-1

Graphical Solution of Linear Equations in Two Variables Consistency/Inconsistency

Quick Review

- **Linear Equation in two variables :** An equation of the form $ax + by + c = 0$, where a, b and c are real numbers and a and b are not both zero, is called a linear equation in two variables x and y.

  General form of a pair of linear equations in two variables is :

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0,$$

  where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers, such that

$$a_1, b_1 \neq 0 \text{ and } a_2, b_2 \neq 0.$$

  e.g.,

$$3x - y + 7 = 0,$$
$$7x + y = 3$$

  are linear equations in two variables x and y.

  - There are two methods of solving simultaneous linear equations in two variables :
    1. Graphical method,
    2. Algebraic method.

1. Graphical Method :

(i) Express one variable say y in terms of the other variable x, $y = ax + b$, for the given equation.

(ii) Take three values of independent variable x and find the corresponding values of dependent variable y, take integral values only.

(iii) Plot these values on the graph paper in order to represent these equations.

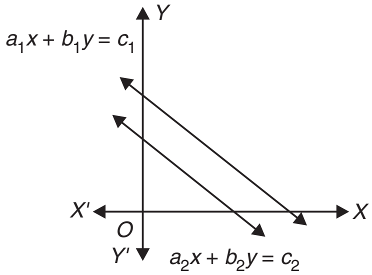
(iv) If the lines intersect at a distinct point, then point of intersection is the unique solution of the two equations. In this case, the pair of linear equations is consistent.

(v) If the lines representing the linear equations coincides, then system of equations has infinitely many solutions. In this case, the pair of linear equations is consistent and dependent.

(vi) If the lines representing the pair of linear equations are parallel, then the system of equations has no solution and is called inconsistent.

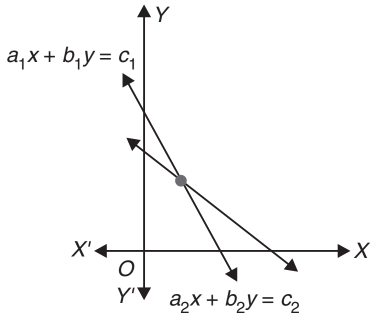
Parallel Lines :

(i) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent with no solution.



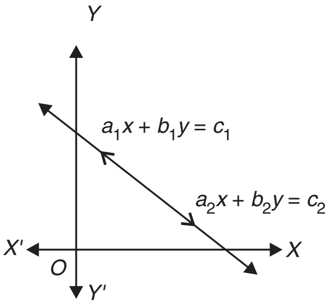
Intersecting Lines :

(ii) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistent with a unique solution.



Coincident Lines :

(iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is consistent with infinitely many solutions.



➢ If $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0,$

is a pair of linear equations in two variables x and y such that :

Possibilities of solutions and Inconsistency :

Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation	Conditions for solvability
$x - 2y = 0$ $3x - 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{-4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution Unique solution	System is consistent
$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	System is consistent
$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	System is inconsistent

How it is done on GREENBOARD?

Q. From the given pair of linear equations $3x + ay = 50$ and $9x - 21y = 15$. find the value of a for them to be parallel

Sol. : Step I : $3x + ay = 50$ and $9x - 2y = 15$

$\Rightarrow a_1 = 3, b_1 = a, c_1 = -50$ and $a_2 = 9, b_2 = -21, c_2 = -15$

Step II : For lines to be parallel

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Step III :

$$\frac{3}{9} = \frac{a}{-21} \neq \frac{-50}{-15}$$

or, $a = \frac{1}{3} \times (-21)$

So, $a = -7$

TOPIC-2
Algebraic Methods to solve pair of Linear Equations and Equations reducible to Linear Equations

Quick Review

➢ **Algebraic Method :** We can solve the linear equations algebraically by **substitution method, elimination method** and by **cross-multiplication method**.

1. Substitution Method :

(i) Find the value of one variable say y in terms of the other variable *i.e.*, x from either of the equations.

(ii) Substitute this value of y in other equation and reduce it to an equation in one variable.

(iii) Solve the equation so obtained and find the value of x.

(iv) Put this value of x in one of the equations to get the value of variable y.

2. Elimination Method :

(i) Multiply given equations with suitable constants, make either the x-coefficient or the y-coefficient of the two equations equal.

(ii) Subtract or add one equation from the other to get an equation in one variable.

(iii) Solve the equation so obtained to get the value of the variable.

(iv) Put this value in any one of the equation to get the value of the second variable.

Note :

(a) If in step (ii), we obtain a true equation involving no variable, then the original pair of equations has infinitely many solutions.

(b) If in step (ii), we obtain a false equation involving no variable, then the original pair of equations has no solution *i.e.*, it is inconsistent.

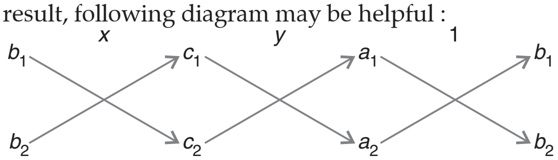
3. Cross-multiplication Method : If two simultaneous linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are given, then a unique solution is given by :

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

or

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Note : To obtain the above result, following diagram may be helpful :



The arrows between the two numbers indicate that they are to be multiplied. The product with upward arrows are to be subtracted from the product with downward arrows.

Remember :

1. If the equations

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

are in the form

$$a_1x + b_1y = -c_1 \text{ and } a_2x + b_2y = -c_2$$

Then, we have by cross-multiplication

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{-1}{a_1b_2 - a_2b_1}$$

➢ **Equations reducible to a Pair of Linear Equations in two variables :** Sometimes, a pair of equations in two variables is not linear but can be reduced to linear form by making some suitable substitutions. Here, first we find the solution of new pair of linear equations and then find the solution for the given pair of equations.

Steps to be followed for solving word problems

S. No.	Problem type	Steps to be followed
1.	Age Problems	If the problem involves finding out the ages of two persons, take the present age of one person as x and of the other as y. Then 'a' years ago, age of 1st person was '$x - a$' years and that of 2nd person was '$y - a$' and after 'b' years, age of 1st person will be '$x + b$' years and that of 2nd person will be '$y + b$' years. Formulate the equations and then solve them.
2.	Problems based on Numbers and Digits	Let the digit in unit's place be x and that in ten's place be y. The two-digit number is given by $10y + x$. On interchanging the positions of the digits, the digit in unit's place becomes y and in ten's place becomes x. The two digit number becomes $10x + y$. Formulate the equations and then solve them.
3.	Problems based on Fractions	Let the numerator of the fraction be x and denominator be y, then the fraction is $\frac{x}{y}$. Formulate the linear equations on the basis of conditions given and solve for x and y to get the value of the fraction.
4.	Problems based on Distance, Speed and Time	We know that Speed $= \frac{\text{Distance}}{\text{Time}}$, $\Rightarrow$ Distance = Speed × Time and Time $= \frac{\text{Distance}}{\text{Speed}}$. To solve the problems related to speed of boat going downstream and upstream, let the speed of boat in still water be x km/h and speed of stream be y km/h. Then, the speed of boat downstream $= x + y$ km/h and speed of boat upstream $= x - y$ km/h.
5.	Problems based on commercial Mathematics	For solving specific questions based on commercial mathematics, the fare of 1 full ticket may be taken as ₹ x and the reservation charges may be taken as ₹ y, so that one full fare $= x + y$ and one half fare $= \frac{x}{2} + y$. To solve the questions of profit and loss, take the cost price of 1st article as ₹ x and that of 2nd article as ₹ y. To solve the questions based on simple interest, take the amount invested as ₹ x at some rate of interest and ₹ y at some other rate of interest.
6.	Problems based on Geometry and Mensuration	Make use of angle sum property of a triangle ($\angle A + \angle B + \angle C = 180°$) in case of a triangle. In case of a parallelogram, opposite angles are equal and in case of a cyclic quadrilateral, opposite angles are supplementary.

How it is done on GREENBOARD ?

Q. Solve the following system of equations by substitution method.

$$x + 2y = 5,\ 2x + y = 4$$

Sol. : Step I : Given equations are $x + 2y = 5$ and $2x + y = 4$

Step II : From $x - 2y = 5$, we get

$$x = 5 - 2y$$

Step III : Putting this value of x in other equation $2x + y = 4$ we get

$$2(5 - 2y) + y = 4$$
$$10 - 4y + y = 4$$
$$3y = 10 - 4$$
$$y = \frac{6}{3} = 2$$

Setp IV : On putting this value of y in (iii)

Also $x = 5 - 2(2)$

$x = 1$

$\therefore$ $x = 1$ and $y = 2$.

Chapter - 4 : Quadratic Equations

TOPIC-1
Solution of Quadratic Equations

Quick Review

- A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.
- The values of x that satisfies an equation is called the solution or roots or zeros of the equation.
- A real number α is said to be a solution/root or zero of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.
- **A quadratic equation can be solved by the following algebraic methods :**
  (i) Splitting the middle term
  (ii) Making perfect squares
  (iii) Using quadratic formula.
- If $ax^2 + bx + c = 0$, $a \neq 0$ can be reduced to the product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
- Method for splitting the middle term of the equation $ax^2 + bx + c = 0$, where $a \neq 0$.
  (i) Form the product "ac"
  (ii) Find a pair of numbers b_1 and b_2 whose product is "ac" and whose sum is "b" (if you can't find such number, it can't be factorised)
  (iii) Split the middle term using b_1 and b_2, that expresses the term bx as $b_1x + b_2x$. Now factor by grouping the pairs of terms.
- Roots of the quadratic equation can be found by equating each linear factor to zero. Since product of two numbers is zero, if either or both of them are zero.
- Any quadratic equation can be converted in to the form $(x + a)^2 - b^2 = 0$ by adding and subtracting some terms. This method of finding the roots of quadratic equation is called the method of making the perfect the square.
- Method of making the perfect square for quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$.

  (i) Dividing throughout by a, we get $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

  (ii) Multiplying and dividing the coefficient of x by 2

$$x^2 + 2\frac{b}{2a}x + \frac{c}{a} = 0$$

  (iii) Adding and subtracting $\frac{b^2}{4a^2}$

$$x^2 + 2\frac{b}{2a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

or,
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

or,
$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2$$

If $(b^2 - 4ac) \geq 0$, then by taking square root

or,
$$\left(x + \frac{b}{2a}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

or,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Roots of $ax^2 + bx + c = 0, a \neq 0$ are $\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $\dfrac{-b-\sqrt{b^2-4ac}}{2a}$, where $b^2 - 4ac > 0$.
- Roots of $ax^2 + bx + c = 0, a \neq 0$ are $\dfrac{-b}{2a}$ and $\dfrac{-b}{2a}$, where $b^2 - 4ac = 0$
- **Quadratic identities :**
  (i) $(a + b)^2 = a^2 + 2ab + b^2$
  (ii) $(a - b)^2 = a^2 - 2ab + b^2$
  (iii) $a^2 - b^2 = (a + b)(a - b)$

Know the Facts

- The Old-Babylonians (1700 BC) stated and solved problems involving quadratic equations.
- The Greek mathematician Euclid developed a geometrical approach for finding out roots, which are solution of quadratic equations.
- In Vedic manuscripts, procedures are described for solving quadratic equations by geometric methods related to completing a square.
- Brahmagupta (C.E. 598-665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx + c = 0$
- Sridharacharya (C.E. 1025) derived the quadratic formula for solving a quadratic equation by the method of completing the perfect square.
- An Arab mathematician Al-Khwarizmi (about C.E. 800) studied quadratic equations of different types.
- Abraham bar Hiyya Ha-nasi, in his book '*Liber embadorum*' published in Europe in C.E. 1145 gave complete solutions of different quadratic equations.
- Edden ratio ϕ is the root of quadratic equation $x^2 - x - 1 = 0$.

Q. Two water taps together can fill a tank in $2\frac{11}{12}$ hrs. the tap of smaller diameter takes 2 hours more than the larger one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Sol. : Step 1 : Let time taken by tap of larger diameter be x hrs. and time taken by tap of smaller diameter be $(x + 2)$ hrs.

Together they fill the tank in $2\frac{11}{12}$ hrs

$= \frac{35}{12}$ hrs.

Together they fill in an hour $= \frac{12}{35}$ tank

Tap of larger diameter will fill the tank in one hour $= \frac{1}{x}$ tank

and other tap $= \frac{1}{x+2}$ tank

Step 2 : According to question,

$$\frac{1}{x} + \frac{1}{x+2} = \frac{12}{35}$$

$$\frac{x+2+x}{x(x+2)} = \frac{12}{35}$$

or, $$\frac{2x+2}{x^2+2x} = \frac{12}{35}$$

Step 3 : Cross multiplying

$35(2x + 2) = 12(x^2+2x)$

or, $70x + 70 = 12x^2 + 24x$

or, $12x^2 - 46x - 70 = 0$

or, $6x^2 - 23x - 35 = 0$

Step 4 : Now factorizing by splitting the middle term.

$6x^2 - 30x + 7x - 35 = 0$

or, $6x(x - 5) + 7(x - 5) = 0$

or, $(x - 5)(6x + 7) = 0$

or, $x = 5$ or $x = \frac{-7}{6}$

Rejecting $x = \frac{-7}{6}$ as time cannot be negative.

Therefore larger tap takes 5 hrs and smaller tap takes 7 hrs.

TOPIC-2
Discriminant and Nature of Roots

Quick Review

- For the quadratic equation $ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is known as discriminant *i.e.*, Discriminant $D = b^2 - 4ac$
- Nature of roots of a quadratic equation :
  (i)If $b^2 - 4ac > 0$, the quadratic equation has two distinct real roots.
  (ii)If $b^2 - 4ac = 0$, the quadratic equation has two equal real roots.
  (iii)If $b^2 - 4ac < 0$, the quadratic equation has no real roots.

R **Q. Find the value of k for which the equation $4x^2 + kx + 25 = 0$ has equal roots.**

Sol. Step 1 : $4x^2 + kx + 25 = 0$

Comparing above equation with $ax^2 + bx + c = 0$

$a = 4,\ b = k$ and $c = 25$

Step 2 : Condition for equal roots is $D = 0$

i.e., $b^2 - 4ac = 0$

Step 3 : Substituting values in the above condition.

$(k^2) - 4(4)(25) = 0$

or, $k^2 - 400 = 0$

or, $k^2 - (20)^2 = 0$

or, $(k - 20)(k + 20) = 0$

or, $k = 20, -20$

Chapter - 5 : Arithmetic Progression

TOPIC-1
To find n^{th} Term of the Arithmetic Progression

Quick Review

- An arithmetic progression is a sequence of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term.
- The difference between the two successive terms of an A.P. is called the common difference.
- Each number in the sequence of arithmetic progression is called the term of an A.P.
- The arithmetic progression having finite number of terms is called a finite arithmetic progression.
- The arithmetic progression having infinite number of terms is called an infinite arithmetic progression.
- A list of numbers $a_1, a_2, a_3, \ldots\ldots$ is an A.P., if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \ldots$ give the same value *i.e.*, $a_{k+1} - a_k$ is same for all different values of k.
- The general form of an A.P. is $a, a + d, a + 2d, a + 3d\ldots$..
- If the A.P. $a, a + d, a + 2d, \ldots\ldots\ldots, l$ is reversed to $l, l - d, l - 2d, \ldots\ldots\ldots, a$, then the common difference changes to negative of original sequence common difference.
- The general term of an A.P. is expressed as :

$$a_n = a + (n - 1)d.$$

  where, a is the first term and d is the common difference.
- The general term of an A.P. $l, l - d, l - 2d\ldots\ldots$.. a is given by :

$$a = l + (n - 1)(-d),$$

  where, l is the last term, d is the common difference and n is the number of terms.

Know the Terms

- A sequence is defined as an ordered list of numbers.
  The first, second and third terms of a sequence are denoted by t_1, t_2, t_3 respectively.
- A sequence is said to be a finite sequence if it has finite number of terms. As : 1, 3, 5, 7.
- A sequence is said to be an infinite sequence if it has infinite number of terms. As : 1, 3, 5, 7............... .
- If the terms of sequence are connected with plus (+) or minus (–), then the numbers are called a series.
  Example : 2 + 4 + 6 + 8 + ... is a series.
- Arithmetic sequence were used by Babylonians 4000 years ago.
- The sequence of numbers 1, 1, 2, 3, 5, 8, 13,...... was discovered by a famous Italian Mathematician Leonasalo Fibonacci, when he was dealing with the problem of rabbit population.
- A sequence in which the difference between any two consecutive terms is the same constant, is called an arithmetic progression.
- If the terms of a sequence or a series are written under specific conditions, then the sequence or series is called a progression.
- If a constant is added or subtracted from each term of an A.P. the resulting sequence is also an A.P.
- If each term of an A.P. is multiplied or divided by an constant, the resulting sequence is also an A.P.
- If the n^{th} term is in linear form *i.e.*, $an + b = a_n$ then the sequence is in A.P.
- If the terms are selected at a regular interval then the given sequence is in A.P.
- If three consecutive number a, b and c are in A.P., then the sum two numbers is twice the middle number *i.e.*, $2b = a + c$.

U **Q.** Which term of the A.P. 6, 13, 20, 27, ... is 98 more than its 24^{th} term ?

Sol. **Step I :** The given A.P. is 6, 13, 20, 27, ...

here first term $a = 6$

Common difference $d = 13 - 6 = 7$

Step II : The 24^{th} term $a_{24} = a + (24 - 1)d$

or, $a_{24} = 6 + 23 \times 7$

$= 6 + 161$

$= 167$

Step III : Now according to question,

$$a_{24} + 98 = a_n$$
$$167 + 98 = a + (n - 1)d$$
$$265 = 6 + (n - 1)7$$
$$259 = (n - 1)7$$
$$\frac{259}{7} = n - 1$$
$$37 = n - 1$$

or $n = 38$

Hence 38^{th} term.

TOPIC-2
Sum of *n* terms of an Arithmetic Progression

Quick review

- Sum of n terms of an A.P is given by :

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

  where, a is the first term, d is the common difference and n is the total number of terms.
- Sum of n terms of an A.P when first and last term is given :

$$S_n = \frac{n}{2}[a + l]$$

  where, a is the first term and l when first and last term in given.
- The n^{th} term of an A.P is the difference of the sum of first n terms and the sum to first $(n-1)$ terms of it. *i.e.*,

$$a_n = S_n - S_{n-1}.$$

How it is done on GREENBOARD ?

U **Q.** Find the number of terms in the A.P 54, 51, 48. ..... whose sum is 513.
Also given the reason of double answer.

Sol. Step I : The given A.P. is 54, 51, 48.....
here $a = 54$, $d = 51 - 54 = -3$
Sum required is 513.

Step II : Applying the sum formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$513 = \frac{n}{2}[2 \times 54 + (n-1)(-3)]$$

$$1026 = n[108 - 3n + 3]$$

$$1026 = n[111 - 3n]$$

$$1026 = 111n - 3n^2$$

or, $3n^2 - 111n + 1026 = 0$

or, $3[n^2 - 37n + 342] = 0$

or, $n^2 - 37n + 342 = 0$

Step III : Factorizing the quadratic equation

$$n^2 - 19n - 18n + 342 = 0$$

$$n(n-19) - 18(n-19) = 0$$

or, $(n-19)(n-18) = 0$

or, $n = 18$ or 19

Since 19th term of given A.P is O

Hence $S_{18} = S_{19}$

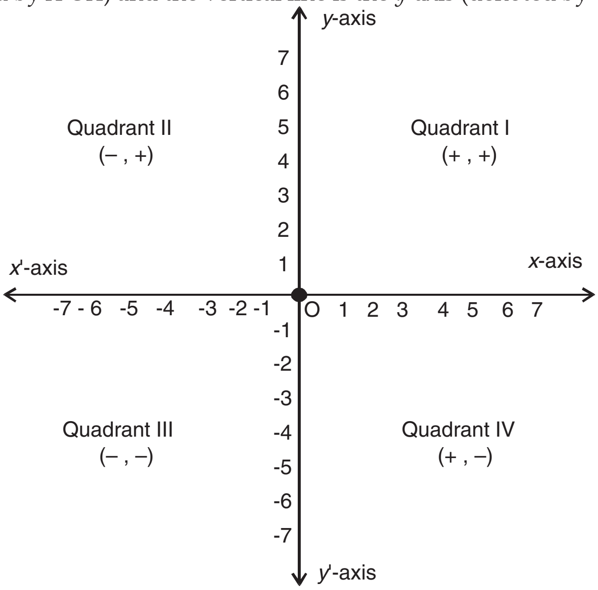
Unit -III : Coordinate Geometry

Chapter - 6 : Lines (In Two-Dimensions)

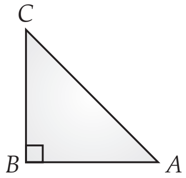
TOPIC-1
Distance between two points and section formula

Quick Review

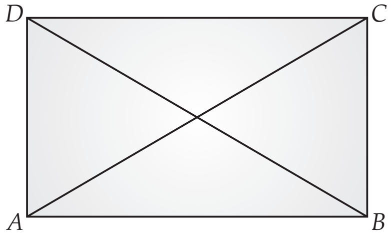
➢ Two perpendicular number lines intersecting at point zero are called co-ordinate axes. The horizontal number line is the x-axis (denoted by $X'OX$) and the vertical line is the y-axis (denoted by $Y'OY$).



- The point of intersection of x-axis and y-axis is called origin and denoted by O.
- Cartesian plane is a plane obtained by putting the co-ordinate axes perpendicular to each other in the plane. It is also called co-ordinate plane or xy-plane.
- The x-co-ordinate of a point is its perpendicular distance from y-axis.
- The y-co-ordinate of a point is its perpendicular distance from x-axis.
- The point where the x-axis and the y-axis intersect, is co-ordinate point (0, 0).
- The abscissa of a point is the x-co-ordinate of the point.
- The ordinate of a point is the y-co-ordinate of the point.
- If the abscissa of a point is x and the ordinate of the point is y, then (x, y) are called the co-ordinates of the point.
- The axes divide the Cartesian plane into four parts called the quadrants (one fourth part), numbered I, II, III and IV anti clockwise from OX.
- The co-ordinates of a point on the x-axis are of the form $(x, 0)$ and that of the point on y-axis are $(0, y)$.
- Sign of co-ordinates depicts the quadrant in which it lies. The co-ordinates of a point are of the form (+, +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant.
- Three points A, B and C are collinear if the distances AB, BC, CA are such that the sum of two distances is equal to the third.
- Three points A, B and C are the vertices of an equilateral triangle if $AB = BC = CA$.
- The points A, B and C are the vertices of an isosceles triangle if $AB = BC$ or $BC = CA$ or $CA = AB$.

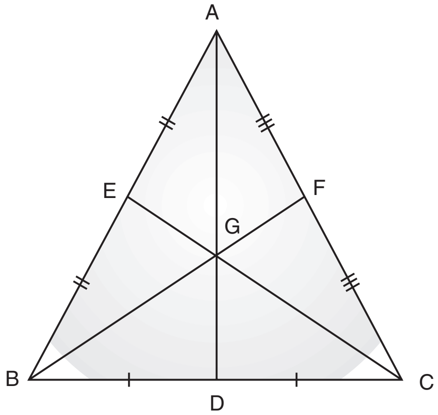


- Three points A, B and C are the vertices of a right triangle, if $AB^2 + BC^2 = CA^2$.
- For the given four points A, B, C and D :



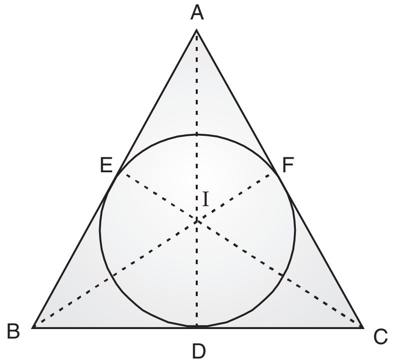
1. $AB = BC = CD = DA$; $AC = BD$ or, $ABCD$ is a square.
2. $AB = BC = CD = DA$; $AC \neq BD$ or, $ABCD$ is a rhombus.
3. $AB = CD$, $BC = DA$; $AC = BD$ or, $ABCD$ is a rectangle.
4. $AB = CD$, $BC = DA$; $AC \neq BD$ or, $ABCD$ is a parallelogram.

- Diagonals of a square, rhombus, rectangle and parallelogram always bisect each other.
- Diagonals of rhombus and square bisect each other at right angle.
- All given points are collinear, if the area of the obtained polygon is zero.
- Centroid is the point of intersection of the three medians of a triangle. In the figure, G is the centroid of the triangle.

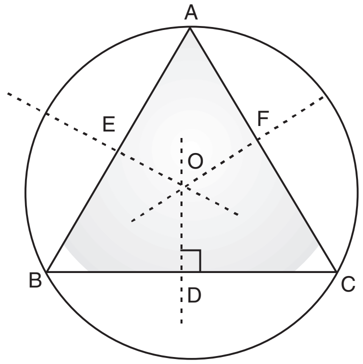


- Centroid divides each median of a triangle in a ratio of $2 : 1$.

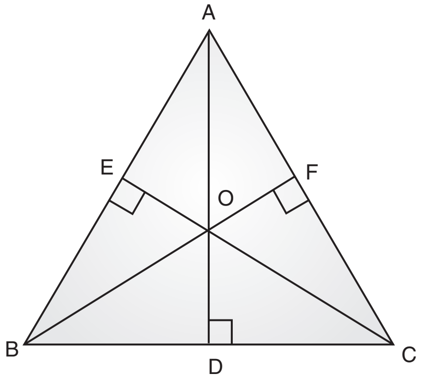
- The incentre is the point of intersection of bisector of the internal angles of a triangle. It is also the centre of the circle touching all the sides of a triangle. Here, I is the incentre of the triangle.



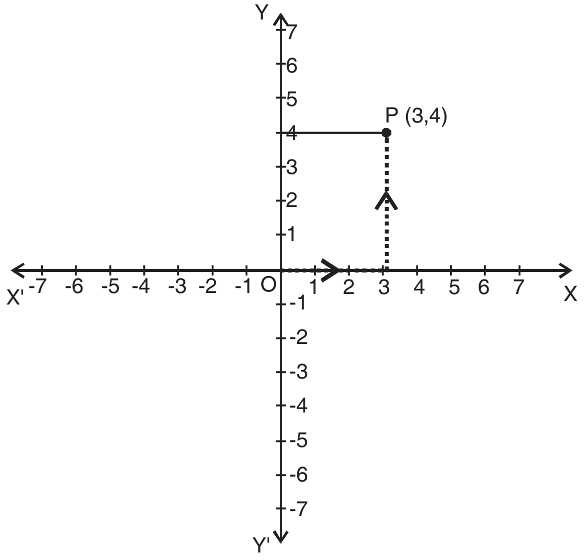
- Circumcentre is the point of intersection of the perpendicular bisectors of the sides of the triangle.



- Orthocentre is the point of intersection of perpendicular drawn from the vertices on opposite sides (called altitudes) of a triangle and can be obtained by solving the equations of any two altitudes. Here, O is the orthocentre.



- If the triangle is equilateral, the centroid, incentre, orthocentre, circumcentre coincides.
- If the triangle is right angled triangle, then orthocentre is the point where right angle is formed.
- If the triangle is right angled triangle, then circumcentre is the mid-point of hypotenuse.
- Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining the orthocentre and circumcentre in the ratio of $2:1$.
- In an isosceles triangle, centroid, orthocentre, incentre, circumcentre lie on the same line.
- Angle bisector divides the opposite sides in the ratio of remaining sides.
- Three given points are collinear, if the area of triangle is zero.
- If $x \neq y$, then $(x, y) \neq (y, x)$ and if $(x, y) = (y, x)$, then $x = y$.
- The distance between two points *i.e.*, $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$.
- The distance of a point $P(x, y)$ from origin is $\left|\sqrt{x^2 + y^2}\right|$.
- To plot a point P(3, 4) in the cartesian plane.
  (i) A distance of 3 units along X-axis.
  (ii) A distance of 4 units along Y-axis.



- Co-ordinates of point (x, y) which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ internally are

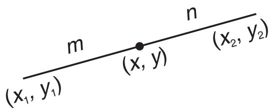
$$x = \left(\frac{mx_2 + nx_1}{m+n}\right) \text{ and } y = \left(\frac{my_2 + ny_1}{m+n}\right)$$

- Co-ordinates of mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) are

$$x = \left(\frac{x_2 + x_1}{2}\right) \text{ and } y = \left(\frac{y_2 + y_1}{2}\right)$$

- Co-ordinates of a point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ externally are :

$$x = \frac{mx_2 - nx_1}{m-n} \text{ and } y = \frac{my_2 - ny_1}{m-n}$$

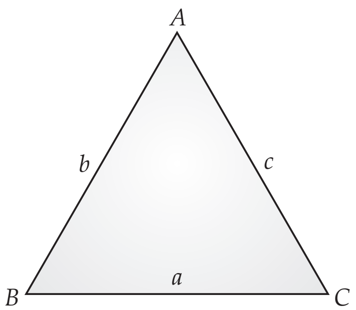


- If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of a triangle, then the co-ordinates of centroid are

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

- If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of a triangle and a, b and c are the lengths of sides BC, CA and AB respectively. then the co-ordinates of incentre are

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + ay_2 + ay_3}{a+b+c}\right)$$



- If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of a triangle, then the area of triangle ABC is given by the numerical value of

$$\text{Area of } \Delta ABC = \frac{1}{2}\left|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

- Area of quadrilateral $= \frac{1}{2}\left|[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_1 - x_1y_4)]\right|$

Know the Facts

- Co-ordinate geometry is the system of geometry where the position of points on the plane is described using an ordered pair of numbers.
- Cartesian plane was discovered by Rene Descarte .
- The other name of co-ordinate geometry is Analytical Geometry.
- Co-ordinate Geometry acts as a bridge between the Algebra and Geometry.
- Medians of a triangle are concurrent. The point of concurrency is called the centroid.
- Trisection of a line segment means dividing it into 3 equal parts, so 2 points are required.
- Centroid of a triangle divides its median in the ratio of 2 : 1.

Q. Find the point of trisection of the line segment joining the points (5, -6) and (-7, 5).

Sol. Step I : Diagrammatic representation.

Points of trisections divides line segment into three equal parts.

i.e., $AP = PQ = QB$

Step II : The ratio of $AP : PQ : QB = 1 : 1 : 1$

or, $\frac{AP}{PB} = \frac{1}{2}$ and $\frac{AQ}{QB} = \frac{2}{1}$

Step III : finding co-ordinates of P using section formula.

1 : 2

$$x_1 = \frac{1(-7)+2(5)}{1+2} = \frac{3}{3} = 1$$

$$y_1 = \frac{1(5)+2(-6)}{1+2} = \frac{-7}{3}$$

$\therefore$ $P(x_1, y_1)$ is $\left(1, \frac{-7}{3}\right)$.

Step IV : Finding co-ordinates of Q using section formula

2 : 1

$$x_2 = \frac{2(-7)+1(5)}{2+1} = \frac{-9}{3} = -3$$

$$y_2 = \frac{2(5)+1(-6)}{2+1} = \frac{4}{3}$$

$\therefore$ Q (x_2, y_2) is $\left(-3, \frac{4}{3}\right)$.

TOPIC-2
Area of Triangle

U **Q. Find k, if the point $A(2, 3)$, $B(5, K)$ and $C(7, 9)$ are collinear**

Sol. Step I : If points A, B, and C are collinear then the area of triangle formed by them is zero.

Step II : Area of triangle formed by the vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Step III : According to question

$$0 = \frac{1}{2}[2(k-9) + 5(9-3) + 7(3-k)]$$

or, $2k - 18 + 45 - 15 + 21 - 7k = 0$

or, $-5k + 33 = 0$

or, $5k = 33$

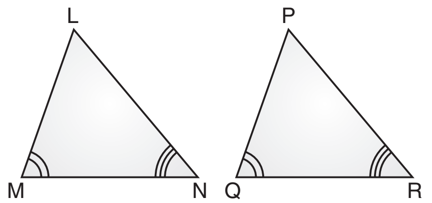
Or $k = \frac{33}{5}$

Unit -IV : Geometry

Chapter - 7 : Triangles

Quick Review

- A triangle is one of the basic shapes of geometry. It is a polygon with 3 sides and 3 vertices/corners.
- Two figures are said to be congruent if they have the same shape and the same size.
- Those figures which have the same shape but not necessarily the same size are called similar figures. Hence, we can say that all congruent figures are similar but all similar figures are not congruent.
- **Similarity of Triangles :** Two triangles are similar, if :
  (i) their corresponding sides are proportional.
  (ii) their corresponding angles are equal.
  If ΔABC and ΔDEF are similar, then this similarity can be written as $\Delta ABC \sim \Delta DEF$.
- **Criteria for Similarity of Triangles :**



In ΔLMN and ΔPQR, if

(i) $\angle L = \angle P, \angle M = \angle Q, \angle N = \angle R$ **(ii)** $\dfrac{LM}{PQ} = \dfrac{MN}{QR} = \dfrac{LN}{PR}$,

then $\Delta LMN \sim \Delta PQR$.

(i) *AAA*-**Criterion :** In two triangles, if corresponding angles are equal, then The triangles area similar and hence their corresponding sides are in the same ratio.

Remark : If two angles of a triangle are respectively equal to the two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, *AAA* similarity criterion can also be stated as follows :

AA-**Criterion :** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

(ii) *SSS*-**Criterion :** In two triangles if the sides of one triangle are proportional to the sides of another triangle, then their corresponding angles are equal and the two triangles are similar and hence corresponding angles are equal.

(iii) *SAS*-**Criterion :** If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.

Some theorems based on similarity of triangles :

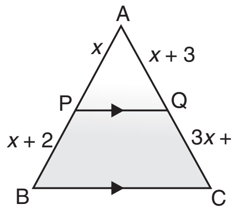
(i) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. It is known as '**Basic Proportionality Theorem**' or '**Thales Theorem**'.

(ii) If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. It is the **Converse of Basic Proportionality Theorem.**

(iii) If two triangles are similar, then the ratio of areas of these triangles is equal to the ratio of squares of their corresponding sides.

- **Theorems Based on Right Angled Triangles :**
  (i) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.
  (ii) In a right angled triangle, the square of hypotenuse is equal to the sum of the squares of the other two sides. It is known as **Pythagoras Theorem.**
- In any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with the twice of the square of the median which bisects the third side.
- Three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the median of the triangle.
- Three times of the square of any side of an equilateral triangle is equal to four times the square of the altitude.

Q. In the given triangle, find x if PQ || BC



Sol. : **Step I :** Given that PQ || BC

Hence by using Basic Proportionality theorem

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Step II :

$$\frac{x}{x+2} = \frac{x+3}{3x+1}$$

or, $x(3x + 1) = (x + 2)(x + 3)$

or, $3x^2 + x = x^2 + 5x + 6$

or, $2x^2 - 4x - 6 = 0$

or, $x^2 - 2x - 3 = 0$

or, $(x - 3)(x + 1) = 0$

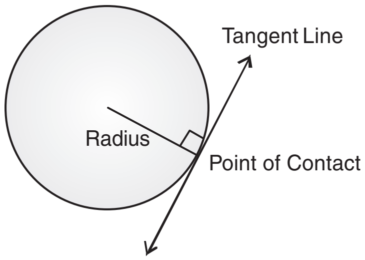
or, $x = 3$ or $x = -1$

or, $x = 3.$

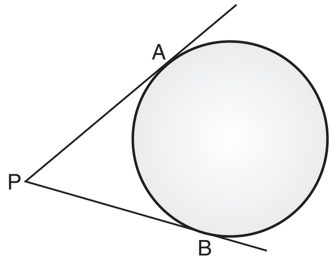
Chapter - 8 : Circles

Quick Review

- A tangent to a circle is a line that intersects the circle at one point only.
- The common point of the circle and the tangent is called the point of contact.
- The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of the tangent.
- **Pythagoras theorem :** In a right triangle, square of the hypotenuse is equal to the sum of the squares of the other two sides.
- A tangent to a circle is a special case of the secant when the two end points of the corresponding chord are coincide.
- There is no tangent to a circle passing through a point lying inside the circle.
- There are exactly two tangents to a circle through a point outside the circle.
- At any point on the circle there can be one and only one tangent.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.



- The lengths of the tangents drawn from an external point to a circle are equal.



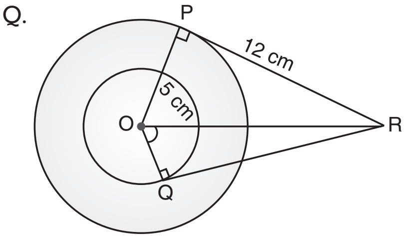
In the figure, $PA = PB$.

Know the Facts

- The word 'tangent' comes from the Latin word 'tangere', which means to touch and was introduced by the Danish mathematician **Thomas Fincke** in 1583.
- The line containing the radius through the point of contact is also called the 'normal' to the circle at that point.
- In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

How it is done on GREENBOARD?

Q.



In the given figure if PR = 12 cm, OP = 5 cm, OQ = 4 cm. find RQ.

Sol. Step I : $OP \perp RP$ (as radius is $\perp$ to tangent at point of contact)

In right triangle OPR,

$$OR^2 = OP^2 + PR^2$$
$$OR^2 = 5^2 + 12^2$$
$$OR^2 = 25 + 144$$
$$OR^2 = 169$$

or, $$OR = \sqrt{169} = 13$$

Step II : Similarly in right triangle OQR

$$OR^2 = OQ^2 + QR^2$$
$$QR^2 = OR^2 - OQ^2$$
$$QR^2 = 13^2 - 4^2$$

or, $$(QR)^2 = 169 - 16$$
$$QR^2 = 153$$

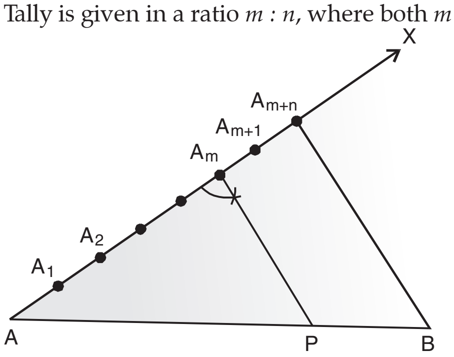
or, $$QR = \sqrt{153} \text{ cm.} = 3\sqrt{17} \text{ cm}$$

Chapter - 9 : Constructions

TOPIC-1
Division of a Line Segment in a Given Ratio.

Quick Review

- The ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle is known as scale factor.
- To divide a line segment internally
  Tally is given in a ratio $m : n$, where both m and n are positive integers, we follow the following steps :



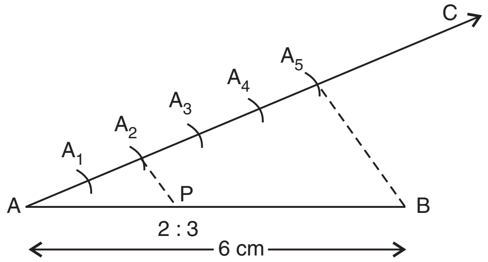
Step 1. Draw a line segment AB of given length by using a ruler.
Step 2. Draw any ray AX making an acute angle with AB.
Step 3. Along AX mark off $(m + n)$ points $A_1, = A_2, \ldots\ldots\ldots, A_m, A_{m+1}, \ldots\ldots\ldots, A_{m+n}$, such that $AA_1 = A_1A_2 = A_{m+n-1} A_{m+n}$.
Step 4. Join BA_{m+n}.

Step 5. Through the point A_m draw a line parallel to $A_{m+n}B$ by making an angle equal to $\angle AA_{m+n}B$ at A_m. *i.e.*, $\angle AA_mP$.
This line meets AB at point P.
The point P is the required point which divides AB internally in the ratio $m : n$.

How it is done on GREENBOARD ?

Q. Draw a line segment $AB = 6$ cm and divide it in the ratio of 2 : 3.

Sol. Step I.



Step II. Steps of construction :

(i) Draw a line segment $AB = 6$ cm.
(ii) At A draw an acute angle BAC.
(iii) Mark A_1, A_2, A_3, A_4, A_5 at equal distance.
(iv) Join A_5 to B.
(v) Draw a line A_2P parallel to A_5B.
(vi) P is the required point on AB which divides it in the ratio 2 : 3.

TOPIC-2
Tangents to a Circle from a Point Outside It

Quick Review

- To draw the tangent to a circle at a given point on it, when the centre of the circle is known.
  Given : A circle with centre O and a point P on it.
  To construct : the tangent to the circle at the point P.
  Steps of construction :
  (i) Join OP.
  (ii) Draw a line segment $AB \perp OP$ at the point P. APB is the required tangent at P.

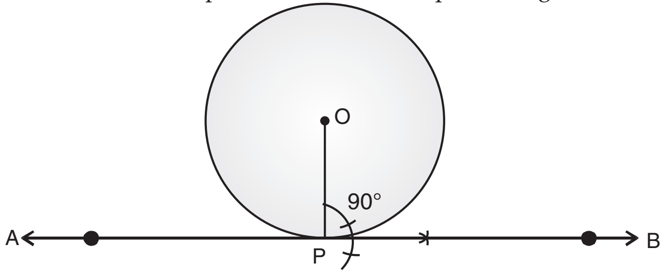


Fig. (9.1)

Steps of construction :
(i) Draw any chord PQ and join P and Q to a point R.
(ii) Draw $\angle QPA$ equal to $\angle PRQ$ on opposite side of chord PQ.
The line segment BPA is the tangent to the circle at P. (Fig. 9.2)

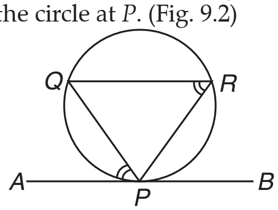


Fig. (9.2)

- **To draw the tangent to a circle from an external point when its centre is known.**
  Given : A circle with centre O and a point P outside it.
  To construct : the tangents to the circle from P.

Steps of construction :

(i) Join OP and bisect it. Let M be the mid-point of OP.

(ii) Taking M as centre and MO as radius, draw a circle to intersect $C(O, r)$ in two points, say A and B.

(iii) Join PA and PB. These are the required tangents from P to the circle. Fig. (9.3)

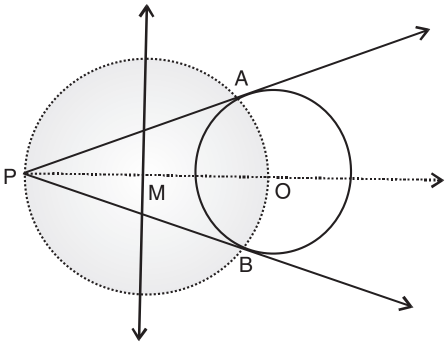


Fig. (9.3)

To draw tangents to a circle from a point outside it (when its centre is not known)

Given : P is a point outside the circle.

Required : To draw tangents from the point P.

Steps of construction :

(i) Draw a secant PAB to intersect the circle at A and B.

(ii) Produce AP to a point C, such that $PA = PC$.

(iii) With BC as a diameter, draw a semi circle.

(iv) Draw $PO \perp CB$, intersecting the semi circle at O.

(v) Taking PO as radius and P as centre, draw an arc to intersect the circle at T and T′.

(vi) Join PT and PT', then PT and PT' are the required tangents. (Fig. 9.4)

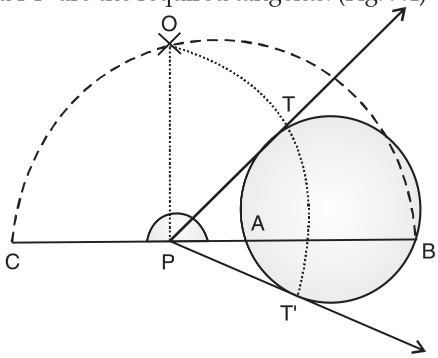


Fig. (9.4)

A **Q.** Draw a pair of tangents to a circle of any convenient radius, which are inclined to the line joining the centre of the circle and the point at which they intersect at an angle of 45°.

OR

Draw a circle of radius 3.5 cm. Draw two tangents to the circle which are perpendicular to each other.

Sol. Steps of construction

Step I. Draw a circle of any convenient radius with O as centre.

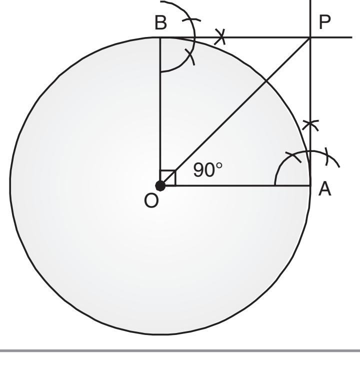
Step II. Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.

Step III. Draw a radius OB, making an angle of 90° with OA.

Step IV. Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P.

Step V. Join OP.

PA and PB are the required tangents, which make an angle of 45° with OP.



TOPIC-3
Construction of a Triangle Similar to a Given Triangle

Quick Review

- Construction of triangles similar to a given triangle :

  (a) Steps of constructions, when $m < n$:

  Step I. Construct the given triangle ABC by using the given data.

  Step II. Take any one of the three sides of the given triangle as base. Let AB be the base of the given triangle.

  Step III. At one end, say A, of base AB. Construct an acute $\angle BAX$ below the base AB.

  Step IV. Along AX mark off n points $A_1, A_2, A_3, \ldots\ldots, A_n$ such that $AA_1 = A_1A_2 = \ldots\ldots\ldots = A_{n-1}\,A_n$

  Step V. Join A_nB.

  Step VI. Draw A_mB' parallel to A_nB which meets AB at B'.

  Step VII. From B' draw $B'C' \parallel CB$ meeting AC at C'.

  Triangle $AB'C'$ is the required triangle each of whose sides is $\left(\frac{m}{n}\right)^{th}$ of the corresponding side of ΔABC.

  (b) Steps of construction, when $m > n$:

  Step I. Construct the given triangle by using the given data.

  Step II. Take any one of the three sides of the given triangle and consider it as the base. Let AB be the base of the given triangle.

  Step III. At one end, say A, of base AB. Construct an acute angle $\angle BAX$ below the base AB *i.e.*, on the opposite side of the vertex C.

  Step IV. Along AX mark off m (larger of m and n) points $A_1, A_2, A_3, \ldots\ldots\ldots A_m$ such that $AA_1 = A_1A_2 = \ldots\ldots\ldots = A_{m-1}A_m$.

  Step V. Join A_nB to B and draw a line through A_m parallel to A_nB, intersecting the extended line segment AB at B'.

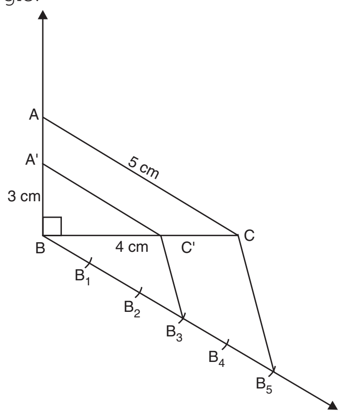
  Step VI. Draw a line through B' parallel to BC intersecting the extended line segment AC at C'.

  $\Delta AB'C'$ so obtained is the required triangle.

How it is done on GREENBOARD ?

Q. Construct a right triangle whose hypotenuse and one side measure 5 cm and 4 cm respectively. Then construct another triangle whose sides are $\frac{3}{5}$ times of the corresponding sides of this triangle.

Sol.



Steps of Construction :

Step I. (i) Draw a line segment BC = 4 cm.

(ii) Draw an angle of 90° at *B*.

(iii) Taking C as centre and radius equal to hypotenuse = 5 cm draw an arc to intersect at *A* Joint *A* to *C*.

Step II. Draw a line making an acute angle with side BC and divide it in five equal sections and mark B_1, B_2, B_3, B_4, B_5.

Step III. Join B_5 to C, and draw a parallel line B_3C' to B_5C which meets BC at C'.

Step IV. Draw a parallel line A'C' to AC

Hence ABC is the required triangle.

Unit -V : Trigonometry

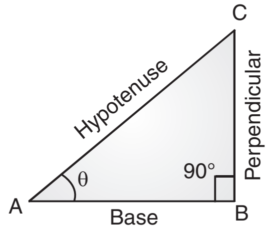
Chapter - 10 : Introduction To Trigonometry And Trigonometric Identities

TOPIC-1

Trigonometric Ratios and Trigonometric Ratios of Complementary Angles

Quick Review

- In fig., a right triangle ABC right angle at B is given and $\angle BAC = \theta$ is an acute angle. Here side AB which is adjacent to $\angle A$ is base, side BC opposite to $\angle A$ is perpendicular and the side AC is hypotenuse which is opposite to the right angle B.



The trigonometric ratios of $\angle A$ in right triangle ABC are defined as

$$\text{sine of } \angle A = \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\text{cosine of } \angle A = \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\text{tangent of } \angle A = \tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\text{cosecant of } \angle A = \text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC} = \frac{1}{\sin\theta}$$

$$\text{secant of } \angle A = \sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{1}{\cos\theta}$$

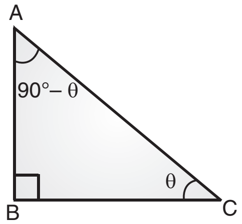
$$\text{cotangent of } \angle A = \cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{1}{\tan\theta}$$

It is clear from the above ratios that cosecant, secant and cotangent are the reciprocals of sine, cosine and tangent respectively.

Also, $$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

and $$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

- The trigonometric ratios of an acute angle in a right triangle express the relationship between the angle and length of its sides.
- The value of trigonometric ratio of an angle does not depend on the size of the triangle but depends on the angle only.
- **Complementary Angles :**
  Two angles are said to be complementary if their sum is 90°. Thus (in fig.) $\angle A$ and $\angle C$ are complementary angles.



➢ **Trigonometric Ratios of Complementary Angles :**

We have, BC = Base, AB = Perpendicular,

AC = Hypotenuse, with respect to θ.

$$\therefore \quad \sin\theta = \frac{AB}{AC}, \cos\theta = \frac{BC}{AC}, \tan\theta = \frac{AB}{BC}$$

$$\text{cosec}\,\theta = \frac{AC}{AB}, \sec\theta = \frac{AC}{BC}, \cot\theta = \frac{BC}{AB}.$$

Again, with respect to the angel (90° – θ) BC = Perpendicular, AB = Base

AC = Hypotenuse

$$\therefore \quad \text{therefore } \sin(90° - \theta) = \frac{BC}{AC} = \cos\theta$$

$$\cos(90° - \theta) = \frac{AB}{AC} = \sin\theta$$

$$\tan(90° - \theta) = \frac{BC}{AB} = \cot\theta$$

$$\text{cosec}\,(90° - \theta) = \frac{AC}{BC} = \sec\theta$$

$$\sec(90° - \theta) = \frac{AC}{AB} = \text{cosec}\,\theta$$

$$\cot(90° - \theta) = \frac{AB}{BC} = \tan\theta$$

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cosec A	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cot A	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Q. If 2 sin θ – 1 = 0, then Prove that sin 3θ = 3 sin θ – 4 sin^3 θ.

Sol. Step I : Given 2 sin θ – 1 = 0

or, $2\sin\theta = 1$

or, $\sin\theta = \frac{1}{2}$ ...(i)

Step II : $\sin 30° = \frac{1}{2}$ ...(ii)

from (i) and (ii), θ = 30°

Step III : L.H.S. = sin 3θ

= sin (3 x 30°)

= sin 90

= 1

R.H.S. = 3 sin θ – 4 sin^3 θ

= 3 x sin 30 – 4(sin 30)^3

$= 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3$

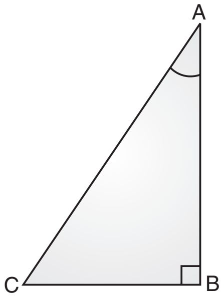
$= \frac{3}{2} - \frac{1}{2}$

$= \frac{2}{2}$

= 1 = L.H.S.

TOPIC-2
Trigonometric Identities

Quick Review



- An equation is called an identity if it is true for all values of the variable(s) involved.
- An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle.

  In $\Delta\, ABC$, right-angled at B, By Pythagoras Theorem.

$$AB^2 + BC^2 = AC^2 \qquad \text{...(i)}$$

Dividing each term of (i) by AC^2,

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

or $$\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = \left(\frac{AC}{AC}\right)^2$$

or $$(\cos A)^2 + (\sin A)^2 = 1$$

or $$\cos^2 A + \sin^2 A = 1 \qquad \text{...(ii)}$$

This is true for all values of A such that $0° \leq A \leq 90°$. So, this is a trigonometric identity. now divide eqn.(1) by AB^2.

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

or $$\left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2 = \left(\frac{AC}{AB}\right)^2$$

or $$1 + \tan^2 A = \sec^2 A \qquad \text{...(iii)}$$

Is this equation true for $A = 0°$? Yes, it is. What about $A = 90°$? Well, tan A and sec A are not defined for $A = 90°$. So, eqn. (iii) is true for all values of A such that $0° \leq A < 90°$.

dividing eqn. (i) by BC^2.

$$\frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

or $$\left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2 = \left(\frac{AC}{BC}\right)^2$$

or $$\cot^2 A + 1 = \operatorname{cosec}^2 A \qquad \text{...(iv)}$$

Note that cosec A and cot A are not defined for all $A = 0°$. Therefore eqn. (iv) is true for all value of A such that $0 < A \leq 90°$.

Using these identities, we can express each trigonometric ratio in terms of other trigonometric ratios, *i.e.*, if any one of the ratios is known , we can also determine the values of other trigonometric ratios.

Q. Prove that. $\dfrac{1}{\sec\theta - \tan\theta} = \dfrac{1 + \sin\theta}{\cos\theta}$

Sol. : Step I : L.H.S. $= \dfrac{1}{\sec\theta - \tan\theta}$

Rationalising with $\sec\theta \times \tan\theta$

L.H.S. $= \dfrac{1}{\sec\theta - \tan\theta} \times \dfrac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$

$= \dfrac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$ $\quad [(a + b)(a - b) = a^2 - b^2]$

$= \sec\theta + \tan\theta$ $\quad [\sec^2\theta - \tan^2\theta = 1]$

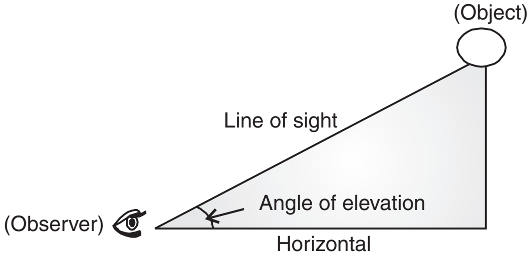
$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1 + \sin\theta}{\cos\theta}$ = R.H.S.

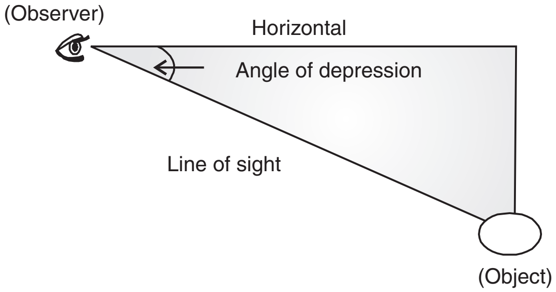
Chapter - 11 : Heights And Distances

Quick Review

- The line of sight is the line drawn from the eye of an observer to a point in the object viewed by the observer.
- The angle of elevation of an object viewed is the angle formed by the line of sight with the horizontal when it is above the horizontal level, *i.e.*, the case when we raise our head to look at the object.



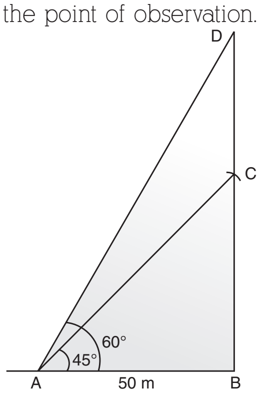
- The angle of depression of an object viewed is the angle formed by the line of sight with the horizontal when it is below the horizontal level, *i.e.*, the case when we lower our head to look at the object.



- The height or object above the water surface is equal to the depth of its image below the water surface.
- The values of the trigonometric ratios of an angle do not vary with the length of the sides of the triangle, if the angles remain the same.

U **Q.** The angle of elevation of top of a tower from a point 50 m away from the base of the tower is 45°. The angle of elevation of top of the flag mounted on the tower is 60°. Find the height of flag.

Sol. : Step I : Let BC be the tower, CD be the flag and A is the point of observation.



Step II : First we will calculate the height of tower BC using triangle ABC.

$$\tan 45° = \frac{BC}{50}$$

$$\Rightarrow \quad 1 = \frac{BC}{50}$$

$$\Rightarrow \quad BC = 50 \text{ metre}$$

Step III : Calculation of height BD.

In ΔABD

$$\tan 60° = \frac{BD}{50}$$

$$\Rightarrow \quad \sqrt{3} = \frac{BD}{50}$$

$$\Rightarrow \quad BD = 50\sqrt{3}$$

Step IV : Calculation for height of flag

$$\text{Height of flag } CD = BD - BC$$

$$= 50\sqrt{3} - 50$$

$$\Rightarrow \quad CD = 50(\sqrt{3} - 1)$$

$$= 50\ (1.73 - 1)$$

$$= 50 \times 0.73$$

$$\Rightarrow \quad \text{Height of flag} = 36.5 \text{ metre}$$

Unit -VI : Mensuration

Chapter - 12 : Areas Related To Circles

Quick Review

- A circle is a collection of all points in a plane which are at a constant distance from a fixed point in the same plane.
- A line segment joining the centre of the circle to a point on the circumference of a circle is called its radius.
- A line segment joining any two points of a circle is called a chord. A chord passing through the centre of circle is called its diameter. A diameter is the largest chord of the circle.
- A part of a circumference of circle is called an arc.
- A diameter of a circle divides a circle into two equal arcs, each known as a semi-circle.
- The region bounded by an arc of a circle and two radii at its end points is called a sector.
- A chord divides the interior of a circle into two parts, each called a segment.
- An arc of a circle whose length is less than that of a semi-circle of the same circle is called a minor arc.
- An arc of a circle whose length is greater than that of a semi-circle of the same circle is called a major arc.
- Circles having the same centre but different radii are called concentric circles.
- Two circles (or arcs) are said to be congruent if on placing one over the other such that they cover each other completely.
- The distance around the circle or the length of a circle is called its circumference or perimeter.
- The mid-point of the hypotenuse of a right triangle is equidistant from the vertices of the triangle.
- Angle subtended at the circumference by a diameter is always a right angle.
- Angle described by minute hand in 60 minutes is 360°.
- Angle described by hour hand in 12 hours is 360°.

Know the Formulae

1. Circumference (perimeter) of a circle = πd or $2\pi r$, where d is diameter and r is the radius of the circle.
2. Area of a circle = πr^2.
3. Area of a semi-circle = $\frac{1}{2}\pi r^2$.
4. Perimeter of a semi-circle = $\pi r + 2r = (\pi + 2)\, r$
5. Area of a ring or an annulus = $\pi(R + r)(R - r)$. Where R is the outer radius.
6. Length of arc $AB = \frac{2\pi r\theta}{360°}$ or $\frac{\pi r\theta}{180°}$. where θ is the angle subtend at centre by the arc.
7. Area of a sector = $\frac{\pi r^2\theta}{360°}$

   or Area of sector = $\frac{1}{2}(l \times r)$. where l is the length of arc.
8. Area of minor segment = $\frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2 \sin\theta$.
9. Area of major segment = Area of the circle – Area of minor segment

   = πr^2 – Area of minor segment.
10. If a chord subtend a right angle at the centre, then

    Area of the corresponding segment = $\left[\frac{\pi}{4} - \frac{1}{2}\right]r^2$
11. If a chord subtend an angle of 60° at the centre, then

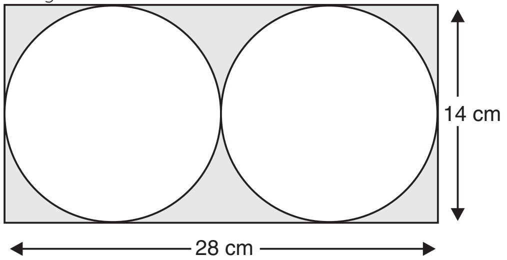
    Area of the corresponding segment = $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)r^2$.
12. If a chord subtend an angle of 120° at the centre, then

    Area of the corresponding segment = $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)r^2$.
13. Distance moved by a wheel in 1 revolution = Circumference of the wheel.
14. Number of revolutions in one minute = $\frac{\text{Distance moved in 1 minute}}{\text{Circumference}}$.
15. Perimeter of a sector = $\frac{\pi r\theta}{180°} + 2r$.

Know the Facts

- An Indian mathematician Srinivas Ramanujan worked out the identity using the value of π correct to million places of decimals.
- The Indian mathematician Aryabhatta gave the value of π as $\frac{62832}{20000}$
- "How I made a greater discovery" this mnemonic help us in getting the value of $\pi = 3.14159$ ------.
- "Can I have a small container of coffee ? " this mnemonic helps us in getting the value of $\pi = 3.1415926$ ------.
- Archimedes calculated the area of a circle by approximating it to a square.
- Area of sector of a circle depends on two parameters-radius and central angle.

U **Q.** Find the area of the shaded region in the given figure.



Sol. Step I : Length of rectangle = 28 cm

Breadth of rectangle = 14 cm

$\therefore$ Diameter of circle = 14 cm

or, radius of circle = 7 cm

Step II : Area of rectangle $= 28 \times 14 \text{ cm}^2$

$= 392 \text{ cm}^2$

Area of each circle $= \frac{22}{7} \times (7)^2$

$= 154 \text{ cm}^2$

Area of both circles $= 2 \times 154 \text{ cm}^2$

$= 308 \text{ cm}^2$

Step III : Shaded area = Area of rectangle − Area of both circles

$= 392 - 308$

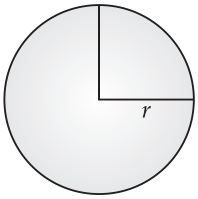
$= 84 \text{ cm}^2$

Chapter - 13 : Surface Areas And Volumes

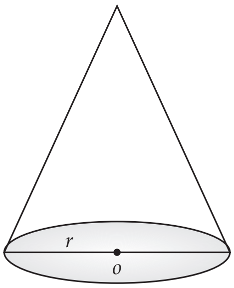
TOPIC-1
Surface areas and Volumes

Quick Review

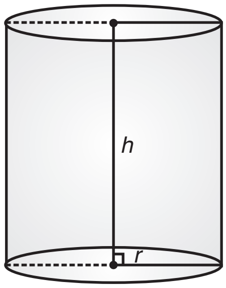
- A sphere is a perfectly round geometrical object in three-dimensional space that is the surface of a completely round ball.



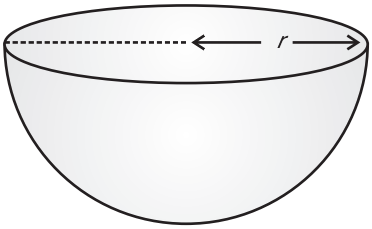
- A Cone is a three dimensional geometric shape that than tapers smoothly from a flat base to a point called the apex or vertex.



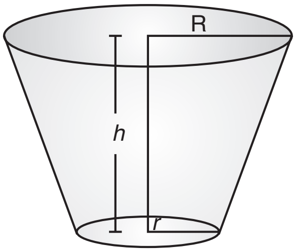
- A cylinder is a solid or a hollow object that has a circular base and a circular top of the same size.
- A right circular cylinder.



- A hemisphere is half of a sphere.



- If a right circular cone is cut off by a plane parallel to its base, then the portion of the cone between the plane and the base of the cone is called a frustum of the cone.



- The total surface area of the solid formed by the combination of solids is the sum of the curved surface areas of each of the individual parts.
- A solid is melted and converted into another solid, volume of both the solids remains the same, assuming there is no wastage during the conversions. The surface area of the two solids may or may not be the same.
- The solids having the same curved surface do not necessarily have the same volume.

  Important Formulae

- **Cuboid :**

Lateral surface area or Area of four walls	$= 2(l + b)h$
Total surface area	$= 2\,(lb + bh + hl)$
Volume	$= l \times b \times h$
Diagonal	$= \sqrt{l^2 + b^2 + h^2}$

Here, l = length, b = breadth and h = height

- **Cube :**

Lateral surface area or Area of four walls	$= 4 \times (\text{edge})^2$
Total surface area	$= 6 \times (\text{edge})^2$
Volume	$= (\text{edge})^3$
Diagonal of a cube	$= \sqrt{3} \times \text{edge}$.

- **Right circular cylinder :**

Area of base or top face	$= \pi r^2$
Area of curved surface or curved surface area	= perimeter of the base × height $= 2\pi rh$
Total surface area (including both ends)	$= 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
Volume	= (Area of the base × height)$= \pi r^2 h$

Here, r is the radius of base and h is the height.

- **Right circular hollow cylinder :**

Total surface area = (External surface + internal sufrace) + (Area of ends)

$$= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)$$
$$= [2\pi h(R + r) + 2\pi(R^2 - r^2)]$$
$$= [2\pi(R + r)(h + R - r)]$$

Curved surface area $= (2\pi Rh + 2\pi rh) = 2\pi h(R + r)$

Volume of the material used = (External volume) – (Internal volume)

$$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2)$$

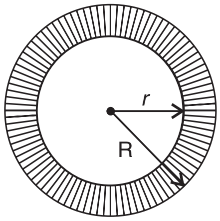
- **Right circular cone :**

$$\text{Slant height, } l = \sqrt{h^2 + r^2}$$

$$\text{Area of curved surface} = \pi r l = \pi r \sqrt{h^2 + r^2}$$

$$\text{Total surface area} = \text{Area of curved surface + Area of base}$$
$$= \pi r l + \pi r^2 = \pi r(l + r)$$

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$



- **Sphere :**

$$\text{Surface area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

- **Spherical shell :**

$$\text{Surface area (outer)} = 4\pi R^2$$

$$\text{Volume of material} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(R^3 - r^3)$$

- **Hemisphere :**

$$\text{Area of curved surface} = 2\pi r^2$$

$$\text{Total surface area} = \text{Area of curved surface + Area of base}$$
$$= 2\pi r^2 + \pi r^2$$
$$= 3\pi r^2$$

$$\text{Volume} = \frac{2}{3}\pi r^3$$

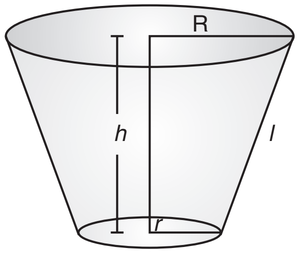
- **Frustum of a cone :**

$$\text{Total surface area} = \pi[R^2 + r^2 + l(R + r)]$$

$$\text{Volume of the material} = \frac{1}{3}\pi h[R^2 + r^2 + Rr]$$

$$\text{Curved surface area} = \pi l(R + r)$$

$$\text{Where } l = \sqrt{h^2 + (R - r)^2}$$



Know the Terms

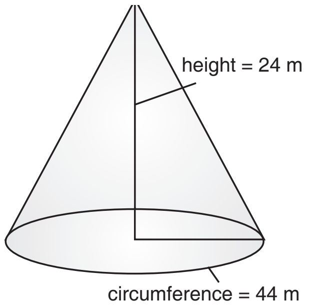
- Frustum : it is a Latin word which means "Piece cut off".
- The platonic solids also called the regular solids or regular polyhedra. 5 such solid cubes are : dodecahedron, icosahedron, octahedron and tetrahedron.
- Greek mathematician Plato equated tetrahedron with the 'element' fire, the cube with earth, the icosahedron with water, the octahedron with air and dodecahedron with the stuff of which the constellations and heavens were made.
- The stone of platonic solids are kept in Ashmolean Museum in Oxford.
- The fonds of Archimedes carried a sculpture consisting of a sphere and cylinder circumscribing it.

U **Q.** The circumference of the base of a conical tent is 44 m. If the height of tent is 24 m, find the length of the canvas used is making the tent, if the width of the canvas is 2 m. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. : Step I. Diagrammatic representation

Given that $h = 24$ m

Circumference $= 44$ m



Step II : Calculation of radius

Given that circumference = 44 m

or, $2\pi r = 44$

or, $r = \dfrac{44}{2\pi} = \dfrac{44\times7}{2\times22} = 7$ m

Step III : Calculation of slant height

$l^2 = r^2 + h^2$

$l^2 = (7)^2 + (24)^2$

$= 49 + 576$

$l^2 = 625$

or, $l = 25$ m

Step IV : Calculation for curved surface area

C.S.A $= \pi rl$

$= \dfrac{22}{7} \times 7 \times 25$ m^2

$= 550$ m^2

Step V : Calculation of length of canvas

We know that area of rectangle = length x breadth

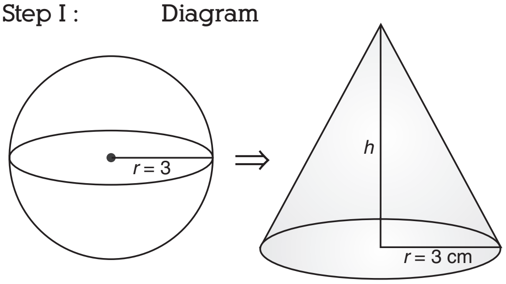
or, $550 = l \times 2$

or, $l = 225$ m

TOPIC-2
Problems Involving Converting One Type of Metallic Solid into Another

A Q. A spherical ball of radius 3 cm is melted and recast into a cone of same radius. Calculate the height of cone

Sol. Step I : **Diagram**



Step II : Volume of spherical ball

$V = \dfrac{4}{3}\pi(3)^3$ cm^3

Volume of cone $V = \dfrac{1}{3}\pi(3)^2 h$

Step III : According to question

$\dfrac{4}{3}\pi\times27 = \dfrac{\pi}{3}\times9\times h$

$\dfrac{4\times27}{9} = h$

$h = 12$ cm

$\therefore$ Height of cone = 12 cm.

Unit -VII : Statistics and Probability

Chapter - 14 : Statistics

TOPIC-1
Mean, Median and Mode

Quick Review

- Statistics deals with the collection, presentation and analysis of numerical data.
- Three measures of central tendency are :
  (i) Mean, **(ii)** Median and
  (iii) Mode
- **Mean :** The *mean of n quantities* $x_1, x_2, x_3, \ldots\ldots, x_n$

$$= \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \Sigma\frac{x_i}{n}$$

where, the Greek letter Σ(sigma) means 'Summation of'.

- **Median :** It is defined as the middle most or the central value of the variable in a set of observations, when the observations are arranged either in ascending or descending order of their magnitudes.

  It divides the arranged series in two equal parts i.e., 50 % of the observations lie below the median and the remaining are above the median.
- **Mode :** The item which occurs most frequently i.e., maximum number of times is called mode.
- **Mean :**

  (a) For Raw Data :

  If n observations $x_1, x_2, ..., x_n$ are given, then their arithmetic mean is given by :

$$x = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

  (b) For Ungrouped Data :

  If there are n distinct observations $x_1, x_2,..., x_n$ of variable x with frequencies $f_1, f_2,..., f_n$ respectively, then the arithmetic mean is given by :

$$x = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + f_3 + ... + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$$

  (c) For Grouped Data :

  (i) To find the mean of grouped data, it is assumed that the frequency of each class-interval is centred around its mid-point.

  (ii) Direct Method :

$$\text{Mean } (x) = \frac{\Sigma f_i x_i}{\Sigma f_i},$$

  where the x_i (class mark) is the mid-point of the i^{th} class interval and f_i is the corresponding frequency.

  (iii) Assumed Mean Method or Short-cut Method :

$$\text{Mean } (x) = a + \frac{\Sigma f_i d_i}{\Sigma f_i},$$

  where a is the assumed mean and $d_i = x_i - a$ are the deviations of x_i from a for each i.

  (iv) Step-Deviation Method :

$$\text{Mean } (x) = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right),$$

  where, a is the assumed mean, h is the class-size and $u_i = \frac{x_i - a}{h}$.

- **Median :**

  (a) For Ungrouped Data :

  If n is odd, $\quad \text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$

  If n is even, $\quad \text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{ term} + \left(\frac{n+1}{2}\right)^{th} \text{ term}}{2}$

  (b) For Grouped Data :

  Let $n = f_1 + f_2 + f_3 + ... + f_n$. First of all find $\frac{n}{2}$ and then the class in which $\frac{n}{2}$ lies. This class is known as the **median class.** Median of the given distribution lies in this class.

  Median of the grouped data can be calculated using the formula :

$$\text{Median } (M_e) = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h,$$

  where, l = lower limit of median class, f = frequency of median class, n = number of observations, $c.f.$ = cumulative frequency of the class preceding the median class, h = class-size or width of the class-interval.

- **Mode of Grouped Data :**

  (i) Mode is the observation which occurred maximum times. In ungrouped data, mode is the observation having maximum frequency. In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class is known as the **modal class**. The mode of the data is a value inside the modal class.

  (ii) Mode of the grouped data can be calculated by using the formula :

$$\text{Mode } (M) = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h,$$

where, l = lower limit of the modal class, h = width or size of the class-interval, f_1 = frequency of the modal class, f_0 = frequency of the class preceding the modal class, f_2 = frequency of the class succeeding the modal class.

Note :

(a) If the series has only one mode, then it is known as **Unimodal.**

(b) If the series has two modes, then it is known as **Bimodal.**

(c) If the series has three modes, then it is known as **Trimodal.**

(d) Mode may or may not be defined for a given series.

➢ **Empirical Relation Between Mean, Median and Mode :**

(i) $$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

(ii) $$\text{Median} = \frac{1}{3}\,\text{Mode} + \frac{2}{3}\,\text{Mean}$$

(iii) $$\text{Mean} = \frac{3}{2}\,\text{Median} - \frac{1}{2}\,\text{Mode}$$

Note : For calculating the mode and median for grouped data, it should be ensured that the class-intervals are continuous before applying the formula. Same condition also apply for construction of an ogive. Further, in case of ogives, the scale may not be the same on both the axes.

➢ **Cumulative Frequency Distribution :**

(i) Cumulative frequency of a particular value of the variable (or class) is the sum (total) of all the frequencies up to than value (or class).

(ii) There are two types of cumulative frequency distributions :

(a) Cumulative frequency distribution of less than type.

(b) Cumulative frequency distribution of more than type.

For example :

Class interval (marks)	**Frequency (No. of Students)**	**Cumulative frequency (c.f.)**	**Less than type**		**More than type**	
			Marks Out of 50	**c.f. less than type**	**Marks Out of 50**	**c.f. less than type**
0 – 10	2	2	Less than 10	2 = 2	0 or More than 0	60 = 60
10 – 20	10	12	Less than 20	2 + 10 = 12	More than 10	60 – 2 = 58
20 – 30	25	37	Less than 30	12 + 25 = 37	More than 20	58 – 10 = 48
30 – 40	20	57	Less than 40	37 + 20 = 57	More than 30	48 – 25 = 23
40 – 50	3	60	Less than 50	57 + 3 = 60	More than 40	23 – 20 = 3

How it is done on GREENBOARD?

Q. Find mean of the following data :

x	1	2	3	4	5
f	6	8	4	5	2

Sol. : Step I :

x	f	fx
1	6	6
2	8	16
3	4	12
4	5	20
5	2	10
Total	25	64

Step II :

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{64}{25}$$

$$\bar{x} = 2{\cdot}56$$

TOPIC-2
Cumulative Frequency Graph

Quick Review

- **Cumulative frequency curve or an Ogive curve :** *The graphical representation of a cumulative frequency distribution is called the cumulative frequency curve or ogive.*

  There are two methods to construct ogives :

1. Less than ogive :

In this method, an ogive is cumulated upward. Scale the cumulative frequencies along the y-axis and exact upper limits along the x-axis. The scale along the y-axis should be such as may accommodate the total frequency.

Step I. Form the cumulative frequency table.

Step II. Mark the actual upper class limits along the x-axis.

Step III. Mark the cumulative frequency of the respective classes along the y-axis.

Step IV. Plot the points (upper limits, corresponding cumulative frequency.)

By joining these points on the graph by a free hand curve, we get an ogive of 'less than' type.

2. More than ogive :

In this method, an ogive is cumulated downward. Scale the cumulative frequencies along the y-axis and the exact lower limits along the x-axis.

Step I. Scale the cumulative frequencies along the y-axis and the actual lower limits along the x–axis.

Step II. Plot the ordered pairs (lower limit, corresponding cumulative frequency). To complete an ogive, we also plot the ordered pair (upper limit of the highest class, 0).

Step III. Join these plotted points by a smooth curve. The curve so obtained is the required 'more than' type ogive.

2. Median :

Ogive can be used to estimate the median of the data. There are two methods to get the median :

(i) Mark a point corresponding to $\frac{N}{2}$, where N is the total frequency on cumulative frequency axis (*i.e., y*-axis). Draw a line parallel to x-axis to cut the ogive at a point. From this point draw a line perpendicular to the x-axis to get another point. The abscissa of this point gives median.

(ii) Draw both the ogives (less than and more than ogive) on the same graph paper which cut each other at a point. From this point draw a line perpendicular to the x-axis, to get another point. The point at which it cuts x-axis, gives the median.

How it is done on GREENBOARD?

Q. Form a more than frequency table of the following data :

x	1 - 2	2 - 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9
f	3	4	1	2	6	2	4	3

Sol. : Step I :

x	f	C.F. (more than)	
1 - 2	3	25	(22 + 3)
2 - 3	4	22	(18 + 4)
3 - 4	1	18	(17 + 1)
4 - 5	2	17	(15 + 2)
5 - 6	6	15	(9 + 6)
6 - 7	2	9	(7 + 2)
7 - 8	4	7	(3 + 4)
8 - 9	3	3	

Chapter - 15 : Probability

Quick Review

- Probability is a quantitative measure of certainty.
- Any activity associated to certain outcome is called a random experiment. *e.g.*,
  (i) tossing a coin, **(ii)** throwing a dice, **(iii)** selecting a card.
- Outcome associated with an experiment is called an event. *e.g.*, **(i)** Getting a head on tossing a coin, **(ii)** getting a face card when a card is drawn from a pack of 52 cards.
- The events whose probability is one are called sure/certain events.
- The events whose probability is zero are called impossible events.
- An events with only one possible outcome is called an elementary event.
- In a given experiment, if two or more events are equally likely to occur or have equal probabilities, then they are called equally likely events.
- Probability of an event always lies between 0 and 1.
- Probability can never be negative.
- A pack of playing cards consist of 52 cards which are divided into 4 suits of 13 cards each. Each suit consists of an ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are spades, hearts, diamonds and clubs.
- King, queen and jack are face cards.
- The sum of the probabilities of all elementary events of an experiment is 1.
- Two events A and B are said to be complementary to of each other if the sum of their probabilities is 1.
- Probability of an event E, denoted as $P(E)$, is given by :

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total possible number of outcomes}}$$

- For an event E, $P(\bar{E}) = 1 - P(E)$, where the event $\bar{E}$ representing 'not E' is the complement of the event E.
- For A and B two possible outcomes of an event,
  (i) If $P(A) > P(B)$, then event A is more likely to occur than event B.
  (ii) If $P(A) = P(B)$, then events A and B are equally likely to occur.

Know the Facts

- The experimental or empirical probability of an event is based on what has actually happened while the theoretical probability of the event attempts to predict what will happen on the basis of certain assumptions.
- As the number of trials in an experiment go on increasing, we may expect the experimental and theoretical probabilities to be nearly the same.
- When we speak of a coin, we assume it to be '**fair** ' *i.e.*, it is symmetrical so that there is no reason for it to come down more often on one side than the other. We call this property of the coin as being '**unbiased**'.
- By the phrase '**random toss**', we mean that the coin is allowed to fall freely without any **bias** or **interference**.
- In the case of experiment we assume that the experiments have equally likely outcomes.
- A deck of playing cards consists of 4 suits : spades (♠), hearts (♥), diamonds (♦) and clubs (♣). Clubs and spades are of black colour, while hearts and diamonds are of red colour.
- The first book on probability '**The Book on Games of Chance**' was written by Italian mathematician **J. Cardan**.
- The classical definition of probability was given by Pierre Simon Laplace.

How it is done on GREENBOARD ?

Q. One dice and one coin are tossed simultaneously. Write the sample space. Find the probability of getting :

(i) Prime number on dice

(ii) head

(iii) head and even number

Sol. Step I : Writing the sample space :

$\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\ (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$

Step II : Finding prime number on dice : prime numbers are 2, 3 and 5

$\therefore$ favourable cases are :

$A = \{(H, 2), (H, 3), (H, 5), (T, 2), (T, 3), (T, 5)\}$

$n(A) = 6$

$$\therefore \text{ Required probability } = \frac{n(A)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

Step III : Finding cases having head

$B = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$

$n(B) = 6$

$$\therefore \text{ Required probability } = \frac{n(B)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$

Step IV : Finding head and even number

$C = \{(H, 2), (H, 4), (H, 6)\}$

$n(C) = 3$

$$\therefore \text{ Required probability } = \frac{n(C)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$