Unit -I : Number System

Chapter - 1 : Real Numbers

TOPIC-1
Rational Numbers

Quick Review

- **Rational Number :** A number '*r*' is called a rational number, if it can be written in the form *p*/*q*, where *p* and *q* are integers and $q \neq 0$, denoted by '*Q*'.

  For example : $\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, -\frac{2}{3}$ etc. are all rational numbers.

  Symbolically, $Q = \left\{\frac{p}{q}, q \neq 0 \text{ and } p, q \in I\right\}$

- **Decimal Expansion of Real Numbers :** The decimal expansion of real number is used to represent a number on the number line.

  If the decimal expansion of a real number is either terminating or non-terminating recurring, then the real number is called a rational number.

- **Cases in Rational Number :**

  Case 1 : When Remainder becomes Zero — Every rational number $\frac{p}{q}$, $(q \neq 0)$ can be expressed as a decimal. On dividing *p* by *q*, when the remainder becomes zero, then the decimal is called a terminating decimal.

  e.g. **: (i)** $\frac{1}{2} = 0.5$

  On dividing 1 by 2, we get value 0.5 *i.e.*, remainder equal to zero, so $\frac{1}{2}$ is a terminating decimal.

  (ii) $\frac{52}{100} = 0.52$

  On dividing 52 by 100, we get value 0.52 *i.e.*, remainder equal to zero, so $\frac{52}{100}$ is a terminating decimal.

  Case 2 : When remainder never becomes Zero — A rational number expressed in the form of *p*/*q* or division of *p* by *q*, when remainder never becomes zero and set of digits repeat periodically then the decimal is called non-terminating recurring or repeating decimal. It is denoted by the bar over it.

  e.g. : (i) $\frac{1}{3} = 0.333.... = 0.\overline{3}$

  On dividing 1 by 3, we get 3 again and again *i.e.*, remainder never becomes zero, so 1/3 is a repeating decimal.

  (ii) $\frac{3}{11} = 0.272727.... = 0.\overline{27}$

  On dividing 3 by 11, we get 27 again and again *i.e.*, remainder never becomes zero. So, 3/11 is a repeating decimal.

- Every integer is a rational number.
- There are infinitely many rational numbers between any two given rational numbers.
- If *x* and *y* are any two rational numbers, then :

  (i) $x + y$ is *a* rational number

  (ii) $x - y$ is *a* rational number

  (iii) $x \times y$ is *a* rational number

  (iv) $x \div y$ is *a* rational number, $(y \neq 0)$.

How it is done on GREENBOARD?

Q. Express $0.\overline{5}$ in the form of $\dfrac{p}{q}$.

Sol. : Step-I : Assume the given decimal expansion as x and count the number of digits which are repeated.

Let $x = 0.\overline{5}$

or $x = 0.555$.................. ...(i)

Thus 1 digit is repeated

Step-II : Multiply both sides by 10 (because one digit is repeating)

On multiplying eqn. (i) by 10, we get

$10x = 5.555$ ...(ii)

Step-III : Solving Eqn. (i) and (ii) we get the value of x subtracting eqn. (i) from (ii), we get

$10x - x = 5.555 - (0.555)$

or $9x = 5$

or $x = \dfrac{5}{9}$

Hence $0.\overline{5} = \dfrac{5}{9}$

TOPIC-2
Irrational Numbers

Quick Review

- If a number cannot be written in the form of p/q, where $q \neq 0$ and $p, q \in$ I, then it is called an irrational number.

  For example : $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{2}+\sqrt{5}, \sqrt{3}-\sqrt{7}, \pi$ etc. are all irrational number.
- The decimal expansion of an irrational number is non-terminating/non-recurring.

  If x and y are two real numbers where x is rational and y is an irrational, then

  (i) $x + y$ is an irrational number.

  (ii) $x - y$ is an irrational number.

  (iii) $x \times y$ is an irrational number.

  (iv) $x \div y$ is an irrational number.
- The addition, subtraction, multiplication and division of rational and irrational number is an irrational number.

How it is done on GREENBOARD?

Q. Locate $\sqrt{17}$ on the number line.

Sol. : Step-I : Write the given number (without root) as the sum of the squares of two natural numbers.

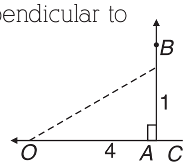
Here $17 = 16 + 1 = 4^2 + 1^2$

Step-II : Draw these two natural numbers on the number lines in which one is perpendicular to other

Draw OA = 4 unit

and AB = 1 unit

Such that AB $\perp$ OA



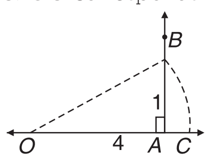
Step-III : By using Pythagoras theorem, find OB

$OB = \sqrt{OA^2 + AB^2} = \sqrt{4^2+1^2} = \sqrt{16+1} = \sqrt{17}$

Step-IV : Take O as centre and draw an arc of radius OB.

Having O as centre and radius OB, draw an arc, which cuts the number line at C. OC corresponds to $\sqrt{17}$.

Hence OC, represents $\sqrt{17}$.



TOPIC-3
n^{th} Root of a Real Number

Quick Review

- **Definition—** In $a^n = b$, a and b are real numbers and n is a positive integer,.
  - **(i)** So a is an n^{th} root of b.
  - **(ii)** It can also be written as $\sqrt[n]{b} = a$.
  - **(iii)** It is also known as a radical.

  Ex : (i) 3 is fourth root of 81 *i.e.,* $3^4 = 81$ *or* $3 = \sqrt[4]{81}$

  (ii) 2 is sixth root of 64 *i.e.,* $2^6 = 64$ *or* $2 = \sqrt[6]{64}$
- **Square root—** The "2^{nd}" root is the square root.
- **Cube root—** The "3^{rd}" root is the cube root.
- $\sqrt{a} \times \sqrt{a} = a$: Square root is used two times in a multiplication to get the original value.
- $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$: Cube root is used three times in a multiplication to get the original value.
- $\underbrace{\sqrt[n]{a} \times \sqrt[n]{a} \times \ldots\ldots\ldots \sqrt[n]{a} = a}_{n \text{ terms}}$ **:** The n^{th} root is used n times in a multiplication to get the original value.
- **Identities used for radicals :** Identities for two positive real numbers r and s :

  (i) $\sqrt{rs} = \sqrt{r} \cdot \sqrt{s}$ **(ii)** $\sqrt{\frac{r}{s}} = \frac{\sqrt{r}}{\sqrt{s}}$

  (iii) $\left(\sqrt{r} + \sqrt{s}\right)\left(\sqrt{r} - \sqrt{s}\right) = r - s$ **(iv)** $\left(r + \sqrt{s}\right)\left(r - \sqrt{s}\right) = r^2 - s$

  (v) $\left(\sqrt{r} - \sqrt{s}\right)^2 = r - 2\sqrt{r}\sqrt{s} + s$
- **Laws of radicals :** Laws for two positive real numbers a and b :

  (i) $\sqrt[n]{a^n} = a$ **(ii)** $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}}$

  (iii) $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$, ($a, b > 0$ be real number) **(iv)** $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

  (v) $\frac{\sqrt[p]{a^n}}{\sqrt[p]{a^m}} = \sqrt[p]{a^{n-m}}$ **(vi)** $\sqrt[p]{a^n \times a^m} = \sqrt[p]{a^{n+m}}$

  (vii) $\sqrt[p]{\left(a^n\right)^m} = \sqrt[p]{a^{nm}}$

  Example :

  (i) $\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = 2$

  (ii) $\left(\sqrt{2} + \sqrt{3}\right)\left(\sqrt{2} - \sqrt{3}\right) = \left(\sqrt{2}\right)^2 - \left(\sqrt{3}\right)^2 = 2 - 3 = -1$

  (iii) $\left(2 - \sqrt{2}\right)\left(2 - \sqrt{2}\right) = 2^2 - \left(\sqrt{2}\right)^2 = 4 - 2 = 2$

  (iv) $\left(\sqrt{5} + \sqrt{7}\right)^2 = \left(\sqrt{5}\right)^2 + \left(\sqrt{7}\right)^2 + 2\sqrt{5}\sqrt{7} = 5 + 7 + 2\sqrt{35} = 12 + 2\sqrt{35}$

  (v) $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

How it is done on GREENBOARD ?

Q. Simplify : $\sqrt[4]{81} - 8\cdot\sqrt[3]{216} + 15\cdot\sqrt[5]{32} + \sqrt{225}$

Sol. : Step-I : Write the exponents in the form of powers

i.e. $(81)^{\frac{1}{4}} - 8\cdot(216)^{\frac{1}{3}} + 15\cdot(32)^{\frac{1}{5}} + (225)^{\frac{1}{2}}$

Step-II : Factorise the radical

Here $(3^4)^{\frac{1}{4}} - 8\cdot(6^3)^{\frac{1}{3}} + 15\cdot(2^5)^{\frac{1}{5}} + (15^2)^{\frac{1}{2}}$

Step-III : Multiplying the powers

i.e. $(3)^{4\times\frac{1}{4}} - 8\cdot(6)^{3\times\frac{1}{3}} + 15\cdot(2)^{5\times\frac{1}{5}} + (15)^{2\times\frac{1}{2}}$

$= 3 - 8\ (6) + 15\ (2) + 15$

Step-IV : Solving the expression

$3 - 48 + 30 + 15 = 0$

TOPIC-4

Laws of Exponents with Integral Powers

Quick Review

➢ Let $a > 0$ be a real number and 'r' and 's' be rational number, then

(i) $a^r.a^s = a^{r+s}$

(ii) $(a^r)^s = a^{rs}$

(iii) $\dfrac{a^r}{a^s} = a^{r-s}, r > s$

(iv) $a^r\,b^r = (ab)^r$

(v) $a^{-r} = \dfrac{1}{a^r}$

(vi) $a^{\frac{r}{s}} = (a^r)^{1/s} = \left(a^{\frac{1}{s}}\right)^r$

(vii) $\left(\dfrac{a}{b}\right)^r = \dfrac{a^r}{b^r}$

(viii) $\left(\dfrac{a}{b}\right)^{-r} = \left(\dfrac{b}{a}\right)^r$

(ix) $a^0 = 1$

➢ **Example :**

(i) $(3)^4 \times (3)^3 = 3^{4+3} = 3^7$

(ii) $\dfrac{(4)^7}{(4)^2} = (4)^{7-2} = 4^5$

(iii) $(3)^2 \times (4)^2 = (12)^2$

(iv) $\left(\dfrac{3}{5}\right)^{-2} = \left(\dfrac{5}{3}\right)^2$

(v) $\left(\dfrac{1}{3}\right)^{-7} = 3^7$

(vi) $(9)^{-2} = \dfrac{1}{9^2}$

How it is done on GREENBOARD ?

Q. Find the value of $\dfrac{3^{40}+3^{39}+3^{38}}{3^{41}+3^{40}-3^{39}}$

Sol. : Step-I : Taking common factor from numerator and denominator as possible we can.

i.e. $\dfrac{3^{40}+3^{39}+3^{38}}{3^{41}+3^{40}-3^{39}} = \dfrac{3^{38}(3^2+3^1+1)}{3^{39}(3^2+3^1-1)}$

Step-II : Shifting the common factor which in denominator and solving the expression which are in bracket.

i.e. $\dfrac{3^{38-39}(9+3+1)}{(9+3-1)}$

Step-III : Solving the expression

i.e. $= \dfrac{3^{-1}\times 13}{11}$

$= \dfrac{13}{3\times 11} = \dfrac{13}{33}$

TOPIC-5
Rationalization of Real Numbers

Quick Review

- If a given number is transformed into an equivalent form, such that the denominator is a rational number then the process is known as Rationalization.
- **Rationalizing the denominator :** If the denominator of a fraction contains a term with root (a number under a radical sign), the process of converting it to an equivalent expression with rational denominator is called as rationalizing the denominator.
- To rationalize the denominator of $\frac{1}{\sqrt{r}+s}$, we multiply this by $\frac{\sqrt{r}-s}{\sqrt{r}-s}$, where r and s are integers .

Example : (i) $\frac{1}{5+\sqrt{3}} = \frac{1}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$

$$= \frac{5-\sqrt{3}}{(5)^2-(\sqrt{3})^2} = \frac{5-\sqrt{3}}{25-3} = \frac{5-\sqrt{3}}{22}.$$

Here, to rationalize the denominator of $\frac{1}{5+\sqrt{3}}$, we should multiply & divide it by $(5-\sqrt{3})$.

- List of Rationalization Factors.

Term	Rationalising Factor
$\frac{1}{\sqrt{r}}$	$\sqrt{r}$
$\frac{1}{\sqrt{r}-s}$	$\sqrt{r}+s$
$\frac{1}{\sqrt{r}+s}$	$\sqrt{r}-s$
$\frac{1}{\sqrt{r}-\sqrt{s}}$	$\sqrt{r}+\sqrt{s}$
$\frac{1}{\sqrt{r}+\sqrt{s}}$	$\sqrt{r}-\sqrt{s}$

Ex. Rationalize the denominator of $\frac{7}{\sqrt{5}-\sqrt{2}}$

Sol.: For rationalize the denominator, we will multiply the numerator and denominator by in conjugate to remove the radical sign from the denominator.

Step-I : Assume the given fraction as x and write the denominator.

Let $x = \frac{7}{\sqrt{5}-\sqrt{2}}$ and denominator $= \sqrt{5}-\sqrt{2}$

Step-II : Find the conjugate of denominator

Here the conjugate of denominator $(\sqrt{5}-\sqrt{2})$ is $(\sqrt{5}+\sqrt{2})$

Step-III : Multiply the numerator and denominator of x by the conjugate of denominator and rationalize it.

$$x = \frac{7}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$$

$$= \frac{7(\sqrt{5}+\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$= \frac{7(\sqrt{5}+\sqrt{2})}{5-2}$$

$$= \frac{7}{3}(\sqrt{5}+\sqrt{2})$$

Unit -II : Algebra

Chapter - 2 : Polynomials

TOPIC-1
Polynomials

Quick Review

- **Definition :** The algebraic expression in which the variables involved have only non-negative integral exponent is called '**Polynomial**'.
  A polynomial $p(x)$ in one variable x is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_2x^2 + a_1x + a_0$.
  where $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers and $a_n \neq 0$. $a_0, a_1, a_2, \ldots\ldots a_n$ are respectively the co-efficient of $x^0, x^1, x^2, \ldots\ldots, x^n$ and n is called the degree of the polynomial.
  This form of polynomial is known as the "Standard Form of Polynomial".
  e.g. : **(i)** $2x^3 - 4x^2 + 5x - 7$ is a polynomial in one variable (x).
  (ii) $3y^3 - 12y^2 + 7y - 9$ is a polynomial in one variable (y).
- **Constant Polynomial :** A polynomial of degree zero is called a constant polynomial.
  e.g. : $4, -\frac{7}{5}, \frac{3}{4}$ are constant polynomials.
- **Zero Polynomial :** Also, the constant polynomial is called the zero polynomial and the degree of the zero polynomial is not defined.
- **Degree of a Polynomial :** Highest power of variable in a polynomial is called the 'degree of polynomial'.
  In One Variable : In polynomial of one variable, the highest power of the variable is called the **degree** of the polynomial.
  e.g. : **(i)** $4x^7 - 3x^3 + 2x^2 + 3x - 6$ is a polynomial in x of degree 7.
  (ii) $\sqrt{3}\,x^2 + 3\sqrt{3}\,x + 3$ is a polynomial in x of degree 2.
  (iii) $\frac{3}{4}x^4 + \frac{2}{5}x^2 + 7x - 3$ is a polynomial in x of degree 4.

  In Two or More Variables : In a polynomial of more than one variable, the sum of the powers of the variable in each term is taken up and the highest sum so obtained is called the degree of the polynomial.
  e.g., : **(i)** $7x^3 - 4x^2y^2 + 3x^2y - 3y + 9$ is a polynomial in x and y of degree 4.
  (ii) $\sqrt{3}\,x^5 - 4y^3 + 7x^3y + 2x - 3$ is a polynomial in x and y of degree 5.
- **Types of Polynomials :**
  On the Basis of Terms in a Polynomial.
  Term : In a polynomial $x^2 + 3x + 4$, the expression x^2, $3x$ and 4 are called terms.
  e.g., **(i)** $x^2 + 3x + 7$. It has three terms.
  (ii) $x^2 - 4$. It has two terms.
  (I) Monomial : A polynomial of one non-zero term, is called a monomial.
  e.g., $2x, -4x^2, 7x^3, 10x$ are monomial.
  (II) Binomial : A polynomial of two non-zero terms, is called a binomial.
  e.g., $(4x^2 + 8), (7y^2 - 3y), (3x - 6), (10x^2 - 4)$ are binomial.
  (III) Trinomial : A polynomial of three non-zero terms, is called a trinomial.
  e.g., $(x^2 + 2x + 4), (4x^2 + \frac{7}{5}x + 14), (3x^2 + 3\sqrt{3}\,x + \sqrt{3})$ are trinomial.

  On the Basis of Degree in a Polynomial.
  (I) Linear Polynomial : A polynomial of degree 1 is called a linear polynomial. It is expressed in the form of $ax + b$, where a & b are real constants and $a \neq 0$.
  e.g., **(i)** $3x + 6$ is a linear polynomial in x.
  (ii) $\sqrt{3}\,x - 3$ is a linear polynomial in x.
  (iii) $\frac{7}{5}y - 10$ is a linear polynomial in y.

(II) Quadratic Polynomial : A polynomial of degree 2 is called a quadratic polynomial. It is expressed in the form of $ax^2 + bx + c$, where a, b & c are real constant and $a \neq 0$.

e.g., **(i)** $3x^2 + 4x + 1$ is a quadratic polynomial in x.

(ii) $\sqrt{3}\,x^2 + 3x + 3\sqrt{3}$ is a quadratic polynomial in x.

(iii) $\frac{7}{5}y^2 + 3y - 1$ is a quadratic polynomial in y.

(III) Cubic Polynomial : A polynomial of degree 3 is called a cubic polynomial. It is expressed in the form of $ax^3 + bx^2 + cx + d$, where a, b, c & d are real constants and $a \neq 0$.

e.g., **(i)** $2x^3 - 4x^2 + 8x - 3$ is a cubic polynomial in x.

(ii) $4y^3 + 3y^2 + 7y - 14$ is a cubic polynomial in y.

➢ **Zeroes of a Polynomial :**

Zero of a polynomial $p(x)$ is a number c such that $p(c) = 0$.

(i) '0' may be a zero of a polynomial.

(ii) Every linear polynomial in one variable has a unique zero of a polynomial.

(iii) A non-zero constant polynomial has no zero of a polynomial.

(iv) Every real number is a zero of the zero polynomial.

(v) Maximum number of zeroes of a polynomial is equal to its degree.

e.g., **(i) Find whether –1 and 1 are the zeroes of the polynomial $x - 1$.**

Solution : Let $p(x) = x - 1$

Then $p(1) = 1 - 1 = 0$, $p(-1) = -1 - 1 = -2$

Therefore, 1 is a zero of the polynomial, but –1 is not.

(ii) Find the zero of the polynomial $p(x) = 3x + 1$.

Solution : Finding the zero of $p(x)$, is the same as solving the equation

$$p(x) = 0$$

Now, $3x + 1 = 0$, given up $x = \frac{-1}{3}$. So, $\frac{-1}{3}$ is the zero of the polynomial $3x + 1$.

How it is done on GREENBOARD?

Q. Verify whether 3 and 0 are zeroes of polynomial $x^2 - 3x$.

Sol. : Step-I : Write the given polynomial equal to $p(x)$

$$p(x) = x^2 - 3x \quad ...(i)$$

Step-II : Putting 3 in place of x and solve it to find the value of x. If this value of x is zero, then 3 will be zero of the given polynomial.

Putting $x = 3$ in (i), we get

$$p(3) = (3)^2 - 3(3) = 9 - 9 = 0$$

Step-III : Similarity putting 0 in place of x and solving it

Putting $x = 0$ in (i) we get

$$p(0) = 0^2 - 3(0) = 0$$

So, 3 and 0 are zeros of given polynomial $p(x)$.

TOPIC-2
Remainder Theorem

Quick Review

➢ If $p(x)$ be any polynomial of degree $n \geq 1$ and a be any real number, then if $p(x)$ is divided by the linear polynomial $(x - a)$, the remainder is $p(a)$, is called the Remainder Theorem.

Division Algorithm For Polynomials : If $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$, in the simple words :

Dividend = Divisor × Quotient + Remainder

Example 1. Divide $p(x)$ by $g(x)$,

where, $p(x) = 3x^2 + x - 1$

and, $g(x), = x + 1$

Solution :

$$
\begin{array}{r|l}
 & \;\;3x-2 \\
x+1 & 3x^2+x-1 \\
 & 3x^2+3x \\
 & \;-\quad\;- \\ \hline
 & \quad -2x-1 \\
 & \quad -2x-2 \\
 & \quad\;\, +\quad + \\ \hline
 & \qquad\quad 1
\end{array}
$$

$\therefore \quad 3x^2 + x - 1 = (x + 1)(3x - 2) + 1$

i.e., Dividend = (Divisor × Quotient) + Remainder

Here, Dividend = $3x^2 + x - 1$, Divisor = $x + 1$, Quotient = $3x - 2$, Remainder = 1.

Example : Divide $p(x) = x^3 + 1$ by $x + 1$.

Solution :

$$
\begin{array}{r|l}
 & x^2-x+1 \\
x+1 & x^3+1 \\
 & x^3+x^2 \\
 & (-)\;(-) \\ \hline
 & \quad -x^2+1 \\
 & \quad -x^2-x \\
 & \quad (+)\;(+) \\ \hline
 & \qquad\quad x+1 \\
 & \qquad\quad x+1 \\
 & \qquad\; (-)\,(-) \\ \hline
 & \qquad\qquad 0
\end{array}
$$

$\therefore \; x^3 + 1 = (x+1)(x^2 - x + 1)$

Here, Dividend = $x^3 + 1$, Divisor = $x + 1$, Quotient = $x^2 - x + 1$, Remainder = 0.

Q. Using remainder theorem, find the remainder when $x^4 + x^3 - 2x^2 + x + 1$ is divided by $x - 1$.

Sol. : Step-I : Consider the given polynomials as $p(x)$ and $g(x)$

Given polynomial is $p(x) = x^4 + x^3 - 2x^2 + x + 1$ ...(i)

and $g(x) = x - 1$

Step-II : Find the zero $g(x)$ by which we have to divide the polynomial $p(x)$.

Putting $g(x) = 0$, to get an equation *i.e.*,

$g(x) = 0$

or, $x = 1$

So it is zero of $g(x)$

Step-III : Put the value of zero of $g(x)$ in the polynomial $p(x)$ which have to be divided by $g(x)$.

On putting $x = 1$ in eqn. (i) we get

$$
\begin{aligned}
p(1) &= (1)^4 + (1)^3 - 2(1)^2 + 1 + 1 \\
&= 1 + 1 - 2 + 1 + 1 \\
&= 2
\end{aligned}
$$

Here the value of $p(1)$ is 2 which is the required remainder obtained by dividing $x^4 + x^3 - 2x^2 + x + 1$ by $(x - 1)$

TOPIC-3
Factor Theorem

Quick Review

- If $p(x)$ is a polynomial of degree $x \geq 1$ and a is any real number, then
  - **(i)** Graph of linear equation is a straight line, while graph of quadratic equation is a parabola.
  - **(ii)** Degree of polynomial = Number of zeroes of polynomial.
  - **(iii)** If remainder $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.
  - **(iv)** $(x + a)$ is a factor of polynomial $p(x)$, if $p(-a) = 0$.
  - **(v)** $(x - a)$ is a factor of polynomial $p(x)$, if $p(a) = 0$.
  - **(vi)** $(x - a)(x - b)$ is a factor of polynomial $p(x)$, if $p(a) = 0$ and $p(b) = 0$
  - **(vii)** $(ax + b)$ is a factor of polynomial $p(x)$, if $p(-b/a) = 0$
  - **(viii)** $(ax - b)$ is a factor of polynomial $p(x)$, if $p(b/a) = 0$

Factorization of a Polynomial :

(i) By Splitting the Middle Term :

Let a quadratic polynomial be $x^2 + lx + m$, where l and m are constants.

Factorise the polynomial by splitting the middle term lx as $ax + bx$, so that $ab = m$. Then,

$$\begin{aligned} x^2 + lx + m &= x^2 + ax + bx + ab \\ &= (x + a)(x + b). \end{aligned}$$

(ii) By Using Factor Theorem :

Consider a quadratic polynomial $ax^2 + bx + c$, where a, b and c are constants. It has two factors $(x - \alpha)$ and $(x - \beta)$.

$\therefore\ ax^2 + bx + c = a(x - \alpha)(x + \beta)$

or, $ax^2 + bx + c = ax^2 - a(\alpha + \beta)x + a\alpha\beta$

On equating the co-efficient of x and constant term, we get $\alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$.

On simplifying, we get the value of α and β.

Ex : Factorise $6x^2 + 17x + 5$ by splitting the middle term, and by using the factor theorem.

Solution :

By Splitting the Method :

If we find the two numbers a and b, such that $a + b = 17$ and $ab = 6 \times 5 = 30$, then we can get the factors. So, factors of 30 are 1 and 30, 2 and 15, 3 and 10, 5 and 6. Of these pairs, 2 and 15 will give us $a + b = 17$.

$$\begin{aligned} \text{So,}\quad 6x^2 + 17x + 5 &= 6x^2 + (2 + 15)x + 5 = 6x^2 + 2x + 15x + 5 \\ &= 2x(3x + 1) + 5(3x + 1) \\ &= (2x + 5)(3x + 1). \end{aligned}$$

By Factor Theorem :

$6x^2 + 17x + 5 = 6\left(x^2 + \dfrac{17}{6}x + \dfrac{5}{6}\right) = 6p(x)$, say. If a and b are the zeroes of $p(x)$.

So, $6x^2 + 17x + 5 = 6(x - a)(x - b)$, then $ab = \dfrac{5}{6}$.

Then, see some possibilities for a and b.

They could be $\pm\dfrac{1}{2}, \pm\dfrac{1}{3}, \pm\dfrac{5}{3}, \pm\dfrac{5}{2}, \pm 1.$

Now, $p\left(\dfrac{1}{2}\right) = \dfrac{1}{4} + \dfrac{17}{6}\left(\dfrac{1}{2}\right) + \dfrac{5}{6} \neq 0,$

But, $p\left(\dfrac{-1}{3}\right) = 0$. So, $\left(x + \dfrac{1}{3}\right)$ is a factor of $p(x)$,

Similarly, we will get $\left(x + \dfrac{5}{2}\right)$ as a factor of $p(x)$.

$$\begin{aligned} \therefore\quad 6x^2 + 17x + 5 &= 6\left(x + \frac{1}{3}\right)\left(x + \frac{5}{2}\right) \\ &= 6\left(\frac{3x+1}{3}\right)\left(\frac{2x+5}{2}\right) = (3x + 1)(2x + 5). \end{aligned}$$

In this example, the use of the splitting method appears more efficient.

How it is done on GREENBOARD ?

Q. Factorise the cubic polynomial $x^3 + 6x^2 + 11x + 6$.

Sol. : Step-I : Consider the given cubic polynomial as $p(x)$ and find the constant term

$$p(x) = x^3 + 6x^2 + 11x + 6$$

Here, Constant term $= 6$

Step-II : Find all the factors of constant term of $p(x)$

All possible factor of 6 are $\pm 1, \pm 2, \pm 3$, and ± 6

Step-III : Check at which factor, $p(x)$, is zero by trial method and get one factor of $p(x)$.

At $x = -1$

$$\begin{aligned} p(-1) &= (-1)^3 + 6(-1)^2 + 11(-1) + 6 \\ &= -1 + 6 - 11 + 6 \\ &= -12 + 12 = 0 \end{aligned}$$

So, $(x + 1)$ is a factor of $p(x)$

Step-IV : Now, write $p(x)$ as the product of this factor and a quadratic polynomial.

On dividing $p(x)$ by $(x + 1)$, we get quotient $x^2 + 5x + 6$

So, $p(x) = (x + 1)(x^2 + 5x + 6)$

Step-V : Now, use splitting method or factor theorem to find the factor of $p(x)$.

Now, by splitting the middle term, we get

$$\begin{aligned} p(x) &= (x + 1)[x^2 + 3x + 2x + 6] \\ &= (x + 1)[x(x + 3) + 2(x + 3)] \\ &= (x + 1)(x + 2)(x + 3) \end{aligned}$$

Hence, the factor of given polynomial are $(x + 1)$, $(x + 2)$ and $(x + 3)$

TOPIC-4
Algebraic Identities

Quick Review

- **Algebraic Identities :**
  An algebraic identity is an algebraic equation that is true for all values of the variables occurring in it.
- **Some useful algebraic identities are given below :**

(i) $(x + y)^2 = x^2 + 2xy + y^2$

(ii) $(x - y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x + y)(x - y)$

(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

(vii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

(viii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

(ix) If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

(x) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(xi) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Ex : Factorize the following :

(i) $x^2 + 4xy + 4y^2$　　**(ii)** $x^3 - 8$

(iii) $4x^2 - 12xy + 9y^2$　　**(iv)** $x^3 + 8y^3 + 6x^2y + 12xy^2$

Sol.

(i) $x^2 + 4xy + 4y^2 = (x + 2y)^2 = (x + 2y)(x + 2y)$

(ii) $x^3 - 8 = x^3 - 2^3 = (x - 2)(x^2 + 2x + 2^2)$

$= (x - 2)(x^2 + 2x + 4)$

(iii) $4x^2 - 12xy + 9y^2 = (2x)^2 - 2(2x)(3y) + (3y)^2$

$= (2x - 3y)^2 = (2x - 3y)(2x - 3y)$

(iv) $x^3 + 8y^3 + 6x^2y + 12xy^2$

$= (x)^3 + (2y)^3 + 3(x)(2y)[x + 2y]$

$= (x + 2y)^3 = (x + 2y)(x + 2y)(x + 2y)$

How it is done on GREENBOARD ?

Q. Evaluate $(102)^3$ by using suitable identities.

Sol. : Step-I : Express the given number without power as the sum or difference of two numbers

Given number without power is 102. Since it is greater than 100, so it can be written as 100 + 2

$\therefore \quad (102)^3 = (100 + 2)^3$

Step-II : Compare the expression with $(x + y)^3$

On comparing $(100 + 2)^3$ with $(x + y)^3$, we get

$x = 100$ and $y = 2$

Step-III : Use the identity $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ to expand it

By using the identity

$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$, we get

$(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

Step-IV : Simplify the above expression.

$(102)^3 = 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200$

$= 1061208$

Chapter - 3 : Linear Equations in Two Variables

TOPIC-1
Introduction of Linear Equation

Quick Review

- In earlier classes, we have studied linear equations in one variable like x + 1 = 0, $x + \sqrt{2} = 0$ etc. We know that such equations have unique solution and solution of these type of equations can be represented on number line.
- **Linear equation in two variables :**
  An equation of the form ax + by + c = 0, where a, b and c are real numbers, such that a and b are both non zero, is called a linear equation in two variables.
  e.g. x + y = 16, p + 4q = 7, $3 = \sqrt{7}x - y$ and 2l + m = 3
  All are linear equations in two variables.
- **Solution of linear equation in Two Variables :**
  Any pair of values of x and y which satisfies the equation ax + by + c = 0, is called its solution. This solution can be written as an ordered pair (x, y), first writing value of x and then value of y.
- **Linear equation in two variable has infinitely many solutions.**
  For finding the solution of linear equation in two variables (i.e. ax + by + c = 0), we use following steps :
  Step 1— Write the given equation in two variables , if not present.
  Step 2— Put an arbitrary value (for convenience put x = 0 or y = 0) of x (or y) in the given equation and then it reduces into linear equation of one variable, which gives a unique solution. Thus, we get one pair of solution of given equation.
  Step 3— Repeat step 2 for another arbitrary value of x (or y) and get another pair of solution of given equation.

How it is done on GREENBOARD?

Q. Find four different solutions of the equation $2x + y = 7$.

Sol. **Step I :** Write the given linear equation.

Given, linear equation in two variables is

$$2x + y = 7 \qquad \text{.... (i)}$$

Step II : Put on arbitrary value of x (or y) in the given equation and find corresponding value of y (or x).

or, On putting $x = 0$ in eq. (i), we get

$$2(0) + y = 7$$

or, $y = 7$

So, (0, 7) is a solution of the given equation **1**

or, On putting $y = 0$ in eq. (i), we get

$$2x + 0 = 7$$

or, $x = \frac{7}{2}$

So, $(\frac{7}{2}, 0)$ is also a solution of the given equation.

Step III : Repeat step 2 for other solutions.

or, On putting $x = 1$ in eq. (i), we get

$$2(1) + y = 7$$

or, $y = 5$

So, (1, 5) is also a solution of the given equation. **1**

or, On putting $y = 1$ in eq. (i), we get

$$2x + 1 = 7$$

or, $2x = 6$

or, $x = 3$

So, (3, 1) is also a solution of the given equation. **1**

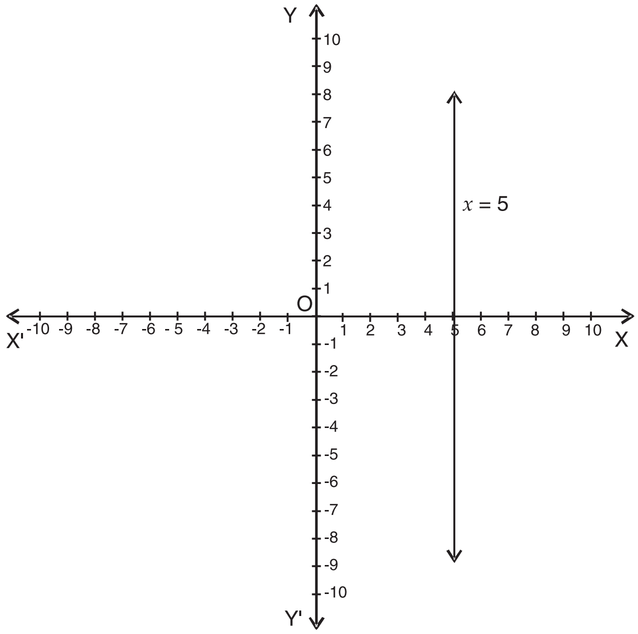
Step IV : Write all the solutions.

(0, 7), $(\frac{7}{2}, 0)$, (1, 5) and (3, 1) are four solutions of the given equation.

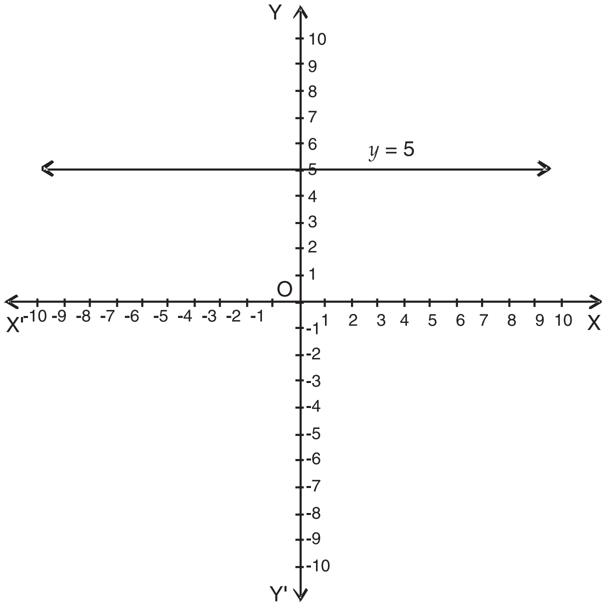
TOPIC-2
Graphical Representation of Linear Equation in Two Variables

Quick Review

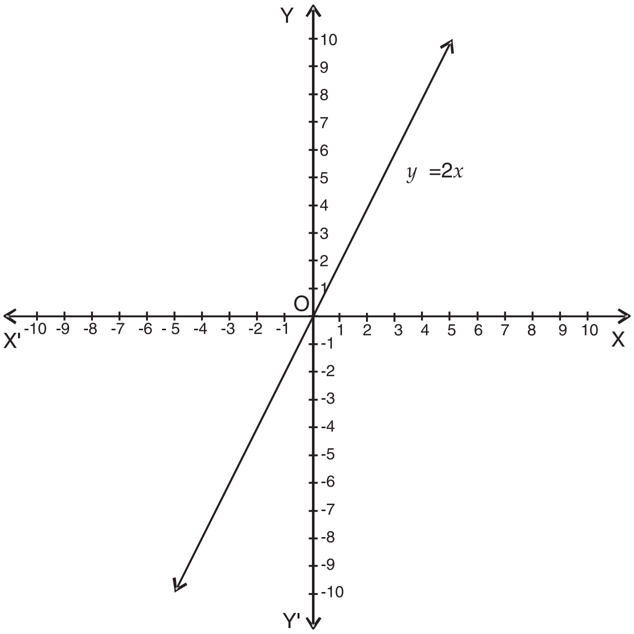
- An equation of the form $ax + by + c = 0$, where a, b and c are real numbers, such that a and b are both non-zero, is called a linear equation in two variables.
- A linear equation in two variables is represented graphically by a straight line, the points of which make up the collection of solutions of equation. This is called the graph of the linear equation.
- A linear equation in two variables has infinitely many solutions.
  The graph of every linear equation in two variables is a straight line.
- $x = 0$ is the equation of the y-axis and $y = 0$ is the equation of the x-axis.
- The graph of $x = k$ is a straight line parallel to the y-axis.
  (As shown in figure) Here, $k = 5$



- The graph of $y = k$ is a straight line parallel to the x-axis. (As shown in figure)
  Here, $k = 5$



- An equation of the type $y = mx$ represents a line passing through the origin, where m is a real number and is called slope of the line.
  (As shown in figure) Here, $m = 2$



- Every point on the line satisfies the equation of the line and every solution of the equation is a point on the line.

Method of plotting the graph of linear equation in two variables :

Let the linear equation in two variables be $ax + by + c = 0$, where $a \neq 0$ and $b \neq 0$. Then, to draw its graph, we use the following steps :

Step 1 : Write the given linear equation and express y in terms of x.

i.e. $by = -(ax + c)$

or, $y = \dfrac{-(ax+c)}{b}$ ...(i)

Step 2 : Put the different arbitrary values of x in *eq.* (i) and find the corresponding values of y.

Step 3 : Form a table as following, by writing the values of y below the corresponding values of x.

x	x_1	x_2	x_3	x_4
y	y_1	y_2	y_3	y_4

Step 4 : Draw the co-ordinate axes on graph paper and take a suitable scale to plot points (x_1, y_1), (x_2, y_2), (x_3, y_3) .... on graph paper.

Step 5 : Join the points (x_1, y_1), (x_2, y_2), (x_3, y_3) ...... by a straight line and produce it on both sides.

Hence, the line so obtained is the required graph of the given linear equation.

Note : It is advisable to choose integral values of x for step 2 in such a way that the corresponding values of y are also integers.

Q. Draw the graph of the equation $2x + y = 3$.

Sol. **Step I :** Write given linear equation and express in terms of x.

Given linear equation is $2x + y = 3$

or, $y = 3 - 2x$ .....(i)

Step II : Put different arbitrary values of x and find the corresponding values of y.

when $x = 0$, then $y = 3 - 2(0) = 3$

when $x = 1$, then $y = 3 - 2(1) = 1$

when $x = -1$, then $y = 3 - 2(-1) = 5$ 2

Step III : Form the table.

From the above step, we get the following table :

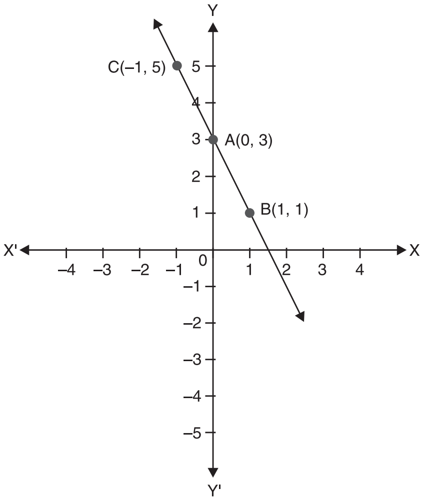
x	0	1	-1
y	3	1	5

Step IV : Draw the co-ordinate axes and plot the points on graph paper.

Draw the co-ordinate axes XOX' and YOY' and plot the points A(0, 3), B(1, 1) and C(-1, 5) by taking suitable scale.

Step V : Join the points by a straight line.

On joining the points A, B and C, we get a straight line BC and produce it on both sides. Thus, the line BC represents the required graph of given linear equation in two variables.



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Hence, line BC represents the required graph of given linear equation.

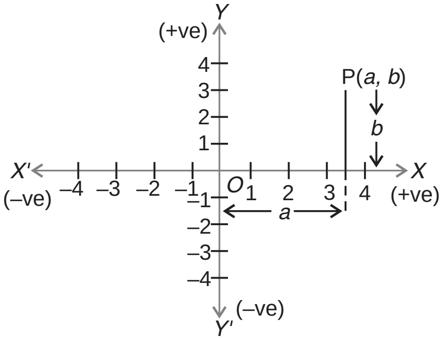
Unit -III : Coordinate Geometry

Chapter - 4 : Coordinate Geometry

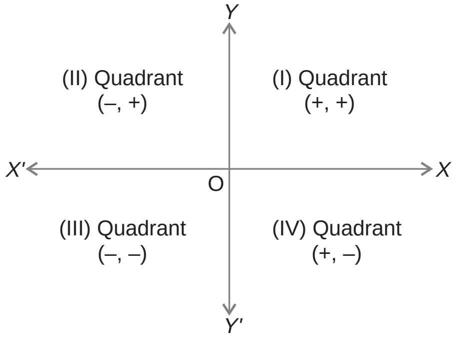
TOPIC-1
Cartesian System

Quick Review

- **Cartesian System :** The system by which we can describe the position of a point in a plane is called Cartesian system.
- In Cartesian System, two mutually perpendicular lines (one horizontal and other vertical) are required to locate the position of a point or an object.
- The plane is called the Cartesian or co-ordinate plane and the lines are called the co-ordinate axes.
- The horizontal line XOX' is called the X–axis and vertical line YOY' is called Y–axis.



- The point where XOX' and YOY' intersect is called the origin, and is denoted by O.
- Location of a point P in cartesian system, written in the form of ordered pair say $P(a, b)$ above figure.
- a is the length of perpendicular of $P(a, b)$ from y-axis and is called abscissa of P.
- b is the length of perpendicular of $P(a, b)$ from x-axis and is called co-ordinate of P.
- Positive numbers lie on the directions OX and OY, OX and OY are called the positive directions of the x–axis and the y-axis, respectively. Similarly, OX' and OY' are called the negative directions of the x-axis and the y-axis respectively.
- The co-ordinate axes divide the plane into four parts called quadrants (one-fourth part) numbered I, II, III and IV anti-clockwise from OX.



How to write the co-ordinates of a point :

- x – co-ordinate (or abscissa) = perpendicular distance of a point from y-axis.
- y – co-ordinate (or ordinate) = perpendicular distance of a point from x-axis.
- If abscissa of a point is x and ordinate is y, then the co-ordinates of the point are (x, y).
- The abscissa of every point on y-axis is zero.
- The ordinate of every point on x-axis is zero.
- Co-ordinate of a point or X-axis are of the form $(x, 0)$.
- Co-ordinate of a point on Y-axis are of the form of $(0, y)$.
- x-axis and y-axis intersect at origin, represented by O and its co-ordinates are (0, 0).

Ex. On which axes do the given points lie.

A (0, 2), B(– 3, 0), C(0, – 3), D(0, 4), E(6, 0), F(3, 0).

Solution : On x-axis : B(–3, 0), E(6, 0), F(3, 0)

On y-axis : A(0, 2), C(0, – 3), D(0, 4)

How it is done on GREENBOARD?

Q. Write the quadrant in which each of the following points lie :

(i) (– 2, – 3), **(ii)** (3, – 4), **(iii)** (– 1, 2)

Sol. If both (abscissa and ordinate) co-ordinates are positive, then points lie in I quadrant. If abscissa is negative and ordinate is positive, then point lies in II quadrant. If both co-ordinates are negative then point lies in III quadrant. If abscissa is positive and ordinate is negative, then point lies in IV quadrant.

Hence (i) the point (– 2, – 3) lies in III quadrant because its both co-ordinates are negative.

(ii) The point (3, – 4) lies in IV quadrant because its x-coordinate is positive and y-coordinate is negative.

(iii) The point (– 1, 2) lies in II quadrant because its x-coordinate is negative and y-coordinate is positive.

TOPIC-2
Plotting a Point in a Plane

Quick Review

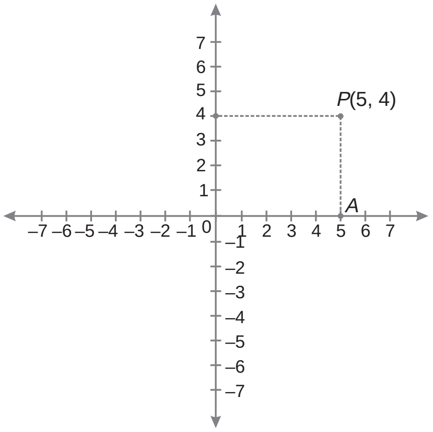
- **Plotting a Point in the plane if its Co-ordinates are given :** Let us suppose the co-ordinates of a point be (5, 4). To plot this point in the co-ordinate plane, following steps are followed :

  Step–1 : Draw the co-ordinate axis and choose units such that one centimetre represents one unit on both the axes.

  Step–2 : Starting from the origin O, count the 5 units on the positive x-axis and mark the corresponding point as A.

  Step–3 : Starting from the point A, count the 4 units on the positive y-axis and mark the corresponding point as P.

  Step–4 : Point P is the position of the point (5, 4), as distance of point P from y-axis is 5 units and distance from x-axis is 4 units.

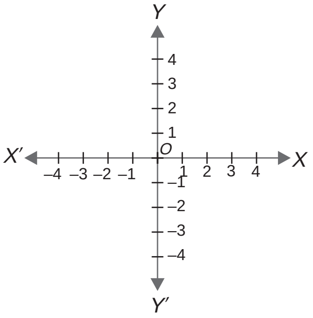


How it is done on GREENBOARD ?

Q. Plot the points (3, 4) and (– 3, 4) on a graph paper

Sol. : Step I : Draw the co-ordinate axes and write unit as both the axes at the same distance as 1 unit on all four directions of both axes.

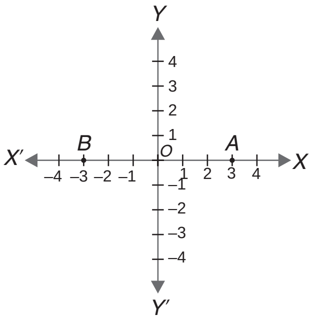
Here XOX' = X axis and YOY' = Y-axis.



Step II : Find the direction and distance of given points corresponding to axes.

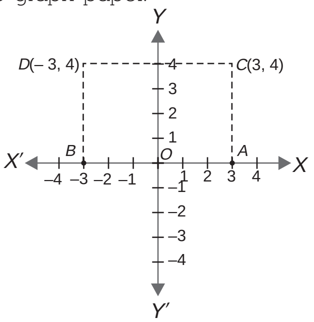
Here the co-ordinates of the point (3, 4) shows that the distance of this point from Y-axis is 3 unit in positive direction and from X-axis is 4 unit in positive direction. Similarity, co-ordinates of the point (– 3, 4) show that distance of this point from y-axis 3 unit in negative direction and from x-axis is 4 unit in positive direction

Step III : Show the x-coordinates of both points. Count 3 unit from origin on the positive x-axis and Mark the point as A. Also count 3 unit from origin on the negative x-axis and mark it B.



Step IV : Show the y-coordinates of both point Start from the point A count 4 units in the positive direction of y-axis and mark it C. Similarity from point B count 4 unit in the positive direction of y-axis and mark it D.

Hence C is the point (3, 4) and D is the point (– 3, 4) on the graph paper.



TOPIC-3
Graph of Linear Equations

Quick Review

➢ **Graph of Linear Equations :** Suppose the Linear equation $ax + by + c = 0$, where a, b are constants.

To draw a line in a co-ordinate plane following steps are followed :

Step–1 : Convert the given equation in the form of $y = mx + c$, *i.e.*, make y as the subject of the formula.

Step–2 : Select at least three values of x, such that $x, y, \in I$.

Step–3 : Draw a table for the ordered pair (x, y).

Step–4 : Plot these ordered pairs (points) on a graph paper selecting the suitable scale.

Step–5 : Draw a straight line passing through the points plotted on the graph paper.

Ex. Draw the graph of the equation : $y = 2x – 2$.

Solution : The given equation is $y = 2x–2$.

Putting $x = 0$, we get $y = (2 \times 0) – 2 = – 2$

Putting $x = 1$, we get $y = (2 \times 1) – 2 = 0$

Putting $x = 2$, we get $y = (2 \times 2) – 2 = 2$

So, the table is :

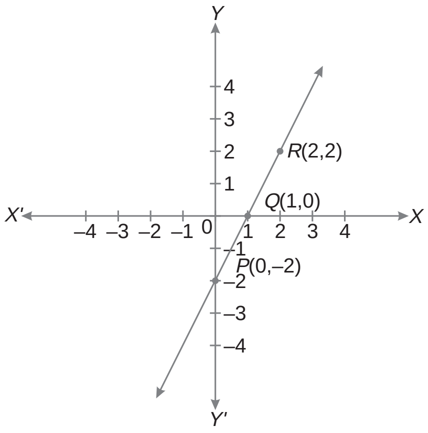
x	0	1	2
y	–2	0	2

On the graph paper, draw the co-ordinate axes.

Now, plot the points $P(0, -2)$, $Q(1, 0)$ and $R(2, 2)$ on the graph paper.

Join P, Q, R and extend it in both the directions.

Then, line PR is the graph of the equation $y = 2x - 2$.



How it is done on GREENBOARD?

Q. Draw the graph of the equation $y = 2x - 1$

Sol. : Step I : Convert the given equation in the form of $y = mx + c$ if it is not in this form

$$y = 2x - 1 \quad \text{...(i)}$$

Step II : Select at least three values of x and put these values in equation (i) and set the corresponding values of y.

Putting $x = 0$, 1 and 2 in eqn. (i) we get

Putting $x = 0$, $y = (2 \times 0) - 1 = -1$

Putting $x = 1$, $y = (2 \times 1) - 1 = 2 - 1 = 1$

Putting $x = 2$, $y = (2 \times 2) - 1 = 3$

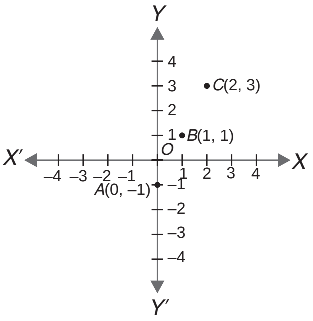
Step III : Draw a table for the ordered pair (x, y)

So, the table is

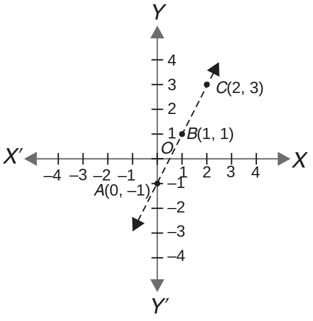
x	0	1	2
y	– 1	1	3

Step IV : Plot these ordered pair (points) on a graph paper selecting suitable scale

Now plot the points $A(0, -1)$, $B(1, 1)$, $C(2, 3)$ on the graph paper



Step V : Draw a straight line passing through the points plotted on the graph paper.



Unit -IV : Geometry

Chapter - 5 : Introduction to Euclid's Geometry

TOPIC-1
Euclid's Geometry

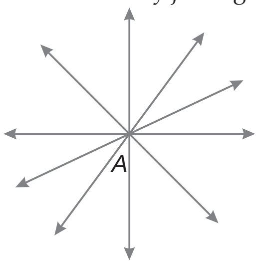
Quick Review

- **Axiom :** Axiom are the assumptions which are obvious universal truths. They are not proved.
  Euclid's Axioms :
- Things which are equal to the same thing are equal to one another.
  e.g. If $\overrightarrow{AB} = \overrightarrow{PQ}$ and $\overrightarrow{PQ} = \overrightarrow{XY}$, then $\overrightarrow{AB} = \overrightarrow{XY}$.
- If equals are added to equals, the wholes are equal.
  e.g., If $M\angle 1 = M\angle 2$, then
  $M\angle 1 + M\angle 3 = M\angle 2 + M\angle 3$.
- If equals are subtracted from equals, the remainders are equal.
  e.g., If $M\angle 1 = M\angle 2$, then
  $M\angle 1 - M\angle 3 = M\angle 2 - M\angle 3$.
- Things which coincide with one another are equal to one another.
  e.g., If $\overrightarrow{AB}$ coincides with $\overrightarrow{XY}$, such that A falls on X and B falls on Y, then $\overrightarrow{AB} = \overrightarrow{XY}$
- The whole is greater than the part.
  e.g., If $M\angle 1 = M\angle 2 + M\angle 3$, then $M\angle 1 > M\angle 2$ & $M\angle 1 > M\angle 3$.
- Things which are double of the same thing are equal to one another.
  e.g., If $a = 2c$ and $b = 2c$, then $a = b$.
- Things which are halves of the same thing are equal to one another.
  e.g., If $a = \frac{c}{2}$ and $b = \frac{c}{2}$, then $a = b$

TOPIC-2
Euclid's Postulates

Quick Review

- **Postulates :** The basic facts which are taken for granted, without proof and which are specific to geometry are called **postulates**.
- **Plane :** A plane is a surface such that the line obtained by joining any two points in it will be entirely in the plane.



- **Incidence Axioms on lines :**

  (i) A line contains infinitely many points.

  (ii) Through a given point A, infinitely many lines can be drawn.

  (iii) One and only one line can be drawn to pass through two given points A and B.

- **Collinear points :** Three or more points are said to be **collinear**, if there is a straight line which passes through all of them.

  In figure I; A, B, C are collinear points, while in figure II; P, Q, R are non-collinear.

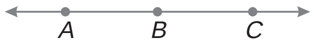


Figure I

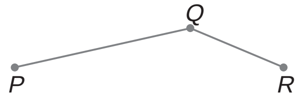
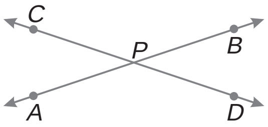
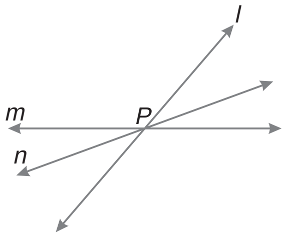


Figure II

- **Intersecting lines :** Two lines which cut at one point are said to be **intersecting lines**. The point P common to two given line segments AB and CD is called their **point of intersection.**

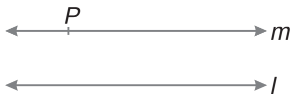


- **Concurrent lines :** Three or more lines intersecting at a same point are said to be concurrent.



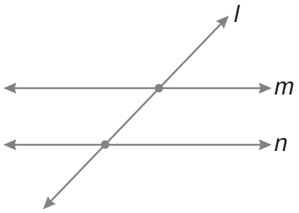
- **Parallel lines :** Two lines l and m in a plane are said to be parallel, if they have no point in common and we write $l \,||\, m$.

  The distance between two parallel lines always remains the same.
- Two distinct lines cannot have more than one point in common.
- **Parallel Line Axiom :** If l is a line and P is a point not on the line l, there is one and only one line which passes through P and is parallel to l.

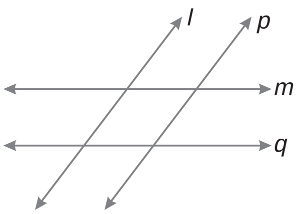


- If two lines l and m are both parallel to the same line n, they will also be parallel to each other.

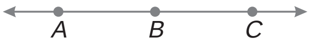
- If l, m, n are lines in the same plane such that l intersects m and $n \,||\, m$, then l also intersects n.



- If l and m are intersecting lines, $l \,||\, p$ and $q \,||\, m$, then p and q also intersect.



- If line segments AB, AC, AD and AE are parallel to a line l, then points A, B, C, D and E will be collinear.
- **Betweenness :** Point B is said to lie between the two points A and C, if :



(i) Points A, B and C are collinear, and

(ii) $AB + BC = AC$.

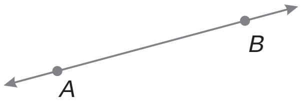
- **Mid-point of a Line-segment :** For a given line segment AB, a point M is said to be the mid-point of AB, if :

(i) M is an interior point of AB, and

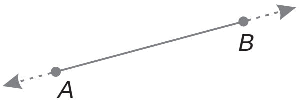
(ii) $AM = MB$.

Euclid's Five Postulates :

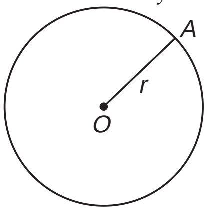
Postulate 1 : A straight line may be drawn from any point to another point.



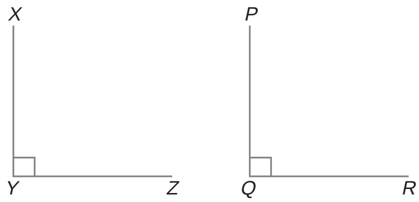
Postulate 2 : A terminated line can be produced indefinitely *i.e.*, 'A line segment can be extended on either side to form a line'.



Postulate 3 : A circle can be drawn with any centre and any radius.



Postulate 4 : All right angles are equal to one another.



If $\angle XYZ = 90°$ and $\angle PQR = 90°$, then $\angle XYZ = \angle PQR$. [congruent angle]

Postulate 5 : If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less then two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

Two equivalent versions of the Euclid's 5th postulates are :

(i) For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l.

(ii) Two distinct intersecting lines cannot be parallel to the same line.

Non-Euclidean geometries :

All the attempts to prove the Euclid's fifth postulate using the first 4 postulates failed. But they led to the discovery of several other geometries, called non-Euclidean geometries.

- **Theorems :** Theorems are statements which are proved using definitions, axioms, previously proved statements and deductive reasoning.

Chapter - 6 : Lines and Angles

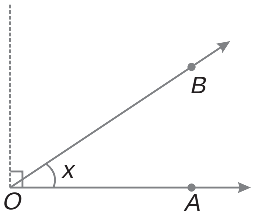
TOPIC-1
Different Types of Angles

Quick Review

- **Line :** Line is a collection of points which has only length neither breadth nor thickness.
- **Line Segment :** A line with two end points.
- **Ray :** A part of line with one end point.
- **Angle :** An angle is formed when two rays originate from the same end point. The rays making an angle are called the arms and the end point is called the vertex.
- **Different Types of Angles :**

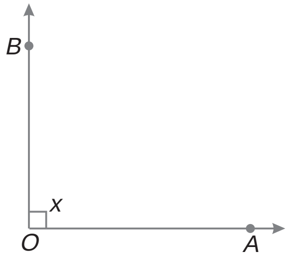
(i) Acute Angle : $0° < x < 90°$

An angle whose measure is more than 0° but less than 90° is called an Acute angle.



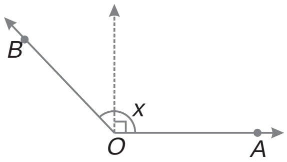
(ii) Right Angle : $x = 90°$

An angle whose measure is 90°, is called a right angle.



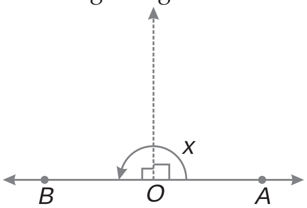
(iii) Obtuse Angle : $90° < x < 180°$

An angle whose measure is more than 90° but less than 180° is called an obtuse angle.



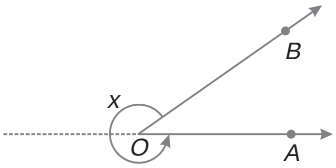
(iv) Straight Angle : $x = 180°$

An angle whose measure is 180° is called a straight angle.



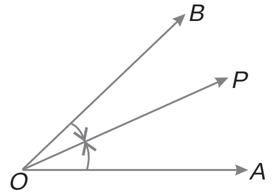
(v) Reflex Angle : $180° < x < 360°$

An angle whose measure is more than 180° but less than 360° is called as reflex angle.



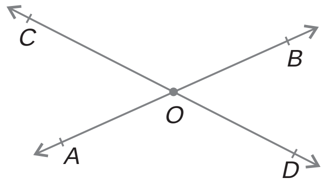
(vi) Complete Angle : $x = 360°$
An angle whose measure is 360° is called a complete angle.

- **Complementary Angles :** Two angles whose sum is 90° are called complementary angles.
  Eg : Complement of 30° angle is 60° angle.
- **Supplementary Angles :** Two angles whose sum is 180° are called supplementary angles.
  Eg : Supplement of 70° angle is 110° angle.
- **Adjacent Angles :** Two angles are called adjacent angles, if :
  (i) they have the same vertex,
  (ii) they have a common arm, and
  (iii) uncommon arms on either side of the common arm.



In the figure, $\angle AOP$ and $\angle BOP$ are adjacent angles.

- **Vertically Opposite Angles :** When two straight lines intersect each other four angles are formed. The pair of angles which lie on the opposite sides of the point of intersection are called vertically opposite angles.



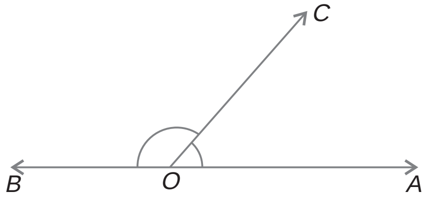
In figure, $\angle AOC$ and $\angle BOD$ are vertically opposite angles and $\angle AOD$ and $\angle BOC$ are also vertically opposite angles.

Vertically opposite angles are always equal.

- **Linear Pair of Angles :** Two adjacent angles are said to form a linear pair of angles, if their non-common arms are two opposite rays.

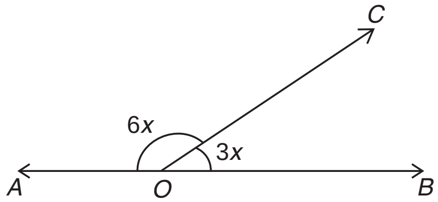
OR

When the sum of two adjacent angles is 180°, then they are called linear pair of angles.



In figure, $\angle AOC$ and $\angle BOC$ form a linear pair of angles.

- Find x in the figure given below

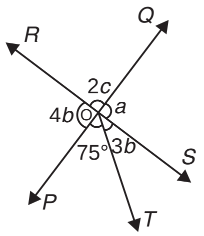


Solution :

$$6x + 3x = 180° \text{ [Linear pair]}$$
$$9x = 180°$$
$$x = \frac{180°}{9} = 20°$$

How it is done on GREENBOARD ?

Q. In the given figure, two straight lines PQ and RS intersect each other at O. If $\angle POT = 75°$, find the values of a, b, c.



Sol. **Step-I :** Identify the straight line and use the suitable property to find the value of b.

Here ROS is a straight line. So by property that sum of all angles on a straight line is 180°, we get

$\therefore \angle ROP + \angle POT + \angle TOS = 180°$

$4b + 75° + 3b = 180°$

or, $7b + 75° = 180°$

or, $7b = 180° - 75° = 105°$

or, $b = \dfrac{105°}{7} = 15°$

and $a + 2c = 180°$ ...(i)

Step-II : Use the property which gives relation for a and find the value of a.

Since, vertically opposite angles are equal

$\therefore a = 4b$

or, $a = 4 \times 15°$

or, $a = 60°$ ...(ii)

Step-III : solve the eqn. (i) and (ii) to find the value of C.

From eqn. (i) we get

$a + 2c = 180°$

or, $60° + 2c = 180°$

or, $2c = 180° - 60° = 120°$

or, $c = \dfrac{120°}{2}$

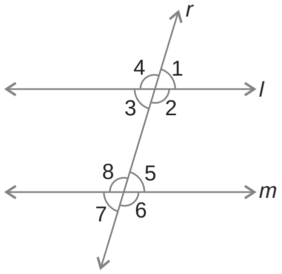
$= 60°$

Hence $a = 60°, b = 15°, c = 60°$

TOPIC-2
Transversal Line

Quick Review

- **Intersecting Lines :** Two lines are said to be intersecting when the perpendicular distance between the two lines is not same everywhere. They meet at one point.
- **Non-Intersecting lines :** Two lines are said to be non-intersecting lines when the perpendicular distance between them is same every where. They do not meet. If these lines are in the same plane these are known as **parallel lines.**
- **Transversal Line :** A straight line which intersects two or more given lines at distinct points is called a transversal of the given lines.



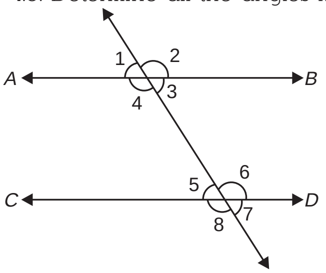
In figure, straight lines l and m are intersected by transversal r.

- **Exterior Angles :** $\angle 1$, $\angle 4$, $\angle 6$ and $\angle 7$
- **Interior Angles :** $\angle 2$, $\angle 3$, $\angle 5$ and $\angle 8$
- **Corresponding Angles :** Two angles on the same side of a transversal are known as corresponding angles if both lie either above the lines or below the lines.

  In figure, $\angle 1$ & $\angle 5$, $\angle 2$ & $\angle 6$, $\angle 3$ & $\angle 7$, $\angle 4$ & $\angle 8$ are the pairs of corresponding angles.
- **Alternate Interior Angles :** $\angle 2$ & $\angle 8$, $\angle 3$ & $\angle 5$ are the pairs of alternate interior angles.
- **Alternate Exterior Angles :** $\angle 1$ & $\angle 7$, $\angle 4$ & $\angle 6$ are the pairs of alternate exterior angles.

- **Consecutive Interior Angles :** The pair of two interior angles on the same side of the transversal are called the pairs of consecutive interior angles.
  In figure, $\angle 2$ & $\angle 5$, $\angle 3$ & $\angle 8$ are the pairs of consecutive interior angles.
- If a transversal intersects two parallel lines, then :
  (i) each pair of corresponding angles is equal.
  (ii) each pair of alternate interior angles is equal.
  (iii) each pair of interior angles on the same side of the transversal is supplementary.
- If a transversal intersects two lines such that, either :
  (i) any one pair of corresponding angles is equal, or
  (ii) any one pair of alternate interior angles is equal, or
  (iii) any one pair of co-interior angles is supplementary, then the lines are parallel.
- Lines which are parallel to a given line are parallel to each other.

Q. In the given figure $AB \parallel CD$ and $\angle 1$ and $\angle 2$ are in the ratio 4:5. Determine all the angles from 1 to 8.



Sol. Step-I : Write the given ratio of angles in terms of variable (say x)

Given ratio = 4:5

Let $\angle 1 = 4x$ and $\angle 2 = 5x$

Step-II : Use the suitable property to find the value of x

We know that, sum of angles on a straight line is $180°$

$\therefore \quad \angle 1 + \angle 2 = 180°$ [AB is a straight line]

$\Rightarrow \quad 4x + 5x = 180°$

$\Rightarrow \quad 9x = 180°$

$\Rightarrow \quad x = \dfrac{180°}{9} = 20°$

Step-III : Put the value of x and get the measure of $\angle 1$ and $\angle 2$.

We have,

$\angle 1 = 4x = 4 \times 20° = 80°$

and $\angle 2 = 5x = 5 \times 20° = 100°$

Step-IV : Find $\angle 3$ and $\angle 4$ by using suitable property.

We know that vertically opposite angles are equal.

$\therefore \quad \angle 3 = \angle 1 = 80°$

and $\angle 4 = \angle 2 = 100°$

Step-V : Find $\angle 5$ and $\angle 6$ by using suitable property.

We know that, alternate interior angles are equal.

$\therefore \quad \angle 5 = \angle 3 = 80°$

and $\angle 6 = \angle 4 = 100°$

Step-VI : Find $\angle 7$ and $\angle 8$ using suitable property.

We know that, vertically opposite angles are equal.

$\therefore \quad \angle 7 = \angle 5 = 80°$

and $\angle 8 = \angle 6 = 100°$

Hence $\angle 1 = \angle 3 = \angle 5 = \angle 7 = 80°$

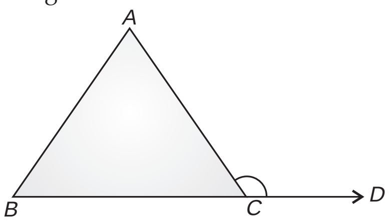
$\angle 2 = \angle 4 = \angle 6 = \angle 8 = 100°$

TOPIC-3
Angle Sum Property of a Triangle

Quick Review

- **Angle Sum Property of a Triangle :**

(i) The sum of all interior angles of a triangle is 180°.



In ΔABC,

$$\angle A + \angle B + \angle C = 180°$$

(ii) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

In ΔABC, $\angle ACD = \angle A + \angle B$

- **Scalene Triangle :** A triangle whose all three sides are unequal in length is called a scalene triangle.
- **Isosceles Triangle :** In a triangle, if two sides are equal in length, then it is called an isosceles triangle.
- **Equilateral Triangle :** In a triangle, if all the sides are equal in length, then it is called an equilateral triangle.
- In a triangle, if

(i) each angle is less than 90°, then it is called an acute angled triangle.

(ii) one of its angle is 90°, then it then it is a right angled triangle or right triangle.

(iii) one of its angle is greater than 90° then it is known as an obtuse angled triangle.

- If all the sides of a polygon are equal, then it is called a regular polygon.
- Sum of all the exterior angles formed by producing the sides of polygon is 360°.
- Each interior angle of a regular polygon of n sides

$$= \frac{(n-2)\times 180°}{n},$$

- Sum of all the interior angles of a polygon of n sides = $(n - 2) \times 180°$.

Q. In ΔPQR, if $\angle P : \angle Q : \angle R = 1:2:3$. Then calculate the measure of $\angle P$, $\angle Q$ and $\angle R$

Sol.` **Step-I :** Write the ratio of angles in term of variable

Given $\angle P : \angle Q : \angle R = 1 : 2 : 3$

$\angle P = x$, $\angle Q = 2x$ and $\angle R = 3x$

Step-II : Use the angle sum property of a triangle to find the value of x.

In ΔPQR, we know that

$\therefore \quad \angle P + \angle Q + \angle R = 180°$

or, $\quad x + 2x + 3x = 180°$

or, $\quad 6x = 180°$

or, $\quad x = 180°$

or, $\quad x = \dfrac{180°}{6} = 30°$

Step-III : Calculate measure of $\angle P$, $\angle Q$ and $\angle R$

Now, $\quad \angle A = x = 30°$

$\angle B = 2x = 2 \times 30° = 60°$

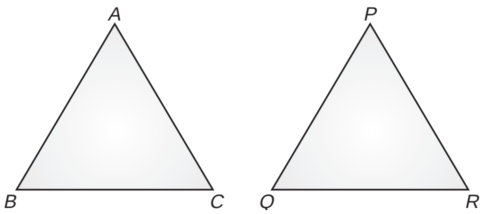
$\angle C = 3x = 3 \times 30° = 90°$

Chapter - 7 : Triangles

TOPIC-1
Criteria for Congruence of Triangles

Quick Review

- **Congruence of Triangle** : The geometrical figures of same shape and size are congruent to each other *i.e.*, two triangles ΔABC and ΔPQR are congruent if and only if their corresponding sides and the corresponding angles are equal.



If two triangles ΔABC and ΔPQR are congruent under the correspondence $A \longrightarrow P$, $B \longrightarrow Q$ and $C \longrightarrow R$, then symbolically it is expressed as

$$\Delta ABC \cong \Delta PQR$$

- *SAS* **Congruence Rule :** Two triangles are congruent if two sides and the included angle of one triangle are equal to the sides and the included angle of the other triangle.
- *ASA* **Congruence Rule :** Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.
- *AAS* **Congruence Rule :** Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.
- *SSS* **Congruence Rule :** If three sides of a triangle are equal to the three sides of another triangle, then the two triangles are congruent.
- *RHS* **Congruence Rule :** If in two right triangles, the hypotenuse and one side of a triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Q. In an isosceles ΔPQR with $PQ = PR$, S and T are points on QR such that $QT = RS$ show that $PS = PT$.

Sol. : Step-I : Read the question carefully and write the given conditions.

ΔPQR is an isosceles triangle in which

$$PQ = PR \quad ...(i)$$

S and T are points on QR such that

$$QT = RS \quad ...(ii)$$

Step-II : Apply the theorems related to given conditions to find other information.

Since $\quad PQ = PR$

So $\quad \angle R = \angle Q \quad ...(iii)$

[Subtracting ST from both sides]

From eqn. (ii) we have $QT = RS$

$\Rightarrow \quad QT - ST = RS - ST$

[Subtracting ST from both sides]

Step-III : Apply the suitable congruence rule in two triangles

In ΔPQS and ΔPRT, $PQ = PR$ [From eqn. (i)]

$\angle Q = \angle R$ [From eqn. (ii)]

and $\quad QS = RT$ [From eqn. (iv)]

$\therefore \quad \Delta PQS \cong \Delta PRT$

(By SAS congruence rule)

Step-IV : Apply the property CPCT *i.e.*, corresponding part of congruence triangles, to get the required result

As $\quad \Delta PQS \cong \Delta PRT$

Then $\quad PS = PT$ (By CPCT)

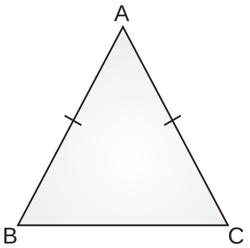
TOPIC-2
Some Properties of Triangles

Quick Review

A triangle is isosceles if any two sides are equal. Here, we prove some properties related to isosceles triangle.

(i) Angles opposite to equal sides of a triangle are equal.

In figure, $\quad \angle B = \angle C$



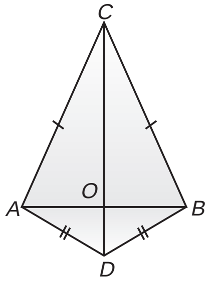
(ii) The sides opposite to equal angles of a triangle are equal.

In figure, $\quad AB = AC$

- In an isosceles triangle, bisector of the vertical angle of a triangle bisect the base.
- The medians of an equilateral triangle are equal in length.
- A point equidistant from two intersecting lines lies on the bisector of the angles formed by the two lines.

How it is done on GREENBOARD?

Q. AB is a line segment. C and D are points on opposite side of AB such that each of them is equidistant from the points A and B. Show that line CD is the perpendicular bisector of AB.



Sol. : Step-I : Read the question carefully and write all given conditions

AB is a line segment. C and B are point on opposite sides of AB such that

$$CA = CB \quad ...(i)$$

and
$$DA = DB \quad ...(ii)$$

CD bisects AB at point O.

Step-II : Find what is given to show

CD is the perpendicular bisector of AB.

Step-III : Show that ΔCAD and ΔCBD are congruent, further use C.P.C.T. to find relation between angles.

In ΔCAD and ΔCBD

$$CA = CB \quad \text{[from eqn. (i)]}$$
$$AD = BD \quad \text{[from eqn. (ii)]}$$

and
$$CD = CD \quad \text{[Common side]}$$

$\therefore$
$$\Delta CAD \cong \Delta CBD$$

[By SSS Congruence rule]

or,
$$\angle ACO = \angle BCD \quad \text{[By C.P.C.T]}$$

or
$$\angle ACO = \angle BCO \quad ...(iii)$$

Step-IV : Show that ΔCAO and ΔCBO are congruent and further use C.P.C.T. to find relation between angles and sides

In ΔCAO and ΔCBO

$$CA = CB \quad \text{[from eqn. (i)]}$$
$$\angle ACO = \angle BCO \quad \text{[from eqn. (ii)]}$$

and
$$CO = CO \quad \text{[Common side]}$$

$\therefore$
$$\Delta CAO \cong \Delta CBO$$

[By SAS Congruence rule]

or,
$$AO = BO \quad \text{[By C.P.C.T]}$$

and
$$\angle AOC = \angle BOC \quad \text{[By C.P.C.T]}$$

Step-V : Since AB is a line segment, so we use the property of linear pair and find the measure of $\angle AOC$ or $\angle BOC$

AB is a line segment So

$$\angle AOC + \angle BOC = 180^\circ$$

or,
$$\angle AOC + \angle AOC = 180^\circ \quad \text{[from eqn. (v)]}$$

or,
$$2\angle AOC = 180^\circ$$

or,
$$\angle AOC = \frac{180^\circ}{2}$$

or,
$$\angle AOC = 90^\circ$$

Thus
$$AO = BO \quad \text{[From eqn. (iv)]}$$

and
$$\angle AOC = 90^\circ$$
$$\cong \angle BOC$$

Hence CD is the perpendicular bisector of AB.

TOPIC-3
Inequalities of a Triangle

Quick Review

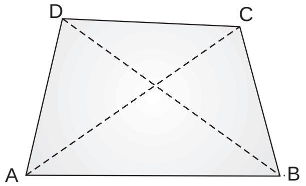
- Angle opposite to the longer side is larger (greater).
- Side opposite to the larger (greater) angle is longer.
- Sum of any two sides is greater than the third side i.e., in ΔABC, AB + BC > CA.
- Difference of any two sides of a triangle is less than the third side i.e., in ΔABC, AB – BC < CA

Chapter - 8 : Quadrilaterals

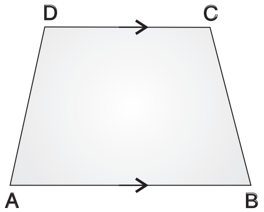
TOPIC-1
Type of Quadrilaterals

Quick Review

- A quadrilateral is a closed figure obtained by joining four points (with no three points collinear) in an order.

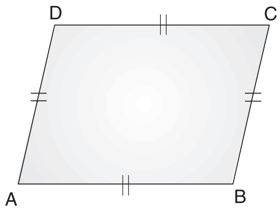


- A diagonal is a line segment obtained on joining the opposite vertices.
- Two sides of a quadrilateral having no common end point are called its opposite sides.
- Two angles of a quadrilateral having common arm are called its adjacent angles.
- Two angles of a quadrilateral not having a common arm are called its opposite angles.
- A trapezium is a quadrilateral in which one pair of opposite sides are parallel. In fig below, ABCD is a trapezium with sides AB || DC and non-parallel sides AD and BC.

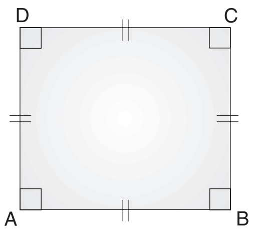


- If the non-parallel sides of a trapezium are equal, then it is known as isosceles trapezium.
- A parallelogram is a quadrilateral in which both the pairs of opposite sides are parallel.
- A rectangle is a quadrilateral each of whose angle is 90°. In fig. below, ABCD is a rectangle with AD || BC, AB || DC and $\angle A = 90° = \angle B = \angle C = \angle D$.

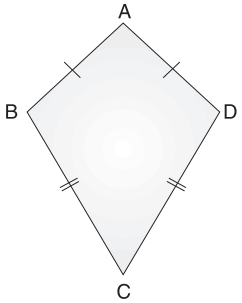
- A rhombus is a parallelogram all the sides of whose are equal. In fig. below, ABCD is a rhombus with $AB = BC = CD = DA$.



- A square is a parallelogram all sides are equal and each angle is 90°. In fig. below, ABCD is a square in which $AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90°$.



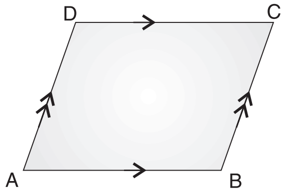
- A kite is a quadrilateral in which two pairs of adjacent sides are equal. A kite ABCD with AB = AD and BC = CD.



TOPIC-2
Properties of a Parallelogram

Quick Review

- Properties of a parallelogram :
  - **(i)** Opposite sides of a parallelogram are parallel.
  - **(ii)** A diagonal of a parallelogram divides it into two congruent triangles.
  - **(iii)** Opposite sides of a parallelogram are equal.
  - **(iv)** Opposite angles of a parallelogram are equal.
  - **(v)** Consecutive angles (conjoined angles) of a parallelogram are supplementary.
  - **(vi)** Diagonals of a parallelogram bisect each other.
- If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.
- If in a quadrilateral each pair of opposite angles is equal, then it is a parallelogram.
- If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.
- A quadrilateral is a parallelogram, if a pair of opposite sides is equal and parallel. In fig. below, ABCD is a parallelogram in which AB || DC and AD || BC.



- Square, rectangle and rhombus are all parallelograms.
- Kite and trapezium are not parallelograms.
- A square is a rectangle.
- A square is a rhombus.
- A parallelogram is a trapezium.
- Every rectangle is a parallelogram; therefore, it has all the properties of a parallelogram. Additional properties of a rectangle are :
  - **(i)** All the interior angles of a rectangle are right angles.
  - **(ii)** The diagonals of a rectangle are equal.
- Every rhombus is a parallelogram; therefore, it has all the properties of a parallelogram. Additional properties of a rhombus are :
  - **(i)** All the sides of a rhombus are equal.
  - **(ii)** Diagonals of a rhombus intersect at right angles.
- Every square is a parallelogram; therefore, it has all the properties of a parallelogram. Additional properties of a square are :
  - **(i)** All sides are equal.
  - **(ii)** All angles are equal to 90°.
  - **(iii)** Diagonals are equal.
  - **(iv)** Diagonals bisect each other at right angle.
  - **(v)** Diagonals bisect the angles of vertex.

TOPIC-3
Mid-Point Theorem

Quick Review

- **Mid Point Theorem :** The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
- **Converse of mid-point theorem :** The line drawn through the mid-point of one side of a triangle parallel to the another side, bisects the third side.
- If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

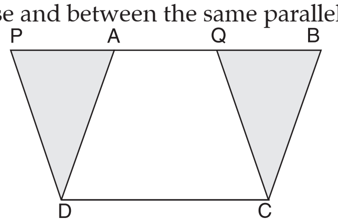
Chapter - 9 : Area Of Parallelograms & Triangles

TOPIC-1
Area of Parallelograms

Quick Review

- Two figures are said to be on the same base and between the same parallels, if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.
- Parallelograms on the same base and between the same parallels are equal in area. Following figure represents the parallelograms on the same base and between the same parallels.

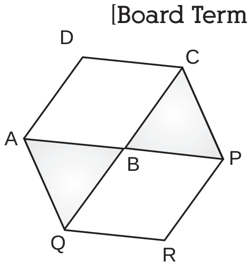
ar ($ABCD$) = ar ($PQCD$)



- Area of a parallelogram is the product of its side which is known as the base and the corresponding altitude.
- Parallelograms on the same base (or equal bases) and having equal areas lie between the same parallels.

Q. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that ar(ABCD) = ar(PBQR)

[Board Term II, 2012, Set 02]



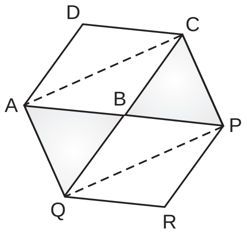
Sol. **Step-I :** Write all the given information.

Given, ABCD and PBQR are parallelogram, CP || AQ

To Prove : ar(ABCD) = ar(PBQR)

Step-II : If any construction is required, then construct it.

Join AC and PQ.



1

Step-III : Use suitable theorem(s), to prove the required result.

ar(△CAQ) = ar(△PAQ) 1

(same base AQ and same parallels AQ ∥ CP)

Subtracting ar(△BAQ) from both sides, we get

ar(△CAQ) – ar(△BAQ) = ar(△PAQ) – ar(△BAQ)

or, ar(△ABC) = ar(△BQP) $\frac{1}{2}$

or, 2ar(△ABC) = 2ar(△BQP)

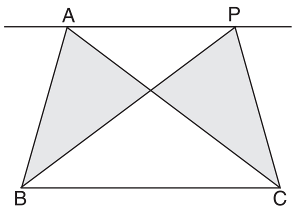
or, ar(ABCD) = ar (PBQR) $\frac{1}{2}$

(∴ Diagonals of ∥ ^{gm} divides it into two triangles of equal areas) **[Hence Proved] 1**

TOPIC-2
Area of Triangles

Quick Review

➢ Two triangles on the same base (or equal base) and between the same parallels are equal in area. Following figure represents the triangles on the same base and between the same parallels.



ar (ΔABC) = ar (ΔPCB)

➢ Area of a triangle is half the product of its base (or any side) and the corresponding altitude (or height).

➢ Two triangles with same base (or equal bases) and equal areas will have equal corresponding altitudes.

➢ Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.

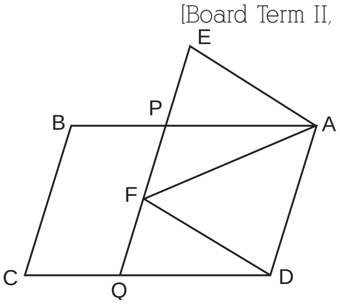
➢ A median of a triangle divides it into triangles of equal areas.

Q. In the given figure, ABCD and AEFD are two parallelograms.

Prove that :

ar(△PEA) = ar(△QFD).

[Board Term II, 2012, Set 02]



Sol. **Step-I :** Write all the given information.

Given, ABCD and AEFD are two parallelograms,

To Prove : ar(△PEA) = ar(△QFD)

Step-II : Use suitable theorem, to prove the required result.

In quadrilateral PQDA,

AP ∥ DQ

(In ∥ ^{gm} ABCD, AB ∥ DC)

and PQ ∥ AD

(In ∥ ^{gm} AEFD, FE ∥ AD)

So, PQDA is a parallelogram. 1

Also, ∥ ^{gm} PQDA and ∥ ^{gm} AEFD are on the same base AD and between same parallels AD and EQ.

∴ ar(∥ ^{gm} PQDA) = ar(∥ ^{gm} AEFD) 1

On subtracting ar (◻ APFD) from both sides, we get

ar(∥ ^{gm} PQDA) – ar(∥ ^{gm} APED)

= ar(∥ ^{gm} AEFD) – ar (∥ ^{gm} APFD)

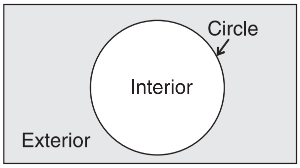
or, ar(△QFD) = ar(△PEA) **Hence Proved. 1**

Chapter - 10 : Circles

TOPIC-1
Basic Properties

Quick Review

- A circle is a collection (set) of all those points in a plane, each one of which is at a constant distance from a fixed point in the plane.
- The fixed point is called the centre and the constant distance is called the radius of the circle.
- All the points lying inside a circle are called its interior points and all those points which lie outside the circle are called its exterior points.
- The collection (set) of all interior points of a circle is called the interior of the circle while the collection of all exterior points of a circle is called the exterior of the circle.



- In a circle, equal chords subtend equal angles at the centre.
- The chords corresponding to congruent arcs are equal.
- If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal.
- If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semi-circular) are congruent.
- One and only one circle can be drawn through three non-collinear points.
- An infinite number of circles can be drawn through a given point P.
- Only one circle be drawn through the two given points.
- Perpendicular bisectors of two chords of a circle intersect each other at the centre of the circle.
- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- Angles in the same segment of a circle are equal.
- An angle in a semi-circle is a right angle.
- The arc of a circle subtending a right angle at any point of the circle in its alternate segment is a semi-circle.
- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line segment, the four points are concyclic, *i.e.*, lie on the same circle.

How it is done on GREENBOARD?

Q. Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Find distance between their centres.

[Board Term II, 2012, 08]

Sol. **Step-I :** Draw a figure as per given information.

Let O and O′ be the centres of the circle of radii 10 cm and 8 cm, respectively. Let PQ be their common chord.

Step-IV : Apply Pythagoras theorem in $\triangle OLP$.

In right angle $\triangle OLP$, we have

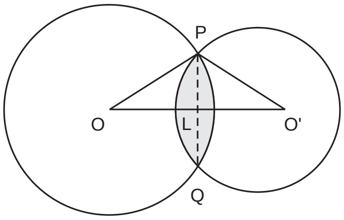
$$OP^2 = OL^2 + PL^2$$

or, $$OL = \sqrt{OP^2 - PL^2}$$

$$= \sqrt{(10)^2 - (6)^2}$$

$$= \sqrt{64}$$

$$= 8 \text{ cm.}$$ 1



Step-II : Write the given information.
Given, OP = 10 cm, O′P = 8 cm and PQ = 12 cm. Find OO′.

Step-III : For finding OO′, first find PL.

$$PL = \frac{1}{2} PQ = 6 \text{ cm}$$

($\because$ Perpendicular from the centre of a circle to a chord bisects the chord) **1**

Step-V : Apply Pythagoras theorem in $\triangle O'LP$.
In right angle $\triangle O'LP$, we have

$$(O'P)^2 = (PL)^2 + (O'L)^2$$

$$O'L = \sqrt{(O'P)^2 - (PL)^2}$$

$$= \sqrt{8^2 - 6^2}$$

$$= \sqrt{64 - 36}$$

$$= \sqrt{28}$$

$$= 5.29 \text{ cm.}$$

$\therefore$ $OO' = OL + O'L$

$$= 8 + 5.29$$

$$= 13.29 \text{ cm.}$$ **1**

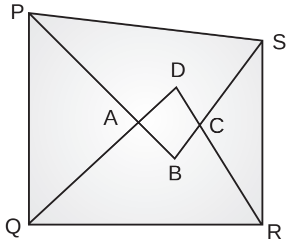
TOPIC-2
Cyclic Quadrilaterals

Quick Review

- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line segment, the four points are concyclic, *i.e.*, lie on the same circle.
- If the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.
- Any exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- **Concentric Circles :** Circles with a common centre are called concentric circles.
- The degree measure of a semi-circle is 180°.
- The degree measure of a circle is 360°.
- The degree measure of a major arc is (360°– θ), where θ is the degree measure of the corresponding minor arc.
- Area of a circle = πr^2 sq. units

Q. Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.

Sol. **Step-I :** Draw a figure according to given information.
Let PQRS be a quadrilateral in which the angle bisectors PB, QD, RP and SB of internal angles P, Q, R and S, respectively form a quadrilateral ABCD



Step-II : Write the proving statement.
ABCD is a cyclic quadrilateral
i.e., $\angle A + \angle C = 180^\circ$
or, $\angle B + \angle D = 180^\circ$ **1**

Step-III : For proving the result, first find the angles ∠BAD and ∠BCD.
Since, $\angle BAD = \angle PAQ = 180^\circ - \angle APQ - \angle AQP$
($\because$ in $\triangle PAQ$, using angle sum property *i.e.*, $\angle PAQ + \angle APQ + \angle AQP = 180^\circ$)

$$= 180^\circ - \frac{1}{2}(2\angle APQ + 2\angle AQP)$$

$$= 180^\circ - \frac{1}{2}(\angle P + \angle Q) \quad \text{...(i)}$$ **1**

[$\because$ PB and QD are bisectors of ∠P and ∠Q, respectively]

Similarly, $\angle BCD = \angle RCS$
$= 180^\circ - \angle CRS - \angle RSC$

$= 180^\circ - \frac{1}{2}(\angle R + \angle S)$

...(ii) $\frac{1}{2}$

Step-IV : Adding the results obtained in step 3 and further use the property of a quadrilateral, which prove the required results.

On adding eqs (i) and (ii), we get

$$\angle BAD + \angle BCD = 180^\circ - \frac{1}{2}(\angle P + \angle Q) + 180^\circ - \frac{1}{2}(\angle R + \angle S)$$

$$= 360^\circ - \frac{1}{2}(\angle P + \angle Q + \angle R + \angle S)$$

$$= 360^\circ - \frac{1}{2} \times 360^\circ$$

($\because$ sum of angles of a quadrilateral is 360°)

$$= 360^\circ - 180^\circ$$

$$= 180^\circ$$ 1

Hence, ABCD is a cyclic quadrilateral because sum of a pair of opposite angles of quadrilateral ABCD is 180°. $\frac{1}{2}$

Chapter - 11 : Geometric Constructions

TOPIC-1
Constructions of Bisectors of Line Segments and Angles

Quick Review

- In a geometrical construction, only two instruments are permitted— an ungraduated ruler (also called a straight edge) and a pair of compasses. In case the measurements are also required, one can use the graduated scale. Geometrical construction of a figure is more accurate than its geometrical drawing.
- **Steps to solve a construction problem :**

  A solution to the construction problem may be divided into the following parts :

  1. **We need to specify clearly :**
     (i) What is given ?
     (ii) What is required ?
  2. **Steps of construction :** This is the sequence of steps that we actually use in drawing the construction. These steps should be specified in the proper order.
  3. **Proof :** For the construction, we need to reason out why the construction is valid.

I. Construction of an angle equal to a given angle :

Given : An $\angle POQ$ and a point A.

Required : To construct an angle at a point A equal to $\angle POQ$.

Steps of Construction :

(i) With O as centre and any (suitable) radius, draw an arc to meet OP at R and OQ at S.

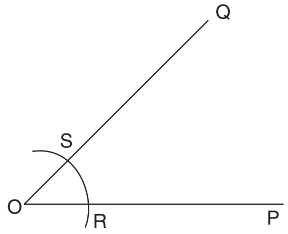
(ii) Through A draw a line AB.

(iii) Taking A as centre and same radius (as in step 1), draw an arc to meet AB at D.

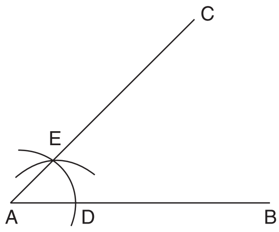
(iv) Measure the segment RS with compasses.

(v) With D as centre and radius equal to RS, draw an arc to meet the previous arc at E.

(vi) Join AE and produce it to C, then $\angle BAC$ is the required angle equal to $\angle POQ$.



(i)



(ii)

II. Construction of angle bisector :

Given : $\angle POQ$.

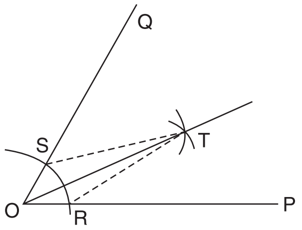
Required : To construct the bisector of $\angle POQ$.

Steps of Construction :

(i) With O as centre and any (suitable) radius, draw an arc to meet OP at R and OQ at S.

(ii) With R as centre and any suitable radius (not necessarily) equal to radius of step 1 (but > ½RS), draw an arc. Also, with S as centre and same radius draw another arc to meet the previous arc at T.

(iii) Join OT and produce it, then OT is the required bisector of $\angle POQ$.



Join ST and RT.

Justification : ΔOST and ΔORT are congruent (SSS criterion).

Hence, $\angle QOT = \angle TOP$.

Concept of bisectior of the angle can be extended to get the division of angle in the powers of 2 like 4, 8, 16, etc. We cannot trisect the angle using the ruler and compasses.

III. Construction of angles with measurements : 60°, 30°, 120°, 90°, 45° :

(i) To construct an angle of 60° :

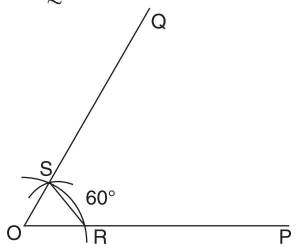
Steps of Construction :

(i) Draw any line OP.

(ii) With O as centre and any suitable radius, draw an arc to meet OP at R.

(iii) With R as centre and same radius (as in step 2), draw an arc to meet the previous arc at S.

(iv) Join OS and produce it to Q, then $\angle POQ = 60°$.

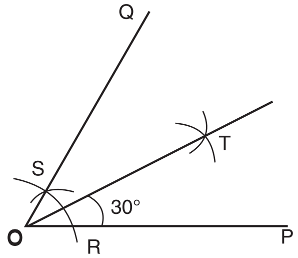


(ii) To construct an angle of 30° :

Steps of Construction :

(i) Construct $\angle POQ = 60°$ (as above).

(ii) Bisect $\angle POQ$ (as in construction II). Let OT be the bisector of $\angle POQ$, then $\angle POT = 30°$.



(iii) To construct an angle of 120° :

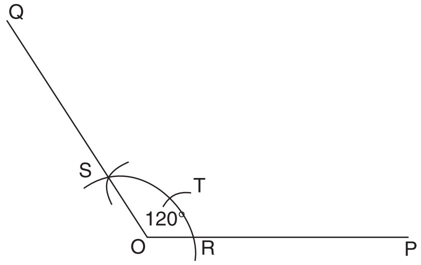
Steps of Construction :

(i) Draw any line OP.

(ii) With O as centre and any suitable radius, draw an arc to meet OP at R.

(iii) With R as centre and same radius (as in step 2), draw an arc to meet the previous arc at T. With T as centre and same radius, draw another arc to cut the first arc at S.

(iv) Join OS and produce it to Q, then $\angle POQ = 120°$.



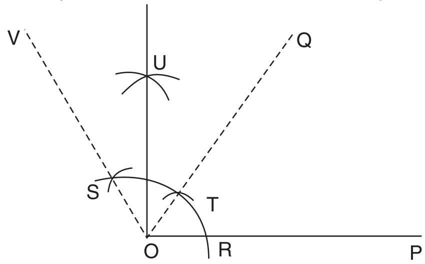
(iv) To construct an angle of 90° :

Steps of Construction :

(i) Construct $\angle POQ = 60°$ [as in construction III (i)]

(ii) Construct $\angle POV = 120°$ [as in construction III (iii)]

(iii) Bisect $\angle QOV$ (as in construction II). Let OU be the bisector of $\angle QOV$, then $\angle POU = 90°$.



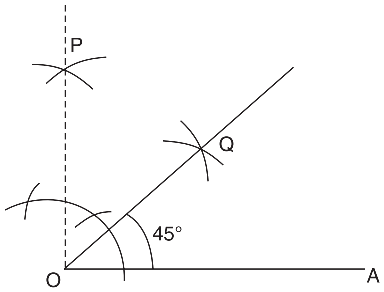
(v) To construct an angle of 45°.

Steps of Construction :

(i) Construct $\angle AOP = 90°$ [as in construction III (iv)]

(ii) Bisect $\angle AOP$ [as in construction (II)]

Let OQ be the bisector of $\angle AOP$, then $\angle AOQ = 45°$.



IV. Construction of the perpendicular bisector of a line segment.

Given : Any line segment *AB*.

Required : To construct perpendicular bisector of line segment *AB*.

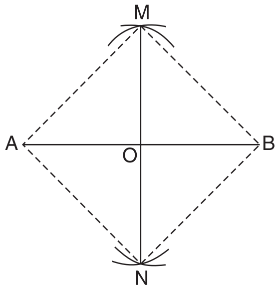
Steps of Construction :

(i) Draw a line segment *AB*.

(ii) Taking *A* and *B* as the centres and radius of more than half the length of *AB* draw arcs on both sides of *AB*.

(iii) Let these arcs intersect each other at points *M* and *N*.

(iv) Join the points of intersection, *i.e.*, *M* and *N*. Thus, *MN* is the required perpendicular bisector of *AB*.



Justification :

Join *A* and *B* to *M* and *N*.

In ΔMAN and ΔMBN,

$MA = MB$ (by construction)

$NA = NB$ (by construction)

$MN = MN$ (common)

$\therefore$ $\Delta MAN \cong \Delta MBN$ (SSS criterion)

$\therefore$ $\angle AMN = \angle BMN$ (c.p.c.t.)

Now, in ΔMOA and ΔMOB,

$MA = MB$ (by construction)

$\angle AMO = \angle BMO$ (proved earlier)

$MO = MO$ (common)

$\Delta MOA \cong \Delta MOB$ (By SAS criterion)

$OA = OB$ (c.p.c.t.)

$\angle MOA = \angle MOB$ (c.p.c.t.)

As $\angle MOA + \angle MOB = 180°$ (Linear pair axiom)

or, $\angle MOA = \angle MOB = 90°$

$\therefore$ MO, *i.e.*, MON is the perpendicular bisector of AB.

V. Construction of bisector of a line segment.

Given : Any line segment AB.

Required : To bisect line segment AB.

Steps of Construction :

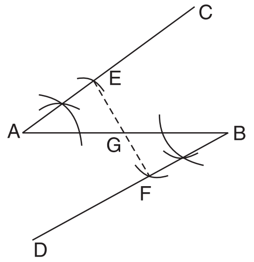
(i) At A, construct any suitable angle BAC.

(ii) At B, construct $\angle ABD = \angle BAC$ on the other side of the line AB.

(iii) With A as centre and any suitable radius, draw an arc to meet AC at E.

(iv) With B as centre, draw an arc BF which is equal to AE on line BD.

(v) Join EF to meet AB at G, then EG is a bisector of the line segment AB and G is the mid-point of AB.



Justification : In ΔAEG and ΔBFG,

$\angle A = \angle B$ (By construction)

$AE = BF$ (By construction)

$\angle AGE = \angle BGF$ (V.O.A.)

$\Delta AEG \cong \Delta BFG$ (By AAS criterion)

and hence $AG = BG$. (By c.p.c.t.)

How it is done on GREENBOARD?

Q. Construct a ΔABC, in which $BC = 5.6$ cm, $\angle B = 30°$ and the difference between the other two sides is 3 cm.

Sol. **Step-1 :** Draw the given base and base angle.

Given, $BC = 5.6$ cm, $\angle B = 30°$

and difference between other two sides *i.e.* $AB - AC = 3$ cm.

First, draw the base $BC = 5.6$ cm.

At point B, draw a ray BX making an $\angle XBC = 30°$.

Step-V : Give justification

Base BC and $\angle B$ are drawn as given. Since, the point A lies on the perpendicular bisector of DC.

$\therefore$ $AD = AC$

Now, $BD = AB - AD$

or, $BD = AB - AC$

Thus, the construction is justified.

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Step-II : Identify the case and according to it, cut the line segment equal to given difference.
Here, AB > AC, *i.e.* the side containing the base angle B is greater than third side.
Hence, cut BD equal to AB – AC *i.e.* BD = 3 cm from the ray BX extended on same side of base BC.
Step-III : Join DC and draw its perpendicular bisector.
Now, join DC and draw its perpendicular bisector PQ.
Step-IV : Join the intersection point of BX and perpendicular bisector to A and get required triangle.
Let A be the point of intersection of BX and perpendicular bisector PQ. Then join AC. Thus, ABC is the required triangle.

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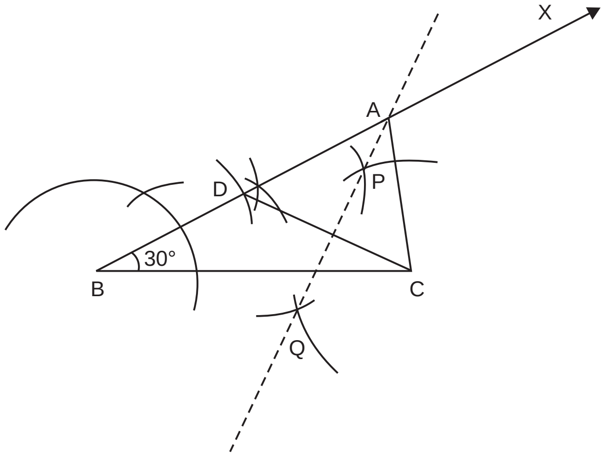

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TOPIC-2
Construction of a Triangle, Given its Base, Sum or Difference of Other Two Sides and One Base Angle

Quick Review

I. Construction of a triangle, given its base, sum of the other two sides and one base angle.

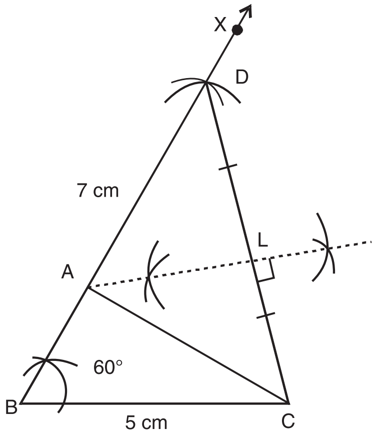
e.g. : **Construct a triangle with base of length 5 cm, sum of the other two sides 7 cm and one base angle of 60°.**

Given : In ABC, base $BC = 5$ cm, $AB + AC = 7$ cm and $\angle ABC = 60°$.

Required : To construct ABC.

Steps of construction :

1. Draw $BC = 5$ cm.
2. At B, construct $\angle CBX = 60°$.
3. From BX, cut off $BD = 7$ cm.
4. Join CD.
5. Draw the perpendicular bisector of CD, intersecting BD at a point A.
6. Join AC. Then ABC is the required triangle.



Justification of construction : In ADC, we have

$AC = AD$ as AL is the perpendicular bisector of DC.

$BD = BA + AD = 7$ cm [by construction]

Now, $BD = BA + AC$

Therefore, $AB + AC = 7$ cm.

II. Construction of a triangle, given its base, difference of the other two sides and one base angle.

e.g., **: Construct a triangle with base of length 7.5 cm, the difference of the other two sides is 2.5 cm, and one base angle of 45°.**

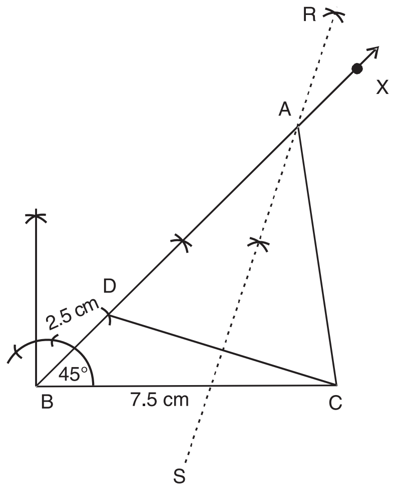
Given : In ΔABC, base $BC = 7.5$ cm, the difference of the other two sides, $AB - AC$ or $AC - AB = 2.5$ cm and one base angle is 45°.

Required : To construct the ABC.

Case (i) : $AB - AC = 2.5$ cm.

Steps of construction :

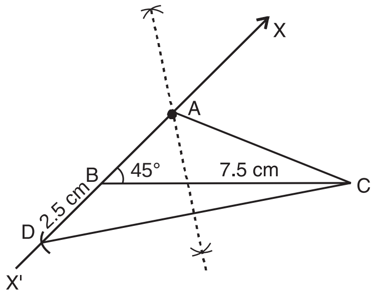
1. Draw $BC = 7.5$ cm.
2. At B, construct $\angle CBX = 45°$.
3. From BX, cut off $BD = 2.5$ cm.
4. Join CD.
5. Draw the perpendicular bisector RS of CD intersecting BX at a point A.
6. Join AC. Then ABC is the required triangle.



Case (ii) : $AC - AB = 2.5$ cm

Steps of construction :

1. Draw $BC = 7.5$ cm.
2. At B, construct $\angle CBX = 45°$ and extend $X\,B$ to X' on opposite side of BC and cut off segment $BD = 2.5$ from ray BX'.
3. Join CD.
4. Draw perpendicular bisector of CD intersecting BX or BX' at a point A.
5. Join AC. Then ABC is the required triangle.



How it is done on GREENBOARD ?

Q. Construct a ΔABC, in which $BC = 6$ cm, $\angle B = 60°$ and the sum of other two sides is 9 cm and justify the construction.

Sol. **Step-I :** Draw the given base and a ray making an angle equal to base angle.

Given, $BC = 6$ cm, sum of other two sides *i.e.* $AB + BC = 9$ cm.

and $\angle B = 60°$

First, draw a base line, $BC = 6$ cm.

Now, draw a ray BX such that $\angle XBC = \angle B = 60°$.

Step-II : Cut a line segment BD equal to the sum of two sides from the ray.

Here, sum of two sides $= AB + AC = 9$ cm

So, cut the line segment $BD = 9$ cm from ray BX.

Step-III : Join DC and draw perpendicular bisector of it.

Now, join DC. Let PQ be the perpendicular bisector of DC, which intersects BD at A.

Step-IV : Join intersection point of BD and perpendicular bisector to C and get required triangle.

Here, intersection point of BD and perpendicular bisector is A. So, on joining A and C, we get a ΔABC which is the required triangle.

Step-V : Give justification.

Base BC and $\angle B$ are drawn as given. Since, PQ is the perpendicular bisector of CD and A lies on it,

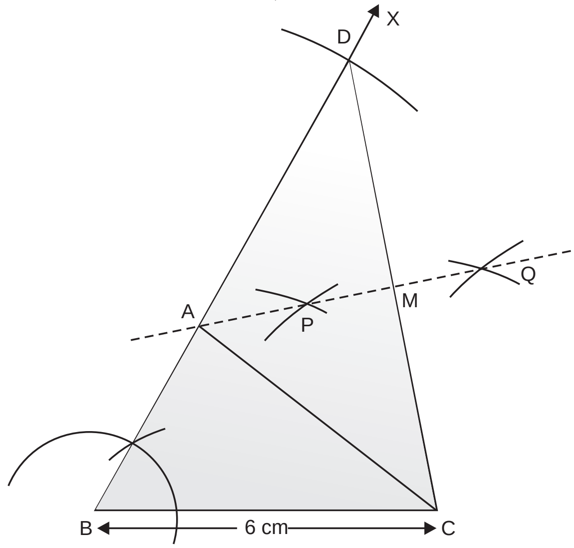
so $AD = AC$

Now, $AB = BD - AD$

$= BD - AC$

$AB + AC = BD$

Thus, construction is justified.



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TOPIC-3
Construction of a triangle of given perimeter and base angle

Quick Review

➢ **Construction of a triangle of given perimeter and base angles.**

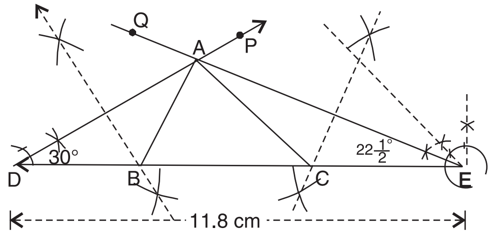
e.g. : **Construct a triangle with perimeter 11.8 cm and base angles 60° and 45°.**

Given : In ΔABC, $AB + BC + CA = 11.8$ cm, $\angle B = 60°$ and $\angle C = 45°$.

Required : To construct the ΔABC.

Steps of construction :

1. Draw $DE = 11.8$ cm.
2. At D, construct $\angle EDP = \frac{1}{2}$ of $60° = 30°$ and at E construct $\angle DEQ = \frac{1}{2}$ of $45° = 22\frac{1}{2}^{\circ}$.
3. Let DP and EQ meet at A.
4. Draw perpendicular bisector of AD to meet DE at B.
5. Draw perpendicular bisector of AE to meet DE at C.
6. Join AB and AC. Then ABC is the required triangle.



How it is done on GREENBOARD ?

Q. Construct a right triangle with perimeter 11 cm and one base angle 30°.

[Board Term II, 2012, (26)]

Sol. **Step-I :** Draw a line segment equal to the perimeter of triangle.

Given, perimeter = 11 cm

So, draw line segment PQ = 11 cm.

Step-II : Draw two rays making angles at *P* and *Q* equal to given angles.

Given, $\angle B = 90°$ and $\angle C = 30°$

So, draw ray LP making an $\angle LPQ = 90°$ at P and ray MQ making an $\angle MQP = 30°$ at Q

Step-III : Draw the bisectors of angles drawn in step 2 and find their intersection point.

Bisect the $\angle LPQ$ and $\angle MQP$. Let the intersection point of these bisectors be A

Step-IV : Draw the perpendicular bisectors of AP and AQ and find the points at which they intersect PQ and get required triangle.

Let the perpendicular bisector of AP be ED which intersect PQ at B and perpendicular bisector of AQ be FG, which intersects PQ at C.

On joining AB and AC, we get the required ΔABC.

Step-V : Give justification.

Since, B lies on perpendicular bisector ED of AP.

So, $AB = BP$,

then $\angle APB = \angle PAB$

Similarly, (C Lies on the perpendicular bisector FG of AQ.)

So, $AC = CQ$,

then $\angle AQC = \angle QAC$

Now,

$$BC + CA + AB = BC + CQ + BP = PQ$$

and $\angle ABC = \angle APB + \angle PAB$

$= 2\angle APB$

$= \angle LPQ = 90°$

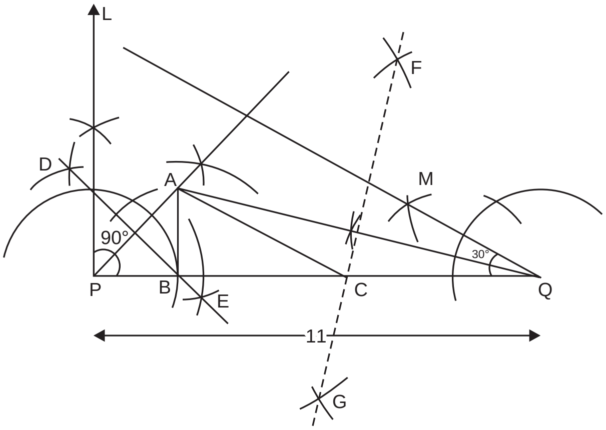
Similarly, $\angle ACB = \angle AQC + \angle QAC$

$= 2\angle AQC$

$= \angle MQC$

$= 30°$

Thus, the construction is justified.



Unit -V : Mensuration

Chapter - 12 : Areas

TOPIC-1
Area of Triangle

Quick Review

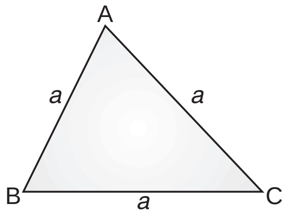
In ΔABC, there are :

(i) three vertices, namely A, B and C.

(ii) three angles, namely $\angle A$, $\angle B$ and $\angle C$.

(iii) three sides, namely AB, BC and CA.

➢ Area $= \frac{1}{2} \times$ base $\times$ corresponding height.

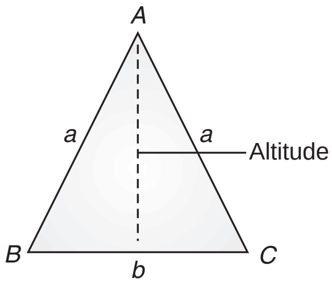


➢ For an equilateral triangle of side 'a'.

(i) Area $= \frac{\sqrt{3}}{4} a^2$

(ii) Perimeter $= 3a$

(iii) Altitude $= \frac{\sqrt{3}}{2} a$

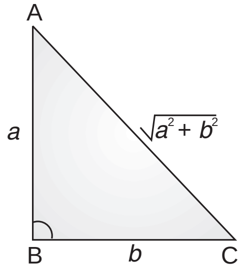


➢For an isosceles triangle with length of two equal sides as 'a' and base 'b'.

(i) Area $= \frac{b}{4}\sqrt{4a^2 - b^2}$

(ii) Perimeter $= 2a + b$

(iii) Altitude $= \frac{1}{2}\sqrt{4a^2 - b^2}$



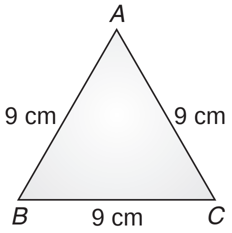
➢ For right angled triangle, with 'a' and 'b' are the sides that includes to the right angle

(i) Area $= \frac{1}{2} \times a \times b$

(ii) Altitude $= a$

(iii) Perimeter $= (a + b + \sqrt{a^2 + b^2})$

Ex. 1. Find the area of an equilateral triangle with side 9 cm.

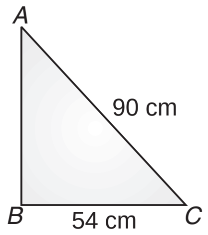


Sol. Area of an equilateral triangle $= \frac{\sqrt{3}}{4} \times a^2$

$$= \frac{81\sqrt{3}}{4} \text{ cm}^2$$

Ex. 2. The longest side of a right triangle is 90 cm and one of the remaining two sides is 54 cm. Find its area.

Sol. By Pythagoras Theorem



$$AB = \sqrt{AC^2 - BC^2}$$

$$= \sqrt{90^2 - 54^2}$$

$$= 72 \text{ cm}$$

Area of triangle $= \frac{1}{2} \times b \times h$

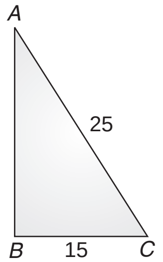
$$= \frac{1}{2} \times 54 \times 72$$

$$= 1944 \text{ cm}^2$$

How it is done on GREENBOARD ?

Q. The base of a right triangle is 15 cm and its hypotenuse is 25 cm, then calculate its area.

Sol. Step-I : We find the height (perpendicular) of the right angled triangle by using pythagoras theorem.



$$AB^2 = AC^2 - BC^2$$
$$= (25)^2 - (15)^2$$
$$= 625 - 225 = 400$$
$$AB = 20 \text{ cm}$$

Step-II : Now we find area of triangle by using

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\left[A = \frac{1}{2} \times b \times h\right]$$

$$= \frac{1}{2} \times 15 \times 20$$
$$= 150 \text{ cm}^2$$

TOPIC-2
Heron's Formula

Quick Review

➢ Consider a triangle with sides a, b and c

Let $AB = c$, $BC = a$ and $CA = b$

So, Perimeter $= a + b + c$

Semi-perimeter $(s) = \dfrac{a+b+c}{2}$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

This formula is known as 'Heron's formula'.

This formula is applicable to all type of triangles whether it is a right triangle or an isosceles or an equilateral triangle.

Ex. Find the area of a triangle when two sides are 24 cm and 10 cm and the perimeter of the triangle is 60 cm.

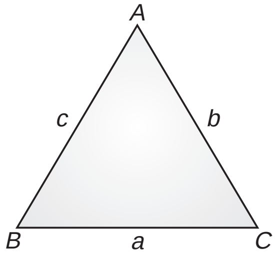
Sol. Let, third side $= x$

Then, $24 + 10 + x = 60$

or, $x = 60 - 34$

$= 26$ cm

$$s = \frac{60}{2} = 30 \text{ cm}$$



$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{30(30-24)(30-10)(30-26)}$$
$$= \sqrt{30 \times 6 \times 20 \times 4} = \sqrt{10 \times 3 \times 3 \times 2 \times 10 \times 2 \times 2 \times 2}$$
$$= 10 \times 3 \times 4 = 120 \text{ cm}^2$$

Q. The side of a triangle are 12 cm, 16 cm, and 20 cm. Find its area.

Sol. : Step I : We find the semiperimeter of a triangle by using Heron's formula

$$S = \frac{a+b+c}{2}$$

$$S = \frac{12+16+20}{2} = 24 \text{ cm}$$

Step II : Now we find the area of triangle by using herons' formula

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{24(24-12)(24-16)(24-20)}$$

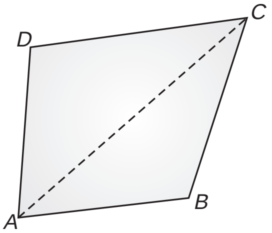
$$= \sqrt{24 \times 12 \times 8 \times 4}$$

$$= 96 \text{ cm}^2$$

TOPIC-3
Application of Heron's Formula in Finding Area of Quadrilaterals

Quick Review

- To find the area of a quadrilateral ABCD, we need to divide the quadrilateral into triangular parts. A diagonal of a quadrilateral divides it into two triangles.



Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔADC

Ex. The sides of a quadrilateral taken in order are 9 m, 40 m, 15 m and 28 m respectively. The angle contained by the first two sides is a right angle. Find its area.

Sol. In ABC,

$$AC^2 = AB^2 + BC^2$$

or,

$$AC^2 = 9^2 + 40^2$$

$$= 81 + 1600 = 1681$$

$$AC = 41 \text{ m}$$

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 9 \times 40 = 180 \text{ m}^2$$

For ΔACD,

$$s = \frac{28+41+15}{2} = \frac{84}{2} = 42 \text{ m}$$

$$\text{Area of } \Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{42(42-28)(42-41)(42-15)}$$

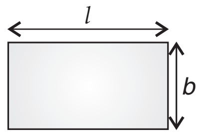
$$= \sqrt{42 \times 14 \times 1 \times 27} = 126 \text{ m}^2$$

$$\text{Area of quadrilateral} = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= 180 + 126 = 306 \text{ m}^2.$$

Formulae for Finding Areas :

1. For a rectangle whose length is 'l' and breadth is 'b', then

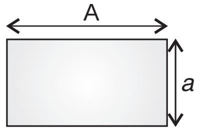


(i) Area of rectangle $= l \times b$

(ii) Perimeter of rectangle $= 2\,(l + b)$

(iii) Diagonal of rectangle $= \sqrt{l^2 + b^2}$

2. For a square whose side is 'a', then

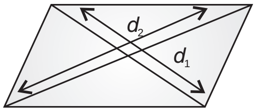


(i) Area of square $= a^2$

(ii) Perimeter of square $= 4\,a$

(iii) Diagonal of square $= \sqrt{2}a$

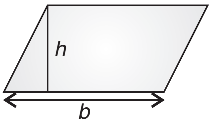
3. For a rhombus whose diagonals are 'd_1' and 'd_2', then



(i) Area of rhombus $= \dfrac{1}{2} \times d_1 \times d_2$

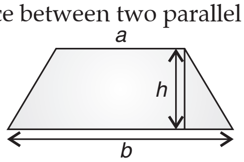
(ii) Perimeter of rhombus $= 2\sqrt{d_1^2 + d_2^2}$

4. For a parallelogram whose base is 'b' and altitude 'h', then



(i) Area of parallelogram $= b \times h$

5. For a trapezium whose parallel sides are 'a' and 'b' and the distance between two parallel sides is h, then

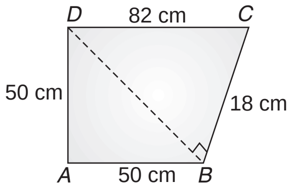


(i) Area of trapezium $= \dfrac{1}{2}\,(a + b) \times h$

How it is done on GREENBOARD?

Q. Find the area of quadrilateral $ABCD$ in which AB = 50 m, BC = 18 m, CD = 82 m, DA = 50 m and $\angle CBD$ = 90°.

Sol. Step-I : Draw a quadrilateral $ABCD$ and join diagonal BD which divide given quadrilateral in two triangles.



First find the diagonal BD by using pythagoras theorem in ΔDBC.

$$BD^2 = CD^2 - BC^2$$
$$= (82)^2 - 18^2$$
$$= 6400$$
$$BD = 80 \text{ m}$$

Step-II : Find the area of right angle ΔBCD by using formula

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \quad \left[A = \frac{1}{2} \times b \times h\right]$$

$$\text{Area of } \Delta BCD = \frac{1}{2} \times 18 \times 80$$
$$= 720 \text{ m}^2$$

Step-III : Now we find the semi perimeter of ΔABD then find the area of ΔABD by using Heron's formula.

$$s = \frac{a + b + c}{2}$$
$$= \frac{50 + 50 + 80}{2} = 90 \text{ m}$$

$$\text{Area of } \Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{90(90-50)(90-50)(90-80)}$$
$$= \sqrt{90 \times 40 \times 40 \times 10}$$
$$= 1200 \text{ m}^2$$

Step-IV : Area of quadrilateral $ABCD$

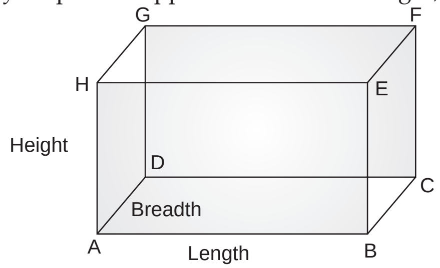
$$= \text{Area of } \Delta ABD + \text{Area of } \Delta BCD$$
$$= 1200 + 720$$
$$= 1920 \text{ m}^2$$

Chapter - 13 : Surface Areas and Volumes

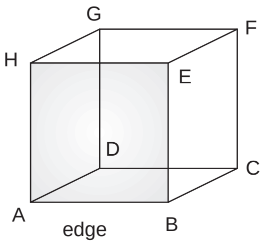
TOPIC-1
Surface Area and Volume of Cube, Cuboid and Sphere (Including Hemisphere)

Quick Review

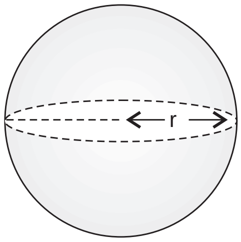
- Cuboid is a solid figure bounded by six parallel opposite faces. It has length, width (breadth) and height.



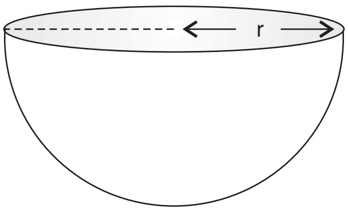
- A cuboid whose all edges equal is called a cube.



- Every cube is a cuboid but every cuboid is not a cube.
- Volume is the capacity or the space occupied by a body.
- A sphere is a perfectly round geometrical object in three-dimensional space, such as the shape of a round ball.



- A hemisphere is half of a sphere



- In case of a room, lateral surface area means the area of the four walls of the room, whereas total surface area means the area of four walls and including the area of the floor and the ceiling.
- The total surface area of any object will be greater than its lateral surface area.
- Volume is the capacity or the space occupied by a body.
- The unit of measurement of both volume and capacity is cubic unit such as cubic feet, cubic cm and cubic m, etc.
- When an object of certain volume is recast into a new shape, the volume of the new shape, formed will always be equal to the volume of the original object.
- The solids having the same curved surface do not necessarily occupy the same volume.
- When an object is dropped into a liquid, the volume of the displaced liquid is equal to the volume of the object that is dipped.

➢ **Cuboid :**

Lateral surface area or area of four walls $= 2(l + b)h$

Total surface area $= 2(lb + bh + hl)$

Volume $= l \times b \times h$

Length of all 12 edges of the cuboid $= 4(l + b + h)$,

where l, b and h are length, breadth and height respectively.

➢ **Cube :**

Lateral surface area or area of four walls $= 4 \times (\text{edge})^2$

Total surface area $= 6 \times (\text{edge})^2$

Volume $= (\text{edge})^3$

Length of all 12 edges of the cube $= 12 \times (\text{edge})$

➢ **Sphere :**

Surface area $= 4\pi r^2$

Volume $= \dfrac{4}{3}\pi r^3$

➢ **Hemisphere :**

Curved surface area $= 2\pi r^2$

Total surface Area $= 3\pi r^2$

Volume $= \dfrac{2}{3}\pi r^3$

How it is done on GREENBOARD ?

Q. The dimensions of a rectangular box are in the ratio of 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rates of ₹ 8 and ₹ 9.50 per m^2 is ₹ 1248. Find the dimensions of the box.

[Board Term II, 2012, (69)]

Sol. **Step-I :** First consider the dimensions of rectangular box in terms of the variable.

Given, the dimension of the rectangular box are in ratio of 2 : 3 : 4. So, let the dimensions of rectangular box be $2x$ m, $3x$ m, and $4x$ m, **1**

Step-II : Use the formula, total surface area of the rectangular box (i.e. cuboid). S = 2 (lb + bh + hl).

The total surface area= $2\,(2x \times 3x + 3x \times 4x + 4x \times 2x) = 52x^2$ **1**

Step-III : Calculate the cost of covering at given different rates.

C_1 = cost of covering @ ₹ 8.00 per m^2 = $52x^2 \times (8)$

C_2 = cost of covering @ ₹ 9.50 per m^2 = $52x^2 \times (9.50)$ **½**

Step-IV : Apply the given condition and simplify it to get the value of a variable.

According to given condition

$$C_2 - C_1 = 52x^2 \times (9.50) - 52x^2 \times (8)$$
$$= 52x^2\,(9.50 - 8)$$
$$= 1248 \text{ (given)}$$

or, $\quad x^2 = \dfrac{1248}{52 \ 1.5} = 16$

or, $\quad x = 4$ m **1**

Step-V : Find the dimensions of the rectangular box.

$\therefore \quad 2x = 2 \times 4 = 8$ m

$3x = 3 \times 4 = 12$ m

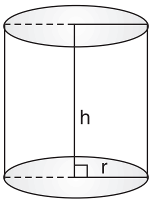
$4x = 4 \times 4 = 16$ m

Hence, the dimensions of rectangular box are 8 m, 12 m and 16 m. **½**

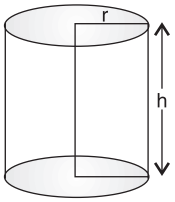
TOPIC-2
Surface Area and Volume of Right Circular Cylinder and Cone

Quick Review

➢ A cylinder is a solid or a hollow object that has a circular base and a circular top of the same size.

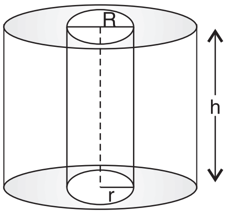


➢ **Right circular cylinder :**



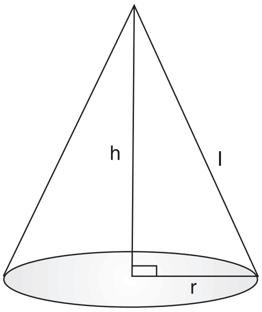
$$\text{Area of each end or base area} = \pi r^2$$
$$\text{Area of curved surface or lateral surface area} = 2\pi rh$$
$$\text{Total surface area (including both ends)} = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$
$$\text{Volume} = \pi r^2 h$$

➢ **Right circular hollow cylinder :**

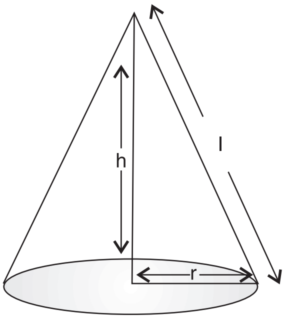


$$\begin{aligned}\text{Area of curved surface} &= (\text{External surface}) + (\text{Internal surface})\\ &= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)\\ &= [2\pi h(R + r) + 2\pi(R^2 - r^2)]\\ &= [2\pi(R + r)(h + R - r)]\\ \text{Volume} &= (\text{External volume}) - (\text{Internal volume})\\ &= (\pi R^2 h - \pi r^2 h) = \pi h\,(R^2 - r^2)\end{aligned}$$

➢ Cone is a pyramid with a circular base.



➢ **Right circular cone :**



$$\begin{aligned}\text{Slant height } (l) &= \sqrt{h^2 + r^2}\\ \text{Area of curved surface} &= \pi rl = \pi r\sqrt{h^2 + r^2}\\ \text{Total surface area} &= \text{Area of curved surface} + \text{Area of base}\\ &= \pi rl + \pi r^2 = \pi r(l + r)\\ \text{Volume} &= \frac{1}{3}\pi r^2 h\end{aligned}$$

How it is done on GREENBOARD ?

Q. There are two cones. The ratio of their radii are 4 : 1 Also, the slant height of the second cone is twice that of the former. Find the relationship between their curved surface area.

[Board Term II, 2013]

Sol. **Step-I :** First consider the unknown variables.

Let r_1 and l_1 be the radius and slant height of first cone.

Let r_2 and l_2 be the radius and slant height of second cone.

Step-II : Write the formula for curved surface area for both the cones.

Curved surface area first cone (CSA_1) = $\pi r_1 l_1$ and curved surface area of second cone (CSA_2) = $\pi r_2 l_2$

Step-III : Use the given condition and simplify it.

According to the question,

$$r_1 : r_2 = 4 : 1 \text{ or, } \frac{r_1}{r_2} = \frac{4_1}{l_2} \text{ and}$$

$$l_2 = 2l_1 \text{ or, } \frac{l_1}{l_2} = \frac{1}{2} \qquad 1$$

$$\therefore \quad \frac{CSA_1}{CSA_2} = \frac{\pi r_1 l_1}{\pi r_2 l_2} = \left(\frac{r_1}{r_2}\right)\left(\frac{l_1}{l_2}\right)$$

$$= \left(\frac{4}{1}\right)\left(\frac{1}{2}\right)$$

$$= \frac{2}{1} \qquad 1$$

$\therefore$ $CSA_1 = 2\ CSA_2$ *i.e.* curved surface area of first cone is twice of the second cone.

Unit -VI : Statistics & Probability

Chapter - 14 : Statistics

TOPIC-1
Mean

Quick Review

- The arithmetic mean of the data is defined as the sum of all the values of the variable divided by the number of values.
- A measure of central tendency tries to estimate the central value which represents the entire data.
- The disadvantage of arithmetic mean is that it is affected by extreme values.
- $$\text{Mean } (\bar{x}) = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{(For raw data)}$$
- $$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} \quad \text{(When frequency } f_i \text{ is given)}$$

How it is done on GREENBOARD?

Q. Find the mean of first seven multiples of 9.

[Board Term II, 2012, Set-10)]

Sol. **Step I :** Write first seven multiples of 9.

First seven multiples of 9 are :

9, 18, 27, 36, 45, 54, 63

Step II : Add all the observations of step 1.

Sum of observations

$= 9 + 18 + 27 + 36 + 45 + 54 + 63$

$= 252$ 1

Step III : Apply the formula to find the mean

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

$$= \frac{252}{7}$$

$$= 36$$

TOPIC-2
Median

Quick Review

- Last cumulative frequency is always the sum total of all the frequencies.
- Median is the value of middle most observation(s).
- The median is to be calculated only after arranging the data in ascending order or descending order.
- If number of observations (n) is odd, Median $= \left(\frac{n+1}{2}\right)^{\text{th}}$ observation.
- If x is even, Median = Mean of the values of the $\left(\frac{n}{2}\right)^{\text{th}}$ and $\left(\frac{n}{2}+1\right)^{\text{th}}$ observations.

i.e., $$\frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ observation} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{ observation}}{2}$$

How it is done on GREENBOARD?

Q. The heights (in cm) of 10 students of a class are given below :
155, 160, 145, 149, 150, 147, 152, 154, 144, 148.
Find the median of the data.
[Board Term II, 2012, Set-15)]

Sol. **Step-I :** First arrange the given data in ascending or increasing order.
Thus, the heights of the 10 students in increasing order is
144, 145, 147, 148, 149, 150, 152, 154, 155, 160.
Step-II : Calculate the no of observations. ½
Here, Number of observations = 10

Step III : Apply the formula of median according to number of observations (even or odd) to find the median.
Here, no of observations are even (*i.e.* n = 10)

$$\therefore \quad \text{Median} = \frac{\frac{n}{2}^{\text{th}} + \left(\frac{n}{2}+1\right)^{\text{th}}}{2}$$ ½

$$\Rightarrow \quad \text{Median} = \text{Mean of } 5^{\text{th}} \text{ and } 6^{\text{th}} \text{ observations}$$

$$= \frac{149+150}{2}$$

$$= 149.50.$$ **1**

TOPIC-3
Mode

Quick Review

- Mode of a statistical data is the value of that variate which has the maximum frequency.
- Disadvantage of the mode is that it is not uniquely defined in many cases.
- The data is symmetric about the mean position when the three averages mean, median and mode are all equal.
- The data is asymmetric when the three measures are unequal.
- In case of grouped data variate corresponding to the highest frequency is to be taken as the mode and not the frequency.

How it is done on GREENBOARD?

Q. Find the mode of the following data
2, 1, 2, 3, 5, 6, 9, 6, 7, 5, 3, 9, 6, 6, 7, 6, 10, 4, 6, 2, 6, 5

Sol. **Step-I :** Arrange the following data is frequency distribution table

x	1	2	3	4	5	6	7	8	9	10
f	1	3	2	1	3	7	2	0	2	1

Step-II : Highest frequency is 7 of the variable 6.
Hence mode of the given data is 6.

TOPIC-4
Frequency Distribution, Bar Graphs, Histogram and Frequency Polygon

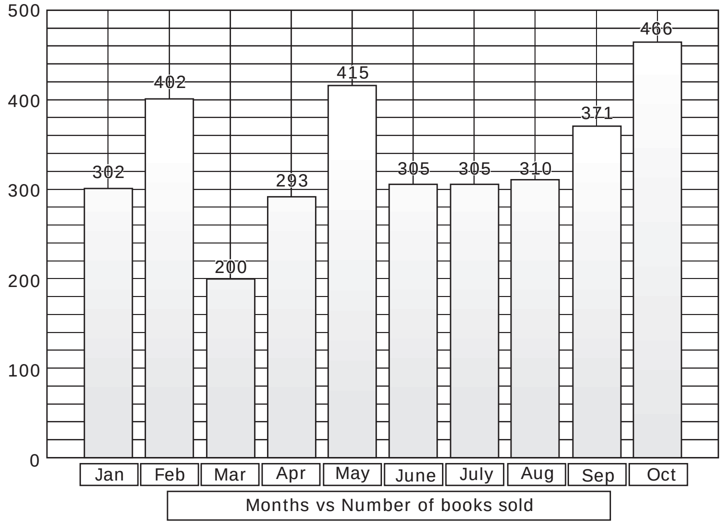
Quick Review

- Class-size or class-width of a class is a measure of the range of the data that can fit in that class.
  It is defined by
  Class width = upper limit of the class – lower limit of the class.
- Class mark of a class is the mid-value of the two limits of that class.

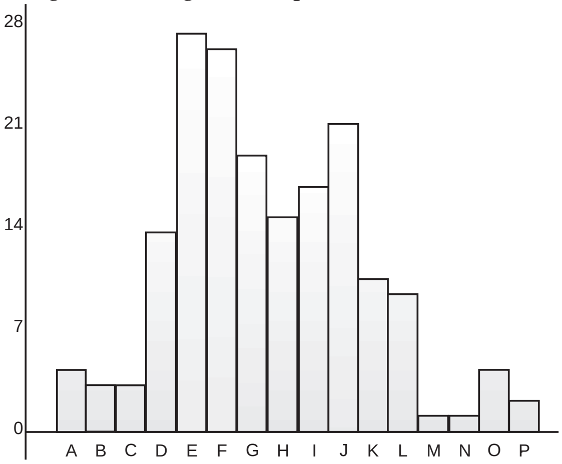
$$\text{Class mark} = \frac{\text{Lower class limit + Upper class limit}}{2}$$

- A frequency distribution in which the upper limit of one class differs from the lower limit of the succeeding class is called an Inclusive or Discontinuous Frequency Distribution.
- A frequency distribution in which the upper limit of one class coincides from the lower limit of the succeeding class is called an Exclusive or Continuous Frequency Distribution.
- In case of continuous frequency distribution, the upper limit of a class is not to be included in that class while in discontinuous both the limits are included.
- For discontinuous classes, the limits are non-overlapping (as in 0 - 9, 10 - 19, 20 - 29, ...). In these cases, it is possible to convert the classes so that they become continuous but have the same width as before. This is done by introducing new lower and new upper limits, these are called true lower limit and true upper limit.
  True lower limit = mean of the lower limit and the upper limit of the preceding class. True upper limit = mean of the upper limit and the lower limit of the succeeding class.
- Data can be represented graphically in following ways :
  (a) Bar Graph **(b)** Histogram **(c)** Frequency polygon.
- A **bar graph** is a pictorial representation of data in which rectangular bars of uniform width are drawn with equal spacing between them on one axis, usually the x-axis. The value of the variable is shown on the other axis that is the y-axis. Following Bar graph depicts number of books sold per month.

Bar Graph

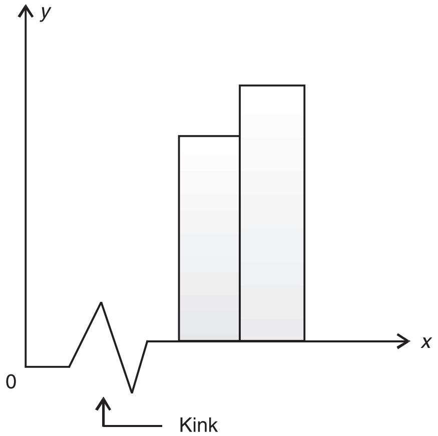


- Bar charts are used for comparing two or more values.
- A **histogram** is one of the most commonly used graphs. A histogram is a vertical bar-graph with no spacing between the bars. The histogram is constructed by the following steps :
  1. The values of the observations are taken on the x-axis with the class-limits clearly marked.
  2. The frequencies are taken along the y-axis.
  3. The base of the rectangle corresponding to a particular class is the line segment (on the x-axis) with the lower limit and the upper limit of the class as the end-points.
  4. The areas of the rectangles are proportional to the frequencies of the classes.
- A histogram is a set of adjacent rectangles whose areas are proportional to the frequencies of a given continuous frequency distribution. The height of rectangles corresponds to the numerical value of the data.



Average Math Score

- The histogram is drawn only for exclusive/continuous frequency distributions.
- If classes are not of equal width, then the height of the rectangle is calculated by the ratio of the frequency of that class, to the width of that class.
- A histogram is different from a bar chart, as in the former case it is the area of the bar that denotes the value, not the height.
- When the scale on the x-axis starts at a higher value and not from the origin, a **kink** is indicated near the origin to signify that the graph is drawn to a scale beginning at a higher value and not at the origin.



- 'Kinks' are a tool used to express areas in a graph. In this case, the kink tells us that there is no observation which takes the value less than 200 (hours).
- The **frequency polygon** of a frequency distribution is a line-graph drawn by plotting the class marks on the x-axis against the frequencies on the y-axis. In case of grouped data, where the classes are of equal width, the frequency polygon is obtained by joining the mid-points of the top edges of the rectangles in the histogram. Two extra lines are drawn by introducing two extra classes (or values).
- One class is introduced before the first class and the other is introduced after the last class. These classes have zero frequencies.
- Frequency polygons are used for understanding the shape of distributions.
- If both a histogram and a frequency polygon are to be drawn on the same graph, then first draw the histogram and then join the mid-points of the tops of the adjacent rectangles in the histogram with line-segments to get the frequency polygon.
- The cumulative frequency of a class-interval is the sum of frequencies of that class and the classes which precede (come before) it.
- $$\text{Class size} = \frac{\text{Range}}{\text{Number of classes}}$$
- $$\text{Class size} = \text{Upper limit} - \text{Lower Limit}$$

How it is done on GREENBOARD ?

Q. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian Society is given below :

Sections of Society	Number of girls per thousand boys
Schedule Caste (SC)	940
Schedule Tribe (ST)	970
Non-SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.

(ii) In the classroom discuss what conclusions can be arrived at from the graph ?

(iii) What step should be taken to improve the situation ?

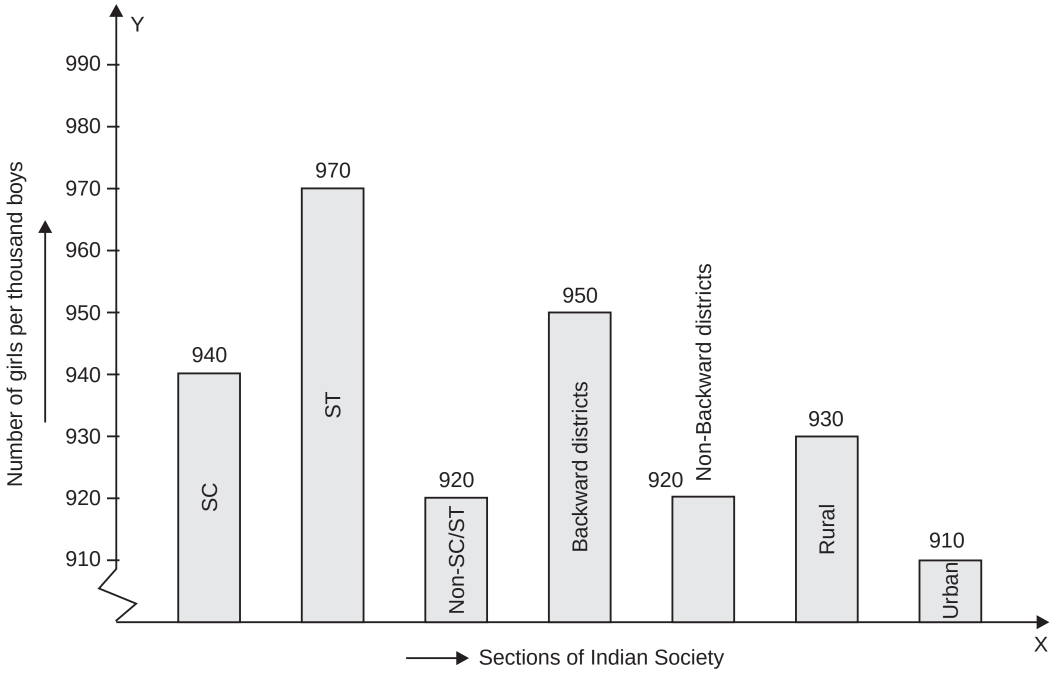
Sol. **Step-I :** Choose the appropriate data for horizontal axis (i.e., x-axis) and vertical axis (i.e., y-axis).

Here, we represent the sections on horizontal axis choosing any scale, since width of bar is not important but for clarity, we take equal widths for all bars and maintain equal gap between. Let one section be represented by one unit.

We represent the number of girls per thousand boys on vertical axis. Here, we can choose the scale as 1 unit = 10.

Step-II : Draw the graph as per given information.

(i) How, the graph is as shown below according to the given data.



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Step-III : Draw the conclusion for part (ii).

(ii) From the graph, we observe that in Scheduled Tribe (ST), there is maximum number of girls per thousand boys among different sections of Indian Society, i.e., 970 whereas there are minimum number of girls per thousand boys in urban area. 1

Step-IV : Suggest one positive step to improve the situation.

(iii) Pre-natal sex determination should be strictly banned in urban. 1

Chapter - 15 : Probaility

TOPIC-1
Experimental Approach

Quick Review

- Probability is a quantitative measure of certainty.
- Any activity associated to certain outcome is called an experiment. *e.g.*, **(i)** tossing a coin, **(ii)** throwing a die, and **(iii)** selecting a card.
- A trial is an action which will result in one or several outcomes.
- An event for an experiment is the collection of some outcomes of the experiment. *e.g.*,
  (i) getting a head on tossing a coin, **(ii)** getting a face card when a card is drawn from a pack of 52 cards.

How it is done on GREENBOARD?

Q. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes.

Find the probability of getting :

(i) 3 heads

(ii) no head

(iii) at least 2 heads **[Board Term II, 2012, (10)]**

Sol. **Step-I :** Analyse the total number of trials.

Here, three coins are tossed simultaneously 200 times.

So, the total number of trials, $n(S) = 200$

Step II : Find the probability for case I i.e., getting 3 heads.

Here frequency of getting 3 heads = 23

Hence, the P(getting 3 heads) = $\frac{23}{200}$. **1**

Step III : Find the probability for Case II i.e., getting no head.

Here, frequency of getting no head = frequency of getting all (three) tails = 22

Hence, the P(getting no head) = $\frac{22}{200} = \frac{11}{100}$.

Step IV : Find the probability for Case III i.e., getting at least 2 heads.

In this case, we include the frequency of getting 2 heads and 3 heads i.e., frequency of getting at least 2 heads = frequency of getting 2 heads + frequency of getting 3 heads

= 84 + 23 = 107

Hence, the P (getting at least 2 heads) = $\frac{107}{200}$ **1**

TOPIC-2
Probability of an Event

Quick Review

- The empirical (experimental) probability of an event E denoted as P(E) is given by :

$$P(E) = \frac{\text{Number of trials in which the event has happened}}{\text{Total number of trials}}$$

- The sum of the probabilities of all elementary events of an experiment is 1.
- Probability of an event lies between 0 and 1.
- Probability can never be negative.
- A pack of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. Each suit consists of one ace, one king, one queen, one jack and 9 other cards numbered from 2 to 10. Four suits are spades, hearts, diamonds and clubs.
- King, queen and jack are face cards. A pack of playing cards consists of 12 face cards.

How it is done on GREENBOARD ?

Q. Out of the past 250 consecutive days, its weather forecasts were correct 175 times.

(i) What is the probability that on a given day it was correct ?

(ii) What is the probability that it was not correct on the given day ?

[Board Term II, 2012, (02, (12)]

Sol. **Step I :** Find the total number of cases.

Here, Total number of days for which the record is available are 250.

$\therefore \quad n(S) = 250.$

Step II : Assume the event (or events) for which we have to calculate probability

let Event E_1 = Number of days when forecast was correct.

and Event E_2 = Number of days when forecast was not correct.

Step III : Find the number of cases in which events E_1 and E_2 happen.

Here, Number of days when forecast was correct = 175

$\therefore \quad n(E_1) = 175$

Number of days when forecast was not correct

$= 250 - 175$

$= 75$

Step IV : Calculate the required probability.

(i) Probability that forecast was correct on a given day.

$$P(E_1) = \frac{175}{250} = 0.7$$ **1**

(ii) Probability that forecast was not correct on a given day.

$$P(E_2) = \frac{75}{250} = 0.3.$$ **1**