SOLVED PAPER With CBSE Marking Scheme

C.B.S.E. 2018 Class–XII Delhi & Outside Delhi Sets

Mathematics

Time : 3 Hours **Max. Marks : 100**

General Instructions :

(i) *All questions are compulsory.*

(ii) *The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.*

(iii) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(iv) *There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*

(v) *Use of calculators in not permitted. You may ask for logarithmic tables, if required.*

SECTION - A

1. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$.

2. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew symmetric, find the values of 'a' and 'b'.

3. Find the magnitude of each of the vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

4. If $a * b$ denotes the large of 'a' and 'b' if $a \circ b = (a * b) + 3$, then write the value of (5) o (10), where * and o are binary operations.

SECTION - B

5. Prove that :

$3\sin^{-1} x = \sin^{-1}(3x - 4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

6. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$.

7. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.

8. The total cost $C(x)$ associated with the production x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

9. Evaluate :

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$$

10. Find the differential equation representing the family of curves $y = a\,e^{bx+5}$, where a and b are arbitrary constants.

11. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find sin θ.

12. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

SECTION - C

13. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$

14. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

OR

If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.

15. If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2 x = 0$.

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16. Find the equations of the tangent and the normal, to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$.

OR

Find the intervals in which the function $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$ is (a) strictly increasing, (b) strictly decreasing.

17. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question ?

18. Find : $\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)} dx$

19. Find the particular solution of the differential equation $e^x \tan y\, dx + (2 - e^x) \sec^2 y\, dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.

OR

Find the particular solution of the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$.

20. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and $\vec{d} \cdot \vec{a} = 21$.

21. Find the shortest distance between the lines

$$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k}) \text{ and}$$

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k}).$$

22. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ?

23. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denote the larger of the two numbers obtained. Find the mean and variance of X.

SECTION - D

24. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A,\}\ |a - b|$ is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

OR

Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall\, x \in R$ is neither one-one nor onto. Also, if $g : R \to R$ is defined as $g(x) = 2x - 1$, find fog(x).

25. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use it to solve the system of equations

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3.$$

OR

Using elementary row transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}.$$

26. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

27. Evaluate : $\int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x} dx$

OR

Evaluate : $\int_1^3 (x^2 + 3x + e^x)dx$, as the limit of the sum.

28. Find the distance of the point (– 1, – 5, – 10) from the point of intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

29. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of ₹ 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit ? Formulate the above LPP and solve it graphically and find the maximum profit.

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CBSE Marking Scheme
(Issued by Board)

SECTION - A

1. $\frac{\pi}{3}-\left(\pi-\frac{\pi}{6}\right)=-\frac{\pi}{3}$ ½+½

Note : $\frac{1}{2}$ m. for any one of the two correct values and $\frac{1}{2}$ m. for final answer.

[CBSE Marking Scheme, 2018]

Detailed Solution :

$$\tan^{-1}(\sqrt{3})-\cot^{-1}(-\sqrt{3})$$
$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)-\cot^{-1}\left(-\cot\frac{\pi}{6}\right)$$
$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right)-\cot^{-1}\left[\cot\left(\pi-\frac{\pi}{6}\right)\right]$$
$$= \frac{\pi}{3}-\left(\pi-\frac{\pi}{6}\right) = \frac{\pi}{3}-\frac{5\pi}{6}$$
$$= -\frac{\pi}{2}$$

2. $a = -2, b = 3$ ½+½

[CBSE Marking Scheme, 2018]

Detailed Solution :

Since, the given matrix is skew symmetric. So, we have $A^T = -A$. (A^T is transpose of A)

$$\therefore \quad \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$$

$\therefore \quad -a = +2$ and $-b = -3$.

Hence, $\quad a = -2$ and $b = 3$.

3. $|\vec{a}| = |\vec{b}| = 3$ ½+½

[CBSE Marking Scheme, 2018]

Detailed Solution :

Given, $\quad$ angle $\theta = 60°$ and $\vec{a}.\vec{b} = \frac{9}{2}$

$$\because \quad \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$$

$$\Rightarrow \quad \cos 60° = \frac{9/2}{|\vec{a}|.|\vec{a}|} \quad (\because |\vec{a}| = |\vec{b}|) \quad ½$$

$$\Rightarrow \quad \frac{1}{2}|\vec{a}|^2 = \frac{9}{2} \Rightarrow |\vec{a}| = 3.$$

$$\therefore \quad |\vec{a}| = |\vec{b}| = 3. \quad ½$$

4. 5 o 10 = (5 * 10) + 3 = 10 + 3 = 13

For 5 * 10 = 10 ½

For Final Answer = 13 ½

SECTION - B

5. In RHS, put $x = \sin\theta$ ½

$$\text{RHS} = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$$
$$= \sin^{-1}(\sin 3\theta) \quad 1$$
$$= 3\theta = 3\sin^{-1}x = \text{LHS} \quad ½$$

6. $|A| = 2,$

$$\therefore \quad A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \quad 1$$

$$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix},$$

$$\text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \quad 1$$

$\therefore \quad$ LHS = RHS

7.
$$f(x) = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$
$$= \tan^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) \quad 1$$
$$= \tan^{-1}\left(\cot\frac{x}{2}\right) = \frac{\pi}{2}-\frac{x}{2} \quad ½$$
$$\therefore \quad f'(x) = -\frac{1}{2} \quad ½$$

[CBSE Marking Scheme, 2018]

Detailed Solution :

Let
$$y = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$
$$= \tan^{-1}\left(\frac{1+2\cos^2\frac{x}{2}-1}{2.\sin\frac{x}{2}.\cos\frac{x}{2}}\right)$$
$$= \tan^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right) \quad 1$$
$$= \tan^{-1}\left(\cot\frac{x}{2}\right)$$

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$$= \tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{x}{2}\right)\right]$$

$$y = \left(\frac{\pi}{2}-\frac{x}{2}\right)$$

Differentiating with respect to x.

$$\frac{dy}{dx} = -\frac{1}{2}. \qquad 1$$

8. Marginal cost $= C'(x) = 0{\cdot}015\,x^2 - 0{\cdot}04x + 30$ 1

At $x = 3$, $C'(3) = 30{\cdot}015$ 1

9.
$$I = \int \frac{1-2\sin^2 x + 2\sin^2 x}{\cos^2 x}dx \qquad ½$$

$$= \int \sec^2 x\, dx \qquad 1$$

$$= \tan x + C \qquad ½$$

10.
$$\frac{dy}{dx} = bae^{bx+5} \qquad ½$$

$$\Rightarrow \quad \frac{dy}{dx} = by \qquad ½$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = b\frac{dy}{dx} \qquad ½$$

$\therefore$ The differential equation is :

$$y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 \qquad ½$$

[CBSE Marking Scheme, 2018]

Detailed Solution :

$$y = a\,e^{bx+5} \qquad ...(1)$$

Differentiate w. r. to x

$$\frac{dy}{dx} = a.b\,e^{bx+5} \qquad ...(2)$$

Again, D.w.r. to x.

$$\frac{d^2y}{dx^2} = a.b^2\,e^{bx+5} \qquad ...(3) \quad 1$$

put $b = \frac{1}{y}.\frac{dy}{dx}$ from (1) and (2) in equation (3). we get

$$\frac{d^2y}{dx^2} = y.\frac{1}{y^2}.\left(\frac{dy}{dx}\right)^2$$

$\therefore$ Required differential equation is

$$\frac{d^2y}{dx^2} = \frac{1}{y}.\left(\frac{dy}{dx}\right)^2 \qquad 1$$

11.
$$\sin\theta = \frac{|(\hat{i}-2\hat{j}+3\hat{k})\times(3\hat{i}-2\hat{j}+\hat{k})|}{|\hat{i}-2\hat{j}+3\hat{j}|\,|3\hat{i}-2\hat{j}+\hat{k}|} \qquad ½$$

$$|(\hat{i}-2\hat{j}+3\hat{k})\times(3\hat{i}-2\hat{j}+\hat{k})|$$

$$= |4\hat{i}+8\hat{j}+4\hat{k}| = 4\sqrt{6} \qquad 1$$

$$\sin\theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7} \qquad ½$$

[CBSE Marking Scheme, 2018]

Detailed Solution :

Let $\vec{a} = \hat{i}-2\hat{j}+3\hat{k} \Rightarrow |\vec{a}| = \sqrt{1^2+(-2)^2+3^2} = \sqrt{14}$

$\vec{b} = 3\hat{i}-2\hat{j}+\hat{k} \Rightarrow |\vec{b}| = \sqrt{3^2+(-2)^2+1^2} = \sqrt{14}$

$$\because \quad \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$$

$$= \frac{(\hat{i}-2\hat{j}+3\hat{k}).(3\hat{i}-2\hat{j}+\hat{k})}{\sqrt{14}.\sqrt{14}} \qquad 1$$

$$= \frac{3+4+3}{14} = \frac{10}{14} = \frac{5}{7}.$$

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta}$$

$$\sin\theta = \sqrt{1-\frac{25}{49}}$$

$$= \frac{\sqrt{24}}{7}. \qquad 1$$

12. A : Getting a sum of 8, B : Red die resulted in no. < 4.

$$P(A/B) = \frac{P(A\cap B)}{P(B)} \qquad 1$$

$$= \frac{2/36}{18/36} = \frac{1}{9} \qquad 1$$

SECTION - C

13.
$$\text{LHS} = \begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} \qquad 1$$

$$= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix} \qquad 1$$

(Using $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$= 1 \times (9yz) + 3x(3z + 9yz + 3y)$

(Expanding along R_1) 1

$= 9(3xyz + xy + yz + zx) =$ RHS 1

14. Differentiating with respect to 'x'

$$2(x^2+y^2)\left(2x+2y\frac{dy}{dx}\right) = x\frac{dy}{dx}+y \qquad 2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y-4x^3-4xy^2}{4x^2y+4y^3-x} \qquad 2$$

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OR

$$\frac{dx}{d\theta} = a(2-2\cos 2\theta) = 4a\sin^2\theta \qquad \mathbf{1}$$

$$\frac{dy}{d\theta} = 2a\sin 2\theta = 4a\sin\theta\cdot\cos\theta \qquad \mathbf{1}$$

$$\therefore \quad \frac{dy}{dx} = \frac{4a\sin\theta\cos\theta}{4a\sin^2\theta} = \cot\theta \qquad \mathbf{1}$$

$$\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{3}} = \frac{1}{\sqrt{3}} \qquad \mathbf{1}$$

[CBSE Marking Scheme, 2018]

Detailed Solution :

$(x^2+y^2)^2 = xy$

Differentiating with respect to x.

$$2.(x^2+y^2)\left[2x+2y.\frac{dy}{dx}\right] = x.\frac{dy}{dx}+y$$

$$4x(x^2+y^2)+4y(x^2+y^2)\frac{dy}{dx}-x\frac{dy}{dx} = y$$

$$[4y(x^2+y^2)-x]\frac{dy}{dx} = y-4x(x^2+y^2)$$

$$\therefore \quad \frac{dy}{dx} = \frac{[y-4x(x^2+y^2)]}{[4y(x^2+y^2)-x]}$$

OR

Given $\quad x = a(2\theta-\sin 2\theta)$

Differentiate with respect to θ.

$$\frac{dx}{d\theta} = a(2-2\cos 2\theta)$$

$$\frac{dx}{d\theta} = 2a(1-\cos 2\theta)$$

and $\quad y = a.(1-\cos 2\theta)$

$$\frac{dy}{d\theta} = 2a\sin 2\theta$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2a\sin 2\theta}{2a(1-\cos 2\theta)}$$

$$\left[\frac{dy}{dx}\right]_{\theta=\frac{\pi}{3}} = \frac{\sin .2\frac{\pi}{3}}{1-\cos\frac{2\pi}{3}} = \frac{\sin\frac{\pi}{3}}{1+\cos\frac{\pi}{3}}$$

$$= \frac{\sqrt{3}/2}{1+\frac{1}{2}} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}.$$

$$\therefore \quad \left[\frac{dy}{dx}\right]_{\theta=\frac{\pi}{3}} = \frac{1}{\sqrt{3}}.$$

15. $\quad y = \sin(\sin x)$

$$\Rightarrow \quad \frac{dy}{dx} = \cos(\sin x)\cdot\cos x \qquad \mathbf{1}$$

and $$\frac{d^2y}{dx^2} = -\sin(\sin x)\cdot\cos^2 x - \sin x\cos(\sin x)$$

1+1

$$\text{LHS} = -\sin(\sin x)\cos^2 x - \sin x\cos(\sin x) + \frac{\sin x}{\cos x}\cos(\sin x)\cos x + \sin(\sin x)\cos^2 x$$

$$= 0 = \text{RHS} \qquad \mathbf{1}$$

16. $\quad x_1 = 2 \Rightarrow y_1 = 3 \ (\because y_1 > 0)$ ½

Differentiating the given equation, we get, ½

$$\frac{dy}{dx} = \frac{-16x}{9y}$$

Slope of tangent at (2, 3) $= \left.\frac{dy}{dx}\right]_{(2,3)} = -\frac{32}{27}$ ½

Slope of Normal at (2, 3) $= \frac{27}{32}$ ½

Equation of tangent: $32x + 27y = 145$ **1**

Equation of Normal: $27x - 32y = -42$ **1**

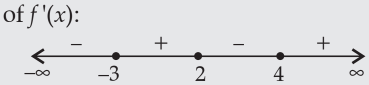
OR

$$f'(x) = x^3 - 3x^2 - 10x + 24 \qquad ½$$

$$= (x-2)(x-4)(x+3) \qquad \mathbf{1}$$

$$f'(x) = 0 \Rightarrow x = -3, 2, 4. \qquad ½$$

sign of $f'(x)$:



$\therefore f(x)$ is strictly increasing on $(-3, 2) \cup (4, \infty)$ **1**

and $f(x)$ is strictly decreasing on $(-\infty, -3) \cup (2, 4)$ **1**

[CBSE Marking Scheme, 2018]

Detailed Solution :

Given, $\quad 16x^2 + 9y^2 = 145 \qquad$ ...(1)

It passes through (x_1, y_1)

$$16x_1^2 + 9y_1^2 = 145$$

put $x_1 = 2$

$$16 \times 4 + 9y_1^2 = 145$$

$$\Rightarrow \quad 9y_1^2 = 81 \Rightarrow y_1 = 3 \qquad (\because y_1 > 0)$$

Diff. equation (1) with respect to x.

$$32x + 18y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{32x}{18y}$$

Slope of the tangent at (2, 3)

$$= \left[\frac{dy}{dx}\right]_{(2,3)} = \frac{-32\times 2}{18\times 3} = \frac{-32}{27}$$

The equation of tangent at (2, 3) is

$$y - 3 = -\frac{32}{27}.(x-2)$$

$$\Rightarrow \quad 27y - 81 = -32x + 64$$

$$\Rightarrow \quad 32x + 27y - 145 = 0$$

Slope of Normal at (2, 3) $= -\dfrac{1}{\left[\frac{dy}{dx}\right]_{(2,3)}} = \dfrac{27}{32}$

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Now, equation of Normal is given by

$$y - 3 = \frac{27}{32}(x-2)$$

$\Rightarrow$ $32y - 96 = 27x - 54$

$\Rightarrow$ $27x - 32y + 42 = 0$

OR

Given $f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$

Differentiating with respect to x.

$$f'(x) = x^3 - 3x^2 - 10x + 24$$

put $f'(x) = 0$

$x^3 - 3x^2 - 10x + 24 = 0$

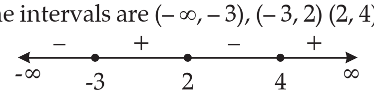
$\Rightarrow$ $x^2(x-2) - x(x-2) - 12(x-2) = 0$

$\Rightarrow$ $(x-2)(x^2 - x - 12) = 0$

$\Rightarrow$ $(x-2)(x+3)(x-4) = 0$

$\Rightarrow$ $x = -3, 2$ and 4

$\therefore$ The intervals are $(-\infty, -3)$, $(-3, 2)$ $(2, 4)$ and $(4, \infty)$.



(a) $f'(x) > 0\ \forall\ x \in (-3, 2) \cup (4, \infty)$

$\therefore$ $f(x)$ is strictly increasing $\forall\ x \in (-3, 2) \cup (4, \infty)$

(b) $f'(x) < 0\ \forall$ x $\in (-\infty, -3) \cup (2, 4)$

$\therefore$ $f(x)$ is strictly decreasing $\forall\ x \in (-\infty, -3) \cup (2, 4)$

17. Let side of base $= x$ and depth of tank $= y$

$$V = x^2 y \Rightarrow y = \frac{V}{x^2},$$

(V = Quantity of water = constant)

Cost of material is least when area of sheet used is minimum.

A (Surface area of tank) $= x^2 + 4xy = x^2 + \frac{4V}{x}$

$$\frac{dA}{dx} = 2x - \frac{4V}{x^2}, \frac{dA}{dx} = 0 \qquad ½+½$$

$\Rightarrow$ $x^3 = 2V$, y $= \frac{x^3}{2x^2} = \frac{x}{2}$ ½+½

$$\frac{d^2A}{dx^2} = 2 + \frac{8V}{x^3} > 0, \qquad ½+½$$

$\therefore$ Area is minimum, thus cost is minimum when

$$y = \frac{x}{2}$$

Value: Any relevant value. **1**

18. Put $\sin x = 1 \Rightarrow \cos x\, dx = dt$ ½

$$\text{Let } I = \int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}$$

$$dx = \int \frac{2}{(1-t)(1+t^2)}dt$$

$$\text{Let } = \frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}, \text{ solving we get}$$

A = 1, B = 1, C = 1 **1½**

$$\therefore \quad I = \int \frac{1}{1-t}dt + \frac{1}{2}\int \frac{2t}{1+t^2}dt + \int \frac{1}{1+t^2}dt$$

$$= -\log|1-t| + \frac{1}{2}\log|1+t^2| + \tan^{-1} t + C \qquad \textbf{1½}$$

$$= -\log(1-\sin x) + \frac{1}{2}\log(1+\sin^2 x) + \tan^{-1}(\sin x) + C \qquad ½$$

[CBSE Marking Scheme, 2018]

Detailed Solution :

$$\int \frac{2\cos x}{(1-\sin x).(1+\sin^2 x)}.dx$$

Let $\sin x = y \Rightarrow \cos x.\, dx = dy$

$$\therefore \int \frac{2\cos x}{(1-\sin x).(1+\sin^2 x)}.dx = \int \frac{2}{(1-y).(1+y^2)}.dy$$

$$\text{Let } \frac{2}{(1-y)(1+y^2)} = \frac{A}{1-y} + \frac{By+C}{1+y^2}$$

$\Rightarrow$ $2 = A(1+y^2) + (By+C)(1-y)$

$\Rightarrow$ $2 = A + Ay^2 + By + C - By^2 - Cy$

$2 = (A-B)y^2 + (B-C).y + A + C$

Comparing the Coefficients of y^2, y and Constant terms.

$A - B = 0$, $B - C = 0$ and $A + C = 2$.

Solving all these equation we get

$$A = B = C = 1$$

$$\therefore \int \frac{2\cos x}{(1-\sin x).(1+\sin^2 x)}.dx = \int \frac{2}{(1-y)(1+y^2)}.dy$$

$$= \int \frac{1}{1-y}.dy + \int \frac{y+1}{1+y^2}.dt$$

$$= \int \frac{1}{1-y}\,dy + \frac{1}{2}\int \frac{2y}{1+y^2}.\,dy + \int \frac{1}{1+y^2}.\,dy$$

$$= -\log|1-y| + \frac{1}{2}\log|1+y^2| + \tan^{-1} y + C$$

$$= \log\left|\frac{\sqrt{1+y^2}}{1-y}\right| + \tan^{-1} y + C$$

$$= \log\frac{\sqrt{1+\sin^2 x}}{1-\sin x} + \tan^{-1}(\sin x) + C$$

19. Separating the variables, we get:

$$\int \frac{\sec^2 y}{\tan y}dy = \int \frac{e^x}{e^x - 2}dx \qquad \textbf{1½}$$

$$\Rightarrow \quad \log|\tan y| = \log|e^x - 2| + \log C \qquad \textbf{1½}$$

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$$\Rightarrow \quad \tan y = C(e^x - 2), \text{ for } x = 0, y = \pi/4,$$

$C = -1$ ½

$\therefore$ Particular solution is: $\tan y = 2 - e^x$. ½

OR

Integrating factor $= e^{\int 2\tan x\,dx} = \sec^2 x$ 1

$\therefore$ Solution is: $y\cdot\sec^2 x = \int \sin x\cdot\sec^2 x\,dx$ 1

$$= \int \sec x\cdot\tan x\,dx$$

$y\cdot\sec^2 x = \sec x + C$, for $x = \frac{\pi}{3}$, $y = 0$, $\therefore$ C = –2

1+½

$\therefore$ Particular solution is: $y\cdot\sec^2 x = \sec x - 2$

or $y = \cos x - 2\cos^2 x$ ½

[CBSE Marking Scheme, 2018]

Detailed Solution :

Given differential equation is

$e^x.\tan y.dx + (2 - e^x).\sec^2 y.dy = 0$

$$\Rightarrow \quad \frac{\sec^2 y}{\tan y}.dy = \frac{e^x}{(e^x - 2)}.dx$$

put $\tan y = t$ $e^x - 2 = z$

$\Rightarrow \quad \sec^2 y.\,dy = dt \qquad \Rightarrow e^x\,dx = dz$

$$\int \frac{1}{t}.dt = \int \frac{1}{z}.dz$$

$$\log|t| = \log|z| + \log C$$

$$\log|t| = \log|C.z|$$

$$t = C.z$$

$$\tan y = C.(e^x - 2)$$

$y = \frac{\pi}{4}$ when $x = 0$

$$\tan\frac{\pi}{4} = C.(e^0 - 2) = -C$$

$\Rightarrow \quad C = -1$

$\therefore$ Required particular solution is

$$y = \tan^{-1}(2 - e^x)$$

OR

Given $\frac{dy}{dx} + 2\tan x.y = \sin x$

(it is linear in y)

Comparing with $\frac{dy}{dx} + P.y = Q$.

Here, $P = 2\tan x, Q = \sin x$

Integrating factor (I.F.) $= e^{\int 2\tan x.dx}$

$$= e^{2\log|\sec x|}$$

$$= \sec^2 x$$

The solution is

$$y \times (\text{I.F.}) = \int (\text{I.F.})\times Q.dx + c$$

$$y.\sec^2 x = \int \sec^2 x.\sin x.dx + c$$

$$y\sec^2 x = \int \sec x.\tan x.dx + c$$

$$y\sec^2 x = \sec x + c$$

$y = 0$ when $x = \frac{\pi}{3}$

$$0 = \sec\frac{\pi}{3} + c$$

$\Rightarrow \quad 0 = 2 + c \Rightarrow c = -2$

$\therefore$ required particular solution is

$$y\sec^2 x = \sec x - 2$$

20. $\vec{d} = \lambda(\vec{c}\times\vec{b}) = \lambda\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -1\\ 1 & -4 & 5\end{vmatrix}$ 1

$\therefore \quad \vec{d} = \lambda\hat{i} - 16\lambda\hat{j} - 13\lambda\hat{k}$ 1

$$\vec{d}\cdot\vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21$$

$\Rightarrow \quad \lambda = -\frac{1}{3}$ 1

$\therefore \quad \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$ 1

[CBSE Marking Scheme, 2018]

Detailed Solution :

Since, vector $\vec{d}$ is perpendicular to both $\vec{c}$ and $\vec{b}$.

$\therefore \quad \vec{d}.\vec{b} = 0$ and $\vec{d}.\vec{c} = 0$

Let $\quad \vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

Now, $\because \quad \vec{d}.\vec{b} = 0$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}).(\hat{i} - 4\hat{j} + 5\hat{k}) = 0$

$\Rightarrow \quad x - 4y + 5z = 0$ ...(i)

$$\vec{d}.\vec{c} = 0$$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}).(3\hat{i} + \hat{j} - \hat{k}) = 0$

$\Rightarrow \quad 3x + y - z = 0$ ...(ii)

and $\quad \vec{d}.\vec{a} = 21$

$\Rightarrow \quad (x\hat{i} + y\hat{j} + z\hat{k}).(4\hat{i} + 5\hat{j} - \hat{k}) = 21$

$\Rightarrow \quad 4x + 5y - z = 21$ ...(iii)

Solving equation (i), (ii) and (iii) we get

$$x = -\frac{1}{3},\ y = \frac{16}{3} \text{ and } z = \frac{13}{3}.$$

So, $\quad \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$.

21. Here $\vec{a_1} = 4\hat{i} - \hat{j}$, $\vec{a_2} = \hat{i} - \hat{j} + 2\hat{k}$ 1

$\vec{a_2} - \vec{a_1} = -3\hat{i} + 2\hat{k}$ 1

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$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i}-\hat{j}$$ **1**

$$\text{Shortest distance} = \frac{\left|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$$

$$= \left|\frac{-6}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}} \text{ or } \frac{6\sqrt{5}}{5}$$ **1**

22. E_1 : She gets 1 or 2 on die.
E_2 : She gets 3, 4, 5 or 6 on die.
A : She obtained exactly 1 tail **1**

$$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{2}{3}$$
$$P(A/E_1) = \frac{3}{8},\ P(A/E_2) = \frac{1}{2}$$ **1**

$$P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)}$$ **1**

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{8}{11}$$ **1**

23. Let X denote the larger of two numbers

X	2	3	4	5	½
$P(X)$	1/10	2/10	3/10	4/10	**1**
$X\cdot P(X)$	2/10	6/10	12/10	20/10	½
$X^2\cdot P(X)$	4/10	18/10	48/10	100/10	½

$$\text{Mean} = \Sigma X\cdot P(X) = \frac{40}{10} = 4$$ ½

$$\text{Variance} = \Sigma X^2\cdot P(X) - [\Sigma X\cdot P(X)]^2$$

$$= \frac{170}{10} - 4^2 = 1$$ **1**

[CBSE Marking Scheme, 2018]

Detailed Solution :

We have, the first five positive integers are 1, 2, 3, 4 and 5.

We can selected two numbers from 5 numbers is

$$^5P_2 = \frac{\lfloor 5}{\lfloor 5-2} = \frac{120}{6} = 20.$$

Here, given X denote the larger of two numbers we observe that X can be take values 2, 3, 4, 5.

$P(X = 2)$ = Probability that larger number is 2.

$P(X = 2)$ = Probability of getting 1 in first selection and 2 in second selection or getting 2 in first selection and 1 in second selection

$$\Rightarrow \quad P(X=2) = \frac{1}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4} = \frac{2}{20}$$

$P(X = 3)$ = Probability that the larger of two number is 3.

$$\Rightarrow \quad P(X=3) = \frac{2}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{2}{4} = \frac{4}{20}$$

$P(X = 4)$ = Probability that the larger of two number is 4

$$\Rightarrow \quad P(X=4) = \frac{3}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4} = \frac{6}{20}$$

$$P(X=5) = \frac{4}{5}\times\frac{1}{4}+\frac{1}{5}\times\frac{4}{5} = \frac{8}{20}.$$

Thus, the probability distribution of X is

X :	2	3	4	5
$P(X)$:	$\frac{2}{20}$	$\frac{4}{20}$	$\frac{6}{20}$	$\frac{8}{20}$

$$\therefore \quad \text{Mean } [E(X)] = 2\times\frac{2}{20}+3\times\frac{4}{20}+4\times\frac{6}{20}+5\times\frac{8}{20}$$

$$= \frac{4}{20}+\frac{12}{20}+\frac{24}{20}+\frac{40}{20}$$

$$= \frac{80}{20} = 4.$$

$$E(X^2) = 2^2\times\frac{2}{20}+3^2\times\frac{4}{20}+4^2\times\frac{6}{20}+5^2\times\frac{8}{20}$$

$$= \frac{8}{20}+\frac{36}{20}+\frac{96}{20}+\frac{200}{20} = \frac{340}{20}$$

$$= 17$$

$$\text{Variance of } X = E(X^2) - [E(X)]^2$$
$$= 17 - 4^2 = 1.$$

Hence, $E(X) = 4$ and Var $(X) = 1$.

SECTION - D

24. Reflexive : $|a-a| = 0$, which is divisible by 4, $\forall\, a \in A$ **1**

$\therefore (a, a) \in R, \forall\, a \in A \therefore R$ is reflexive

Symmetric : let $(a, b) \in R$

$\Rightarrow |a-b|$ is divisible by 4

$\Rightarrow |b-a|$ is divisible by 4 $(\because |a-b| = |b-a|)$ **1**

$\Rightarrow (b, a) \in R \therefore R$ is symmetric

Transitive: let $(a, b), (b, c) \in R$

$\Rightarrow |a-b|$ & $|b-c|$ are divisible by 4

$\Rightarrow a - b = \pm 4m,\ b - c = \pm 4n,\ n \in Z$

Adding we get, $a - c = 4(\pm m \pm n)$ **2**

$\Rightarrow (a - c)$ is divisible by 4

$\Rightarrow |a-c|$ is divisible by 4 $\therefore (a, c) \in R$

$\Rightarrow R$ is transitive

Hence R is an equivalence relation in A **1**

set of elements related to 1 is {1, 5, 9} **1**

and [2] = {2, 6, 10}.

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OR

Here $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$ **2**

$\therefore f$ is not 1 – 1

for $y = \frac{1}{\sqrt{2}}$ let $f(x) = \frac{1}{\sqrt{2}} \Rightarrow x^2 - \sqrt{2}x + 1 = 0$

As $D = \left(-\sqrt{2}\right)^2 - 4(1)(1) < 0,$

$\therefore$ No real solution

$\therefore f(x) \neq \frac{1}{\sqrt{2}}$, for any $x \in R\ (D_f)$ $\therefore f$ is not onto **2**

$$\text{fog}(x) = f(2x-1) = \frac{2x-1}{(2x-1)^2+1} = \frac{2x-1}{4x^2-4x+2}$$ **2**

[CBSE Marking Scheme, 2018]

Detailed Solution :

Given $R = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$

Where $A = \{x \in Z : 0 \leq x \leq 12\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,\}$

1. Reflexivity : For any $a \in A$, we have $|a - a| = 0$ is divisible by 4.

$\Rightarrow (a, a) \in R$

Thus R is Reflexive.

2. Symmetry : Let $(a, b) \in R$ then $(a, b) \in R$

$\Rightarrow |a - b|$ is divisible by 4.

$\Rightarrow |b - a|$ is divisible by 4.

$\Rightarrow (b, a) \in R.$

Thus, $(a, b) \in R \Rightarrow (b, a) \in R$

$\therefore R$ is Symmetric.

3. Transitivity : Let $(a, b) \in R$ and $(b, c) \in R$ then $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow |a - b|$ is divisible by 4 and $|b - c|$ is divisible by 4.

$\Rightarrow |a - b| = 4\lambda$ and $|b - c| = 4\mu$ for some $\lambda, \mu \in N$

$\Rightarrow a - b = \pm 4\lambda$ and $b - c = \pm 4\mu.$

$\Rightarrow a - c = \pm 4\lambda + 4\mu$

$\Rightarrow |a - c|$ is divisible by 4.

$\Rightarrow (a, c) \in R$

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R.$

$\therefore R$ is transitive.

Hence, R is an equivalence Relation

Now, Let x be an element of A such that $x R 1$ or $(x, 1) \in R$

then, $|x - 1|$ is divisible by 4.

$\Rightarrow |x - 1| = 0, 4, 8, 12$

$\Rightarrow x - 1 = 0, 4, 8, 12$

$\Rightarrow x = 1, 5, 9\ (\because 13 \notin A)$

Hence, the set of all element which is related to 1 is $\{1, 5, 9\}$.

Also, equivalence class [2].

$= \{2, 6, 10\}.$

OR

Given, $f : 1R \to R; f(x) = \frac{x}{1+x^2}$

$g : 1R \to 1R; g(x) = 2x - 1.$

To show that f is neither one-one nor onto

(i) f is one-one : Let $x_1, x_2 \in R$ (domain)

and $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{1+x_1^2} = \frac{x_2}{1+x_2^2}$

$\Rightarrow x_1(1 + x_2^2) = x_2(1 + x_1^2)$

$\Rightarrow x_1 + x_1.x_2^2 - x_2 - x_2x_1^2 = 0$

$\Rightarrow (x_1 - x_2).(x_1.x_2 - 1) = 0$

Taking $x_1 = 4, x_2 = \frac{1}{4} \in R.$

$f(4) = f(x_1) = \frac{4}{17}$

$f(x_2) = f\left(\frac{1}{4}\right) = \frac{4}{17}$

$\therefore f$ is not one-one.

(ii) f is onto : Let $y \in R$ (co-domain)

$f(x) = y$

$\Rightarrow \frac{x}{1+x^2} = y \Rightarrow y.(1 + x^2) = x$

$\Rightarrow yx^2 + y - x = 0$

$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

since, $x \in R$, $\therefore$ $1 - 4y^2 \geq 0$

$\Rightarrow (1 + 2y)(1 - 2y) \geq 0$

$\therefore -\frac{1}{2} \leq y < \frac{1}{2}$

So Range $(f) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Range $(f) \neq R$ (Co-domain)

$\therefore f$ is not onto.

Hence f is neither one-one nor not.

Now, fog$(x) = f(g(x))$

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$$= \frac{2x-1}{(2x-1)^2+1} = \frac{2x-1}{4x^2+1-4x+1}$$

$$= \frac{2x-1}{4x^2-4x+2}$$

25. $|A| : -1 \neq 0 \therefore A^{-1}$ exists **1**

Co-factors of A are :

$$\left.\begin{array}{llll} A_{11} = 0; & A_{12} = 2; & A_{13} = 1 & \text{1 m for} \\ A_{21} = -1; & A_{22} = -9; & A_{23} = -5 & \text{4 correct} \\ A_{31} = 2; & A_{32} = 23; & A_{23} = 13 & \text{cofactors} \end{array}\right\} \quad \mathbf{2}$$

$$\text{adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\Rightarrow \qquad A^{-1} = \frac{1}{|A|}.adj(A)$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \qquad \tfrac{1}{2}$$

For, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$, the system of equation

is $A \cdot X = B$ ½

$$\therefore X = A^{-1} \cdot B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \mathbf{1}$$

$\therefore x = 1, y = 2, z = 3$ **1**

OR

Using elementary Row operations :

Let : $A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \mathbf{1}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \qquad \mathbf{1}$$

$\{$Using, $R_2 \to R_2 - 2R_1$; $R_3 \to R_3 + 2R_1\}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

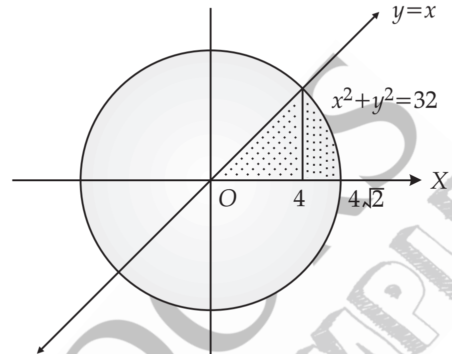
$\{$Using, $R_1 \to R_1 - 2R_2\}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A \qquad \mathbf{3}$$

$\{$Using, $R_1 \to R_1 - R_3$; $R_2 \to R_2 - R_3\}$

$$\therefore \qquad A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} \qquad \mathbf{1}$$

26. Correct figure : **1**



1

Point of intersection, $x = 4$ **1**

Area of shaded region $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$

1

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32-x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}} \qquad \mathbf{1}$$

$$= 8 + 16\frac{\pi}{2} - 8 - 4\pi = 4\pi \text{ Sq. units.} \qquad \mathbf{1}$$

27. Put $\sin x - \cos x = t$, $(\cos x + \sin x)\,dx = dt$, $1 - \sin 2x = t^2$ **1**

$$\left.\begin{array}{ll} \text{when} & x = 0, t = -1 \\ \text{and} & x = \pi/4, t = 0 \end{array}\right\} \qquad \tfrac{1}{2}$$

$$\therefore \qquad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x}\,dx$$

$$= \int_{-1}^{0} \frac{1}{16 + 9(1-t^2)}\,dt$$

$$= \int_{-1}^{0} \frac{1}{25 - 9t^2}\,dt \qquad \mathbf{2}$$

$$\Rightarrow \qquad I = \left[\frac{1}{30}\log\left|\frac{5+3t}{5-3t}\right|\right]_{-1}^{0} \qquad \mathbf{1\tfrac{1}{2}}$$

$$= \frac{1}{30}\left[0 - \log\frac{1}{4}\right]$$

$$= -\frac{1}{30}\log\frac{1}{4} \text{ or } \frac{1}{15}\log 2 \qquad \mathbf{1}$$

OR

Here, $f(x) = x^2 + 3x + e^x$, $a = 1$, $b = 3$, $nh = 2$

$$\therefore \int_1^3 (x^2 + 3x + e^x)\,dx \qquad \mathbf{1}$$

$$= \lim_{h \to 0}\left[f(1) + f(1+h) + \ldots + f(1 + \overline{n-1}h)\right] \qquad \mathbf{1}$$

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$$= \lim_{h\to 0}\left[4(nh)+\frac{(nh-h)(nh)(2nh-h)}{6} + \frac{5(nh-h)(nh)}{2}+\frac{h}{e^h-1}\times e\times(e^{nh}-1)\right]$$ **3**

$$= 8+\frac{8}{3}+10+e(e^2-1)$$

$$= \frac{62}{3}+e^3-e$$ **1**

[CBSE Marking Scheme, 2018]

Detailed Solution :

Given $\int_1^3 (x^2+3x+e^x).dx$

Let $I = \int_1^3 (x^2+3x+e^x)dx$

and $f(x) = x^2 + 3x + e^x$

$a = 1, b = 3$

$\therefore \quad h = \frac{b-a}{n} = \frac{2}{n}$

$I = \int_1^3 (x^2+3x+e^x).dx$

$= \lim_{h\to 0} h\ [f(1) + f(1 + h) + f(1 + 2h)$
$+ .... f(1 + (n-1)h]$

$= \lim_{h\to 0} h\ [4 + e + (1 + h)^2 + 3(1 + h) + e^{1+h}$
$+ (1 + 2h)^2 + 3.(1 + 2h) + e^{1+2h} + ...$
$+ (1 + (n-1)\ h)^2 + 3(1 + (n-1) - h) + e^{(1 + (n-1).h)}$

$= \lim_{h\to 0} h\ [4n + e + h^2(1^2 + 2^2 + 3^2 + ... + (n-1)^2]$
$+ 2h(1+ 2+3+........+(n-1) + 3h.(1 + 2+ 3 + ...$
$+ (n-1)) + e.(e^h + e^{2h} + e^{3h} + ... e^{(n-1).h}]$

$$= \lim_{h\to 0} h.\left[4n+e+h^2\frac{(n-1)(n-2)(2n-3)}{6} +2h\frac{n(n-1)}{2}+3h\frac{n(n-1)}{2}+e\frac{e^h[e^{h.(n-1)}-1]}{e^h-1}\right]$$

$$= 8+\frac{8}{3}+4+6+e^3-e$$

$$= \frac{62}{3}+e^3-e$$

28. General point on the line is : $(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$ **1½**

As the point lies on the plane

$\therefore 2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5 \Rightarrow \lambda = 0$ **1½**

$\therefore$ Point is $(2, -1, 2)$ **1**

Distance

$$= \sqrt{(2-(-1))^2+(-1-(-5))^2+(2-(-10))^2} = 13$$ **2**

[CBSE Marking Scheme, 2018]

Detailed Solution :

Here, the line

$$\vec{r} = 2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k}) \quad ...(1)$$

meet the plane $\vec{r}(\hat{i}-\hat{j}+\hat{k}) = 5$ ...(2)

Put the value of vector $\vec{r}$ from (1) in eq^n (2)

$[(2\hat{i}-\hat{j}+2\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})].(\hat{i}-\hat{j}+\hat{k}) = 5.$

or $[(2\hat{i}-\hat{j}+2\hat{k}).(\hat{i}-\hat{j}+\hat{k})+\lambda(3\hat{i}+4\hat{j}+2\hat{k})].(\hat{i}-\hat{j}+\hat{k})$
$= 5$

or $(2 + 1 + 2) + \lambda.(3 - 4 + 2) = 5$

or $\quad 5 + \lambda = 5 \Rightarrow \lambda = 0$

putting the value of λ in equation (i)

$$\vec{r} = (2\hat{i}-\hat{j}+2\hat{k})$$

$\therefore$ The points are $(2, -1, 2)$_ and $(-1, -5, -10)$

The distance between the points

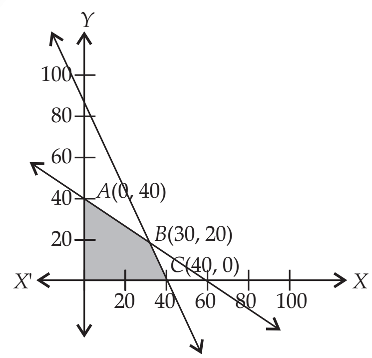
$(2, -1, 2)$ and $(-1, -5, 10)$ in

$= \sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$= \sqrt{9+16+144} = \sqrt{169}$

$= 13.$

29.



Let number of packets of type $A = x$

and number of packets of type $B = y$

$\therefore$ L.P.P. is: Maximize, $Z = 0.7x + y$ **1**

Subject to constraints:

$4x + 6y \le 240$ or $2x + 3y \le 120$
$6x + 3y \le 240$ or $2x + y \le 80$ **2**

$x \ge 0, y \ge 0$

Correct graph **2**

$Z(0, 0) = 0, Z(0, 40) = 40$

$Z(40, 0) = 28, Z(30, 20) = 41$ (Max.)

$\therefore$ Max. profit is ₹ 41 at $x = 30, y = 20.$ **1**

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