Strictly based on the latest ICSE Curriculum

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For 2019 Exam

ICSE

SOLVED PAPER 2018

CLASS 10

MATHEMATICS

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0504

LATEST SYLLABUS

Mathematics

CLASS 10

There will be ***one*** *paper of* ***two and a half*** *hours duration carrying 80 marks and Internal Assessment of 20 marks.*
The paper will be divided into ***two*** *sections, Section I (40 marks), Section II (40 marks).*
Section I: *Will consist of compulsory short answer questions.*
Section II: *Candidates will be required to answer* ***four*** *out of* ***seven*** *questions.*

1. Commercial Mathematics

(i) Value Added Tax

Computation of tax including problems involving discounts, list-price, profit, loss, basic/cost price including inverse cases.

(ii) Banking

Recurring Deposit Accounts : *computation of interest and maturity value using the formula:*

$$I = P\frac{n(n+1)}{2\times 12}\times\frac{r}{100}$$

MV = P x n + I

(iii) Shares and Dividends

(a) Face/Nominal value, Market Value, Dividend, Rate of Dividend, Premium.

(b) Formulae

- *Income = number of shares × rate of dividend × FV.*
- *Return = (Income/Investment) × 100.*

Note : *Brokerage and fractional shares* ***not included.***

2. Algebra

(i) Linear Inequations

Linear Inequations in one unknown for $x \in$ N, W, Z, R. Solving

- *Algebraically and writing the solution in set notation form.*
- *Representation of solution on the number line.*

(ii) Quadratic Equations in one variable

(a) *Nature of roots*

- *Two distinct real roots if $b^2 - 4ac > 0$*
- *Two equal real roots if $b^2 - 4ac = 0$*
- *No real roots if $b^2 - 4ac < 0$*

(b) *Solving Quadratic equations by:*

- *Factorisation*
- *Using Formula.*

(c) *Solving simple quadratic equation problems.*

(iii) Ratio and Proportion

(a) *Proportion, Continued proportion, mean proportion.*

(b) *Componendo, dividendo, alternendo, invertendo properties and their combinations.*

(c) *Direct simple applications on proportions only.*

(iv) Factorisation of polynomials:

(a) *Factor Theorem.*

(b) *Remainder Theorem.*

(c) *Factorising a polynomial completely after obtaining one factor by factor theorem.*

Note : f (x) not to exceed degree 3.

(v) Matrices

(a) *Order of a matrix. Row and column matrices.*

(b) *Compatibility for addition and multiplication.*

(c) *Null and Identity matrices.*

(d) *Addition and subtraction of 2×2 matrices.*

(e) *Multiplication of a 2×2 matrix by*

- *a non-zero rational number*
- *a matrix.*

(vi) Arithmetic and Geometric Progression

- *Finding their General term.*
- *Finding Sum of their first 'n' terms.*
- *Simple Applications.*

(vii) Co-ordinate Geometry

(a) Reflection

(i) Reflection of a point in a line :
x=0, y =0, x= a, y=a, the origin.

(ii) *Reflection of a point in the origin.*

(iii) *Invariant points.*

(b) Co-ordinates expressed as (x,y), Section formula, Midpoint formula, Concept of slope, equation of a line, Various forms of straight lines.

(i) *Section and Mid-point formula (Internal section only, co-ordinates of the centroid of a triangle included).*

(ii) *Equation of a line:*

- *Slope –intercept form $y = mx + c$*
- *Two- point form $(y-y_1) = m(x-x_1)$*
  Geometric understanding of 'm' as slope/ gradient tanθ where θ is the angle the line makes with the positive direction of the x axis.
  Geometric understanding of 'c' as the y-intercept/the ordinate of the point where the line intercepts the y axis/ the point on the line where x=0.
- *Conditions for two lines to be parallel or perpendicular.*

Simple applications of all the above.

3. Geometry

(a) Similarity

Similarity, conditions of similar triangles.

...contd.

(i) As a size transformation.

(ii) Comparison with congruency, keyword being proportionality.

(iii) Three conditions: SSS, SAS, AA. Simple applications (proof not included).

(iv) Applications of Basic Proportionality Theorem.

(v) Areas of similar triangles are proportional to the squares of corresponding sides.

(vi) Direct applications based on the above including applications to maps and models.

(b) Loci

Loci : Definition, meaning, Theorems and constructions based on Loci.

(i) The locus of a point at a fixed distance from a fixed point is a circle with the fixed point as centre and fixed distance as radius.

(ii) The locus of a point equidistant from two intersecting lines is the bisector of the angles between the lines.

(iii) The locus of a point equidistant from two given points is the perpendicular bisector of the line joining the points.

Proofs not required

(c) Circles

(i) Angle Properties

- *The angle that an arc of a circle subtends at the center is double that which it subtends at any point on the remaining part of the circle.*
- *Angles in the same segment of a circle are equal (without proof).*
- *Angle in a semi-circle is a right angle.*

(ii) Cyclic Properties:

- *Opposite angles of a cyclic quadrilateral are supplementary.*
- *The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle (without proof).*

(iii) Tangent and Secant Properties:

- *The tangent at any point of a circle and the radius through the point are perpendicular to each other.*
- *If two circles touch, the point of contact lies on the straight line joining their centers.*
- *From any point outside a circle two tangents can be drawn and they are equal in length.*
- *If two chords intersect internally orexternally then the product of the lengths of the segments are equal.*
- *If a chord and a tangent intersect externally, then the product of the lengths of segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.*
- *If a line touches a circle and from the point of contact, a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.*

Note : Proofs of the theorems given above are to be taught unless specified otherwise.

(iv) Constructions

(a) Construction of tangents to a circle from an external point.

(b) Circumscribing and inscribing a circle on a triangle and a regular hexagon.

4. Mensuration

Area and volume of solids – Cylinder, Cone and Sphere.

Three-dimensional solids - right circular cylinder, right circular cone and sphere: Area (total surface and curved surface) and Volume. Direct application problems including cost, Inner and Outer volume and melting and recasting method to find the volume or surface area of a new solid. Combination of solids included.

Note : Problems on Frustum are not included.

5. Trigonometry

(a) Using Identities to solve/prove simple algebraic trigonometric expressions

$\sin^2 A + \cos^2 A = 1$

$1 + \tan^2 A = \sec^2 A$

$1 + \cot^2 A = \operatorname{cosec}^2 A;\ 0 \le A \le 90°$

(b) Heights and distances: Solving 2-D problems involving angles of elevation and depression using trigonometric tables.

Note: Cases involving more than two right angled triangles excluded.

6. Statistics

Statistics – basic concepts, Mean, Median, Mode. Histograms and Ogive.

(a) Computation of:

- *Measures of Central Tendency: Mean, median, mode for raw and arrayed data. Mean*, median class and modal class for grouped data. (both continuous and discontinuous).*

** Mean by all 3 methods included:*

Direct : $\dfrac{\Sigma fx}{\Sigma f}$

Short-cut : $A + \dfrac{\Sigma fd}{\Sigma f}$ where $d = x - A$

Step-deviation: $A + \dfrac{\Sigma ft}{\Sigma f} \times i$ where $t = \dfrac{x - A}{i}$

(b) Graphical Representation. Histograms and Less than ogive.

- *Finding the mode from the histogram, the upper quartile, lower Quartile and median etc. from the ogive.*
- *Calculation of inter Quartile range.*

7. Probability

- *Random experiments*
- *Sample space*
- *Events*
- *Definition of probability*
- *Simple problems on single events*

Note: SI units, signs, symbols and abbreviations

(1) Agreed conventions

(a) Units may be written in full or using the agreed symbols, but no other abbreviation may be used.

(b) The letter's is never added to symbols to indicate the plural form.

(c) A full stop is not written after symbols for units unless it occurs at the end of a sentence.

(d) When unit symbols are combined as a quotient, *e.g.* metre per second, it is recommended that they be written as m/s, or as m s^{-1}.

(e) Three decimal signs are in common international use: the full point, the mid-point and the comma. Since the full point is sometimes used for multiplication and the comma for spacing digits in large numbers, it is recommended that the mid-point be used for decimals.

(2) Names and symbols

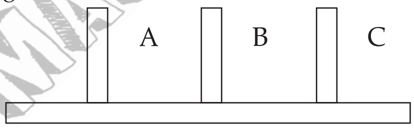
In general			
Implies that	$\Rightarrow$	is logically equivalent to	$\Leftrightarrow$
Identically equal to	$\equiv$	is approximately equal to	$>>$
In set language			
Belongs to	$\in$	does not belong to	$\notin$
Is equivalent to	$\leftrightarrow$	is not equivalent to	$\not\leftrightarrow$
union	$\cup$	intersection	$\cap$
universal set	ξ	is contained in	$\subset$
natural (counting) numbers	N	the empty set	$\varnothing$
		whole numbers	W
integers	Z	real numbers	R
In measures			
Kilometre	km	Metre	m
Centimetre	cm	Millimetre	mm
Kilogram	kg	Gram	g
Litre	1	Centilitre	cl
Square kilometre	km^2	Square meter	m^2
Square centimetre	cm^2	Hectare	ha
Cubic metre	m^3	Cubic centimetre	cm^3
Kilometres per hour	km/h	Metres per second	m/s

INTERNAL ASSESSMENT

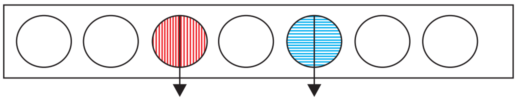
The minimum number of assignments : Two assignments as prescribed by the teacher.

Suggested Assignments

- Comparative newspaper coverage of different items.
- Survey of various types of Bank accounts, rates of interest offered.
- Planning a home budget.
- Conduct a survey in your locality to study the mode of conveyance / Price of various essential commodities/ favourite sports. Represent the data using a bar graph / histogram and estimate the mode.
- To use a newspaper to study and report on shares and dividends.
- Set up a dropper with ink in it vertical at a height say 20 cm above a horizontally placed sheet of plain paper. Release one ink drop; observe the pattern, if any, on the paper. Vary the vertical distance and repeat. Discover any pattern of relationship between the vertical height and the ink drop observed.
- You are provided (or you construct a model as shown) - three vertical sticks (size of a pencil) stuck to a horizontal board. You should also have discs of varying sizes with holes (like a doughnut). Start with one disc; place it on (in) stick A. Transfer it to another stick (B or C); this is one move (*m*). Now try with two discs placed in A such that the large disc is below and the smaller disc is above (number of discs = n=2 now). Now transfer them one at a time in B or C to obtain similar situation (larger disc below). How many moves? Try with more discs (n = 1, 2, 3, etc.) and generalise.



- The board has some holes to hold marbles, red on one side and blue on the other. Start with one pair. Interchange the positions by making one move at a time. A marble can jump over another to fill the hole behind. The move (*m*) equal 3. Try with 2 (n=2) and more. Find the relationship between n and m.



- Take a square sheet of paper of side 10 cm. Four small squares are to be cut from the corners of the square sheet and then the paper folded at the cuts to form an open box. What should be the size of the squares cut so that the volume of the open box is maximum?
- Take an open box, four sets of marbles (ensuring that marbles in each set are of the same size) and some water. By placing the marbles and water in the box, attempt to answer the question: do larger marbles or smaller marbles occupy more volume in a given space?
- An eccentric artist says that the best paintings have the same area as their perimeter (numerically). Let us not argue whether such sizes increases the viewer's

...contd.

appreciation, but only try and find what sides (in integers only) a rectangle must have if its area and perimeter are to be equal (note: there are only two such rectangles).

- Find by construction the centre of a circle, using only a 60-30 setsquare and a pencil.
- Various types of "cryptarithm".

EVALUATION

The assignments/project work are to be evaluated by the subject teacher and by an External Examiner. (The External Examiner may be a teacher nominated by the Head of the school, who could be from the faculty, but not teaching the subject in the section/class. For example, a teacher of Mathematics of Class VIII may be deputed to be an External Examiner for Class X, Mathematics projects.)

The Internal Examiner and the External Examiner will assess the assignments independently.

Award of marks (20 Marks)

Subject Teacher (Internal Examiner) : 10 marks

External Examiner : 10 marks

The total marks obtained out of 20 are to be sent to the Council by the Head of the school.

The Head of the school will be responsible for the entry of marks on the mark sheets provided by the Council.

INTERNAL ASSESSMENT IN MATHEMATICS- GUIDELINES FOR MARKING WITH GRADES

Criteria	Preparation	Concepts	Computation	Presentation	Understanding	Marks
Grade I	Exhibits and selects a well-defined problem. Appropriate of techniques.	Admirable use of mathematical concepts and methods and exhibits competency in using extensive range of mathematical techniques.	Careful and accurate work with appropriate computation, construction and measurement with correct units.	Presents well stated conclusions; mathematical language, symbols, conventions, tables, diagrams, graphs, etc.	Shows strong personal contribution; demonstrate knowledge and understanding of assignment and can apply the same in different situations.	4 marks for each criterion
Grade II	Exhibits and selects routine approach. Fairly good techniques.	Appropriate use of mathematical concepts and methods and shows adequate competency in using limited range of techniques.	Commits negligible errors in computation, construction and measurement.	Some statements of conclusion; uses appropriate math language, symbols, conventions, tables, diagrams, graphs, etc.	Neat with average amount of help; assignment shows learning of mathematics with a limited ability to use it.	3 marks for each criterion
Grade III	Exhibits and selects trivial problems. Satisfactory techniques.	Uses appropriate mathematical concepts and shows competency in using limited range of techniques.	Commits a few errors in computation, construction and measurement.	Assignment is presentable thought it is disorganized in some places.	Lack of ability to conclude without help; shows some learning of mathematics with a limited ability to use it.	2 marks for each criterion
Grade IV	Exhibits and selects an insignificant problem. Uses some unsuitable techniques.	Uses inappropriate mathematical concepts for the assignment.	Commits many mistakes in computation and measurement.	Presentation made is somewhat disorganized and untidy.	Lack of ability to conclude even with considerable help; assignment contributes to mathematical learning to a certain extent.	1 marks for each criterion
Grade V	Exhibits and selects a completely irrelevant problem. Uses unsuitable techniques.	Not able to use mathematical concepts.	Inaccurate computation, construction and measurement.	Presentation made is completely disorganized untidy and poor.	Assignment does not contribute to mathematical learning and lacks practical applicability.	0 marks

ICSE Solved Paper, 2018

Class-X
Mathematics

(Maximum Marks : 80)

(Time allowed : Two hours and a half)

Answers to this Paper must be written on the paper provided separately.

*You will **not** be allowed to write during the first 15 minutes.*

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answer

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*Attempt **all** questions from **Section A** and **any four** questions from **Section B.***

All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer.

Omission of essential working will result in loss of marks.

The intended marks for questions or parts of question are given in brackets [ ].

Mathematical tables are provided.

SECTION- A (40 Marks)

*Attempt **all** questions from this Section.*

1. (a) Find the value of 'x' and 'y' if : 3

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

(b) Sonia had a recurring deposit account in a bank and deposited ₹ 600 per month for $2\frac{1}{2}$ years. If the rate of interest was 10% p.a., find the maturity value of this account. 3

(c) Cards bearing numbers 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is : 4

(i) a prime number.

(ii) a number divisible by 4.

(iii) a number that is a multiple of 6.

(iv) an odd number.

2. (a) The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. Find the 3

(i) radius of the cylinder

(ii) volume of cylinder. (use $\pi = \frac{22}{7}$)

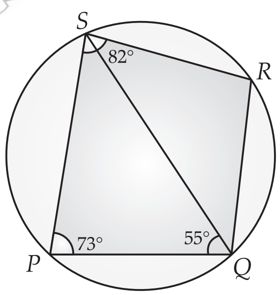
(b) If $(k-3)$, $(2k+1)$ and $(4k+3)$ are three consecutive terms of an *A.P.*, find the value of k. 3

(c) *PQRS* is a cyclic quadrilateral. Given $\angle QPS = 73°$, $\angle PQS = 55°$ and $\angle PSR = 82°$, calculate : 4

(i) $\angle QRS$

(ii) $\angle RQS$

(iii) $\angle PRQ$



3. (a) If $(x + 2)$ and $(x + 3)$ are factors of $x^3 + ax + b$, find the values of 'a' and 'b'. 3

(b) Prove that $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$ 3

(c) Using a graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsmen. Estimate the mode of the data : 3

Runs scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
No. of batsmen	4	18	9	6	7	2	4

4. (a) Solve the following inequation, write down the solution set and represent it on the real number line : 3

$-2 + 10x \le 13x + 10 < 24 + 10x, x \in Z$

(b) If the straight lines $3x - 5y = 7$ and $4x + ay + 9 = 0$ are perpendicular to one another, find value of a. 3

(c) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. 4

SECTION- B (40 Marks)

*Attempt any **four** questions from this Section*

5. (a) The 4^{th} term of a G.P. is 16 and the 7^{th} term is 128. Find the first term and common ratio of the series. 3

(b) A man invests ₹ 22,500 in ₹ 50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate : 3

(i) The number of shares purchased

(ii) The annual dividend received.

(iii) The rate of return he gets on his investment. Give your answer correct to the nearest whole number.

(c) Use graph paper for this question (Take 2 cm = 1 unit along both x and y axis). 4

$ABCD$ is a quadrilateral whose vertices are $A(2, 2)$, $B(2, -2)$, $C (0 - 1)$ and $D(0, 1)$

(i) Reflect quadrilateral $ABCD$ on the y-axis and name it as $A'B'CD$.

(ii) Write down the coordinates of A' and B'.

(iii) Name two points which are invariant under the above reflection.

(iv) Name the polygon $A'B'CD$.

6. (a) Using properties of proportion, solve for x. Given that x is positive : 3

$$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

(b) If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, find $AC + B^2 - 10C$. 3

(c) Prove that $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$ 4

7. (a) Find the value of k for which the following equation has equal roots. 3

$$x^2 + 4kx + (k^2 - k + 2) = 0$$

(b) One map drawn to a scale of 1 : 50,000, a rectangular plot of land $ABCD$ has the following dimensions. AB = 6 cm ; BC = 8 cm and all angles are right angles. Find : 3

(i) the actual length of the diagonal distance AC of the plot in km.

(ii) the actual area of the plot in sq km.

(c) $A(2, 5)$, $B(-1, 2)$ and $C(5, 8)$ are the vertices of a triangle ABC, 'M' is a point on AB such that $AM : MB = 1 : 2$. Find the co-ordinates of 'M'. Hence find the equation of the line passing through the points C and M. 4

8. (a) ₹ 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹ 100 more. Find the original number of children. 3

(b) If the mean of the following distribution is 24, find the value of 'a'. 3

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	7	a	8	10	5

(c) Using ruler and compass only, construct a ΔABC such that BC = 5 cm and AB = 6.5 cm and $\angle ABC = 120°$. 4

(i) Construct a circum-circle of ΔABC

(ii) Construct a cyclic quadrilateral $ABCD$, such that D is equidistant from AB and BC.

9. (a) Priyanka has a recurring deposit account of ₹ 1000 per month at 10% per annum. If she gets ₹ 5550 as interest at the time of maturity, find the total time for which the account was held. **3**

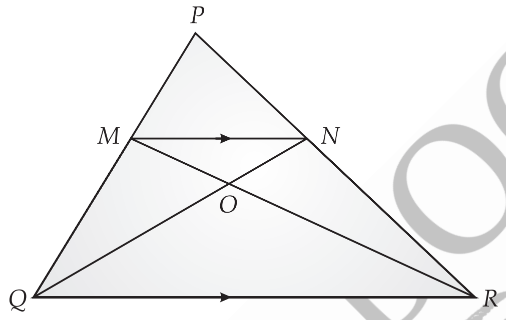
(b) In ΔPQR, MN is parallel to QR and

$$\frac{PM}{MQ} = \frac{2}{3}$$

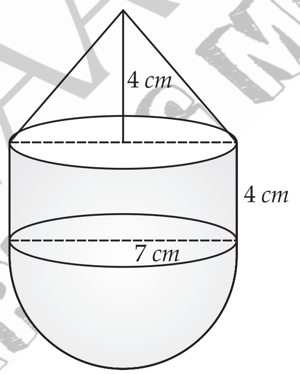
(i) Find $\frac{MN}{QR}$

(ii) Prove that ΔOMN and ΔORQ are similar.

(iii) Find, area of ΔOMN : Area of ΔORQ



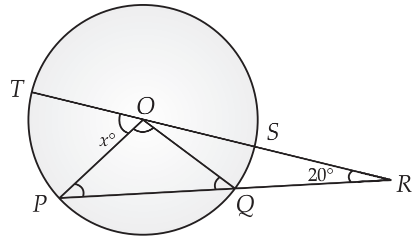
(c) The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find volume of the solid.



10. (a) Use Remainder theorem to factorize the following polynomial : **3**

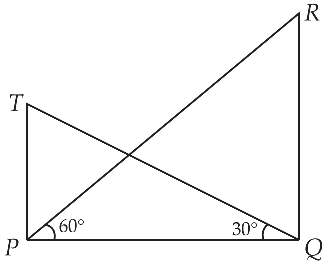
$$2x^3 + 3x^2 - 9x - 10.$$

(b) In the figure given below 'O' is the centre of the circle. If $QR = OP$ and $\angle ORP = 20°$. Find the value of 'x' giving reasons. **3**



(c) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from a point Q is 30°. Find the height of the tower PT, correct to the nearest metre. **4**

To know about more useful books for class-10 [click here]



11. (a) The 4^{th} term of an A.P. is 22 and 15^{th} term is 66. Find the first term and the common difference. Hence find the sum of the series to 8 terms. **4**

(b) Use Graph paper for this question. **6**

A survey regarding height (in cm) of 60 boys belonging to Class 10 of a school was conducted. The following data was recorded :

Height in cm	135-140	140-145	145-150	150-155	155-160	160-165	165-170
No. of boys	4	8	20	14	7	6	1

Taking 2 cm = height of 10 cm along one axis and 2 cm = 10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following :

(i) the median

(ii) lower quartile

(iii) if above 158 cm is considered as the tall boys of the class. Find the number of boys in the class who are tall.

❑❑

SOLUTIONS

SECTION- A

1. (a)

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

or
$$\begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

By equating $2x + 6 = 10$ or $x = 2$

$2y - 5 = 15$ or $y = 10$

(b)

$p = ₹\ 600$

$n = 2½$ year = 30 months

$r = 10\%$ p.a.

$M.V. = ?$

$$M.V. = p \times n + \frac{p \times n\,(n+1) \times r}{2,400}$$

$$= 600 \times 30 + \frac{600 \times 30 \times 31 \times 10}{2,400}$$

$= 18,000 + 2,325$

$= ₹\ 20,325$

(c) (i) Prime number = {2}

No. of favourable cards = 1

Total number of cards = 10

Hence, probability of a getting a prime number card

$$= \frac{\text{Number of favourable cards}}{\text{Total no. of cards}} = \frac{1}{10}$$

(ii) Number divisible by 4 = {4, 8, 12, 16, 20}

No. of favourable cards = 5

Hence, probability of a getting card, where number divisible by 4.

$$= \frac{\text{Number of favourable cards}}{\text{Total no. of cards}} = \frac{5}{10} = \frac{1}{2}$$

(iii) Number which are multiple of 6 = {6, 12, 18}

No. of favourable cards = 3

Hence, probability of getting card, which is multiple of 6.

$$= \frac{\text{Number of favourable cards}}{\text{Total No. of cards}} = \frac{3}{10}$$

(iv) Odd number = 0 (No. odd card)

No. of favourable cards = 0

Hence, probability of getting an odd number card

$$= \frac{\text{Number of favourable cards}}{\text{Total No. of cards}} = \frac{0}{10} = 0$$

2. (a) Given

circumference = 132 cm

height = 25 cm

(i) circumference = $2\pi r$

$$132 = 2\pi r$$

$$132 = \frac{2 \times 22}{7} \times r$$

$$r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Hence, radius of cylinder = 21 cm.

(ii) volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25$$

$$= 34{,}650 \text{ cm}^3.$$

(b) For three consecutive terms of an in A.P., the common difference should be same, *i.e.*

$$(2k + 1) - (k - 3) = (4k + 3) - (2k + 1)$$

$$2k + 1 - k + 3 = 4k + 3 - 2k - 1$$

$$k + 4 = 2k + 2$$

$$k = 2$$

(c) Given $\angle QPS = 73°, \angle PQS = 55°, \angle PSR = 82°$

(i) Since, sum of opposite pairs of angles in cyclic quadrilaterals is 180°

Hence,

$$\angle QRS + \angle QPS = 180°$$

$$\angle QRS + 73° = 180°$$

$$\angle QRS = 180° - 73°$$

$$= 107°$$

(ii) $\angle PQR + \angle PSR = 180°$ (PQRS is a cyclic quadrilateral)

$$\angle PQR + 82° = 180°$$
$$\angle PQR = 180° - 82$$
$$= 98°$$

$\because$ $\angle PQS + \angle RQS = \angle PQR$

$$55° + \angle RQS = 98°$$
$$\angle RQS = 98° - 55°$$
$$= 43°$$

(iii) Join P to R

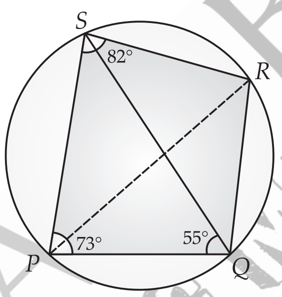
In $\Delta PSQ \angle SPQ + \angle PQS + \angle PSQ = 180°$

$$73° + 55° + \angle PSQ = 180°$$
$$\angle PSQ = 180° - 128°$$
$$= 52°$$

Now,

$\angle PRQ = \angle PSQ$ ($\because$ Angle is same segment)

$\therefore$ $\angle PRQ = 52º$



3. (a) $x^3 + ax + b$

$(x + 2)$ is a factor of given polynomial then $x = -2$ will satisfy the polynomial

$$(-2)^3 + a(-2) + b = 0$$
$$-8 - 2a + b = 0$$
$$b - 2a = 8 \quad ...(i)$$

$(x + 3)$ is also a factor of given polynomial then $x = -3$ will satisfy the polynomial

$$(-3)^3 + a(-3) + b = 0$$
$$-27 - 3a + b = 0$$
$$b - 3a = 27 \quad ...(ii)$$

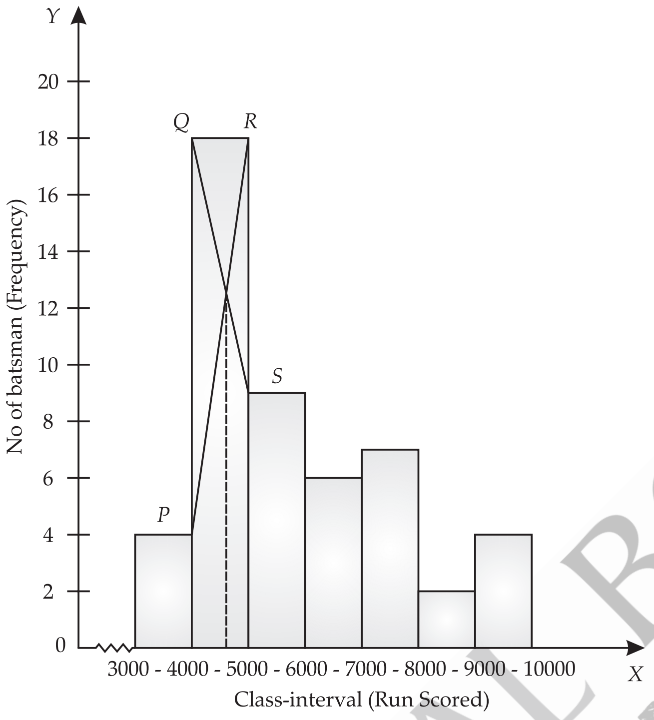
On solving (i) and (ii), we get

$$a = -19, b = -30$$

(b)

$$\text{L.H.S.} = \sqrt{\sec^2\theta + \text{cosec}^2\theta}$$
$$= \sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$$
$$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$$
$$= \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\theta.\cot\theta} \quad [\because \tan\theta.\cot\theta = 1]$$
$$= \sqrt{(\tan\theta + \cot\theta)^2}$$
$$= \tan\theta + \cot\theta = \text{R.H.S.}$$

(c)



$$\text{Mode} = 4600$$

4. (a) $$-2 + 10x \leq 13x + 10 < 24 + 10x$$

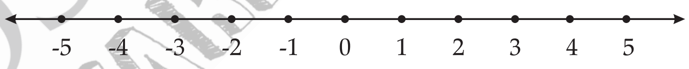
or $$-2 \leq 3x + 10 < 24$$

or $$-12 \leq 3x < 14$$

or $$-4 \leq x < \frac{14}{3}$$

or $$-4 \leq x < 4\frac{2}{3}$$

$\therefore$ The solution set is $x \in [-4, 4]$ and $x \in Z$



(b) Eqn. (i) $$3x - 5y = 7$$

$$y = \frac{3}{5}x - \frac{7}{5}$$

$\therefore$ $$m_1 = \frac{3}{5}$$

Eqn. (ii) $$4x + ay + 9 = 0$$

$$y = \frac{-4}{a}x - \frac{9}{a}$$

$\therefore$ $$m_2 = \frac{-4}{a}$$

both lines are perpendicular to each other.

$\therefore$ $$m_1 \times m_2 = 1$$

or $$\frac{3}{5} \times \left(\frac{-4}{a}\right) = -1$$

or $$\frac{-12}{5a} = -1$$

$$a = \frac{12}{5}$$

(c)
$$x^2 + 7x = 7$$
$$x^2 + 7x - 7 = 0$$

Compare the equation to
$$ax^2 + bx + c = 0$$
$$a = 1, b = 7, c = -7$$
$$D = b^2 - 4ac$$
$$= (7)^2 - 4 \times 1 \times (-7)$$
$$= 49 + 28$$
$$= 77$$

Roots are real, different and irrational number,

or
$$x = \frac{-b \pm \sqrt{D}}{2a}$$

or
$$x = \frac{-7 \pm \sqrt{77}}{2}$$
$$x = \frac{-7 \pm 8.7749}{2}$$

Taking (+) sign
$$x = \frac{-7 + 8.7749}{2}$$
$$= \frac{1.7749}{2} = 0.89$$

taking (–) sign
$$x = \frac{-7 - 8.7749}{2}$$
$$= -\frac{15.7749}{2}$$
$$= -7.89$$

SECTION- B

5. (a) Given, $T_4 = 16$ and $T_7 = 128$

Let First term of G.P. be a and common ratio be r.

so $T_4 = ar^3 = 16$ ....(i)

$T_7 = ar^6 = 128$ ...(ii)

Dividing (ii) by (i), we get
$$\frac{ar^6}{ar^3} = \frac{128}{16}$$
$$r^3 = 8$$
$$r = 2$$

From (i)
$$a.(2)^3 = 16$$
$$a \times 8 = 16$$
$$a = 2$$

(b) Given,

Total investment = ₹ 22,500, Face value = ₹ 50

$$\text{Market value} = ₹\left(50 - \frac{10}{100} \times 50\right) = ₹\ 45$$ ($\because$ 10% discount)

Dividend = 12%

(i) $$\text{No. of Shares} = \frac{\text{Investment}}{\text{Market value of share}}$$

$$= \frac{22500}{45} = 500$$

(ii) $$\text{Annual dividend} = \text{No. of shares} \times \text{Nominal value of share} \times \text{dividend rate}$$

$$= 500 \times 50 \times \frac{12}{100}$$

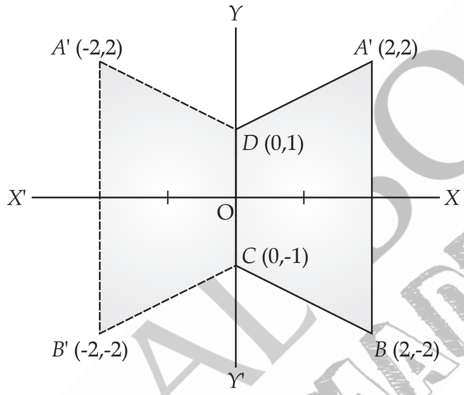
$$= ₹\, 3000$$

(iii) $$\text{Return percentage} = \frac{\text{Total dividend}}{\text{Total investment}} \times 100$$

$$= \frac{3000}{22500} \times 100$$

$$= 13.3\% = 13\%$$

(c) (i)



(ii) Co-ordinates of $A' \rightarrow (-2, 2)$

Co-ordinates of $B' \rightarrow (-2, -2)$

(iii) Two invariant points are $C(0, -1)$ and $D(0, 1)$

(iv) $A'B'CD$ is a trapezium quadrilateral

6. (a) $$\frac{2x + \sqrt{4x^2 - 1}}{2x - \sqrt{4x^2 - 1}} = 4$$

Applying componendo & dividendo

$$\frac{2x + \sqrt{4x^2 - 1} + 2x - \sqrt{4x^2 - 1}}{2x + \sqrt{4x^2 - 1} - 2x + \sqrt{4x^2 - 1}} = \frac{4 + 1}{4 - 1}$$

$$\frac{4x}{2\sqrt{4x^2 - 1}} = \frac{5}{3}$$

$$12x = 10\sqrt{4x^2 - 1}$$

Squaring both sides,

$$144x^2 = 100\,(4x^2 - 1)$$

$$36x^2 = 25(4x^2 - 1)$$

$$36x^2 = 100x^2 - 25$$

$$100x^2 - 36x^2 = 25$$

$$64x^2 = 25$$

$$x^2 = \frac{25}{64}$$

$$x = \pm\frac{5}{8}$$

$\because$ x is positive, Hence $x = \frac{5}{8}$

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(b)

$$AC = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1+3\times-1 & 2\times0+3\times4 \\ 5\times1+7\times-1 & 5\times0+7\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{bmatrix} = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times0+4\times-1 & 0\times4+4\times7 \\ -1\times0+7\times-1 & -1\times4+7\times7 \end{bmatrix}$$

$$= \begin{bmatrix} 0-4 & 0+28 \\ 0-7 & -4+49 \end{bmatrix} = \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}$$

$$10C = 10\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$$\therefore \quad AC + B^2 - 10C = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4-10 & 12+28-0 \\ -2-7+10 & 28+45-40 \end{bmatrix}$$

$$= \begin{bmatrix} -15 & 40 \\ 1 & 33 \end{bmatrix}$$

(c) LHS $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = \left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$

$$= \left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)$$

$$= \frac{(\sin\theta+\cos\theta-1)(\sin\theta+\cos\theta+1)}{\sin\theta\cos\theta}$$

$$= \frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta} \qquad [\because a^2-b^2=(a-b)(a+b)]$$

$$= \frac{(\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta)-1}{\sin\theta.\cos\theta}$$

$$= \frac{(1+2\sin\theta\cos\theta)-1}{\sin\theta.\cos\theta} \qquad [\because \sin^2\theta+\cos^2\theta=1]$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = \text{RHS}$$

Hence proved.

7. (a) $x^2 + 4kx + (k^2 - k + 2) = 0$

Compare the equation to

$$ax^2 + bx + c = 0$$

$$a = 1, b = 4k, c = k^2 - k + 2$$

Since roots of the equation are equal

$$b^2 - 4ac = 0$$
$$(4k)^2 - 4 \times 1 \times (k^2 - k + 2) = 0$$
$$16k^2 - 4k^2 + 4k - 8 = 0$$
$$12k^2 + 4k - 8 = 0$$
$$3k^2 + k - 2 = 0$$
$$3k^2 + 3k - 2k - 2 = 0$$
$$3k(k+1) - 2(k+1) = 0$$

$$(k+1)(3k-2) = 0$$
$$k = -1, k = 2/3$$

(b) (i) Since, $AB = 6$ cm
$BC = 8$ cm

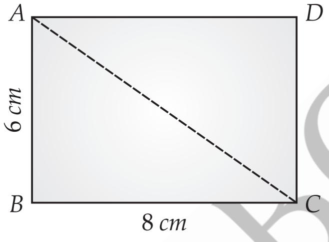
By Pythagorean theorem

$$\text{Diagonal } AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{(6)^2 + (8)^2}$$
$$= \sqrt{100} = 10 \text{ cm}$$

Since, it is given that 1 cm = 50,000 cm (1 : 50000)

$\therefore$ 10 cm = 5,00,000 cm

Diagonal AC = 5 km



(ii)

$$\text{Area} = AB \times BC$$
$$= 6 \times 8$$
$$= 48 \text{ cm}^2$$
$$= 48 \times (50{,}000 \times 50{,}000) \qquad [1 \text{ km}^2 = 10{,}00{,}00{,}00{,}000 \text{ cm}^2]$$
$$= 1{,}20{,}00{,}00{,}00{,}000 \text{ cm}^2$$
$$= 12 \text{ sq. km}$$

(c) Let $M(x_1, y_1)$ be the point which divides the line segment AB in the ratio 1 : 2
Hence, Coordinate of M are

$$M(x_2, y_1) = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$
$$(x_1, y_1) = \left(\frac{1 \times (-1) + 2 \times 2}{1+2}, \frac{1 \times 2 + 2 \times 5}{1+2}\right)$$
$$= \left(\frac{-1+4}{3}, \frac{2+10}{3}\right)$$
$$= (1, 4)$$

Coordinates of M are (1, 4)
Now equation of line passing through C(5, 8) and M(1, 4) is

$$y - 8 = \frac{4-8}{1-5}(x-5)$$
$$y - 8 = \frac{-4}{-4}(x-5)$$
$$y - 8 = x - 5$$
$$y = x + 3$$

8. (a) Let the number of children be x and ₹ y is given to each children then

$$x.y = 7{,}500 \qquad \text{...(i)}$$

and

$$(x-20)(y+100) = 7{,}500$$
$$xy + 100x - 20y - 2{,}000 = 7{,}500$$
$$7{,}500 + 100x - 20y - 2{,}000 = 7{,}500 \qquad [\because xy = 7{,}500]$$
$$100x - 20y - 2{,}000 = 0$$
$$100x - 20 \times \frac{7{,}500}{x} - 2{,}000 = 0 \qquad \left[\text{From (i) } y = \frac{7{,}500}{x}\right]$$
$$100x^2 - 1{,}50{,}000 - 2{,}000x = 0$$
$$x^2 - 20x - 1{,}500 = 0$$
$$x^2 - 50x + 30x - 1{,}500 = 0$$
$$x(x-50) + 30(x-50) = 0$$

$$(x-50)(x+30) = 0$$
$$x = 50, -30$$

The number of children can not be – ve.

$$x = 50$$

$\therefore$ Number of children = 50

(b)

Marks	Mid x_i	f_i	f_ix_i
0-10	5	7	35
10-20	15	a	15a
20-30	25	8	200
30-40	35	10	350
40-50	45	5	225
		$30 + a$	$810 + 15a$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$
$$24 = \frac{810+15a}{30+a}$$
$$720 + 24a = 810 + 15a$$
$$24a - 15a = 810 - 720$$
$$9a = 90$$
$$a = 10$$

(c) **(a)** **Steps of construction :**

(i) Draw a line segment BC = 5 cm.

(ii) Construct $\angle ABC = 120°$.

(iii) Cut BA = 6.5 cm

(iv) Join A to C.

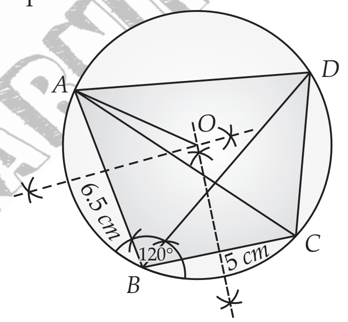
(v) Construct perpendicular bisectors of AB and BC, intersecting at O. Join AO

(vi) Taking O as centre, and OA as radius draw a circle, passing through A, B and C.

(b) **(i)** Draw the bisector of $\angle ABC$ such that it touches the circle at point D.

(ii) Join A to D and C to D.

(iii) $ABCD$ is required cyclic quadrilateral.



9. **(a)** P = ₹ 1,000, R = 10%, p.a., S.I.= ₹ 5,550

$$n = ?$$
$$\text{SI} = \frac{Pn(n+1)}{2} \times \frac{1}{12} \times \frac{R}{100}$$
$$5{,}550 = \frac{1000 \times n(n+1) \times 10}{2 \times 12 \times 100}$$
$$n(n+1) = 1332$$
$$n^2 + n - 1332 = 0$$
$$n^2 + 37n - 36n - 1332 = 0$$
$$n(n+37) - 36(n+37) = 0$$
$$(n+37)(n-36) = 0$$
$$n = 36, -37$$

The value of time can not be – ve, Hence

$$n = 36 \text{ months}$$

or $$n = 3 \text{ years}$$

(i) Given $MN || QR$, then

or $\therefore$ $$\frac{PM}{PQ} = \frac{MN}{QR} \quad \text{(By Thales theorem)}$$

or $$\frac{2}{PM+MQ} = \frac{MN}{QR}$$

or $$\frac{2}{2+3} = \frac{MN}{QR}$$

$\Rightarrow$ $$\frac{MN}{QR} = \frac{2}{5}$$

(ii) $MN || QR$ (Given)

$$\angle OMN = \angle ORQ \quad \text{(Alternate angle)}$$
$$\angle ONM = \angle OQR \quad \text{(Alternate angle)}$$
$$\angle MON = \angle ROQ \quad \text{(Vert. Opp. angles)}$$

$\therefore$ $$\Delta OMN \sim \Delta ORQ \quad \text{(by AAA similarity)}$$

(iii) $\because$ ΔOMN and ΔORQ are similar triangle

We know that ratio of the areas of two similar triangle is equal to ratio of the squares of the corresponding sides.

$\therefore$ $$\frac{ar(\Delta OMN)}{ar(\Delta ORQ)} = \frac{MN^2}{QR^2}$$
$$= \left(\frac{MN}{QR}\right)^2 = \left(\frac{2}{5}\right)^2$$
$$= \frac{4}{25}$$

Hence, $ar(\Delta OMN) : ar(\Delta ORQ) = 4 : 25$

$$\text{common radius } (r) = 7 \text{ cm}$$
$$\text{Height of cylinder} = \text{height of cone}$$
$$= h = 4 \text{ cm}$$
$$\text{volume of solid} = \text{volume of hemisphere} + \text{volume of cylinder} + \text{volume of cone}$$
$$= \frac{2}{3}\pi r^3 + \pi r^2 h + \frac{1}{3}\pi r^2 h$$
$$= \frac{2}{3}\pi \times 7 \times 7 \times 7 + \pi \times 7 \times 7 \times 4 + \frac{1}{3} \times \pi \times 7 \times 7 \times 4$$
$$= \frac{686\pi}{3} + 196\pi + \frac{196\pi}{3}$$
$$= \frac{686\pi + 588\pi + 196\pi}{3}$$
$$= \frac{1,470\pi}{3} = 490\pi \text{ cm}^3$$

10. (a) Let $$f(x) = 2x^3 + 3x^2 - 9x - 10$$

the factors of the constant terms are $\pm 1, \pm 2, \pm 5$ and ± 10.

We have

$$f(-1) = 2(-1)^3 + 3(-1)^2 - 9(-1) - 10$$
$$= -2 + 3 + 9 - 10$$
$$= 0$$

So, $(x + 1)$ is a factor of $f(x)$

Let, we now divide $f(x) = 2x^3 + 3x^2 - 9x - 10$ by $(x + 1)$ to get the other factor of $f(x)$

$$\begin{array}{r|l} & 2x^2 + x - 10 \\ x+1 & 2x^3 + 3x^2 - 9x - 10 \\ & 2x^3 + 2x^2 \\ & - \quad - \\ \hline & \quad x^2 - 9x - 10 \\ & \quad x^2 + x \\ & \quad - \quad - \\ \hline & \quad\quad -10x - 10 \\ & \quad\quad -10x - 10 \\ & \quad\quad + \quad + \\ \hline & \quad\quad\quad 0 \end{array}$$

$\therefore$
$$\begin{aligned} 2x^3 + 3x^2 - 9x - 10 &= (x+1)(2x^2 + x - 10) \\ &= (x+1)[2x^2 + 5x - 4x - 10] \\ &= (x+1)[x(2x+5) - 2(2x+5)] \\ &= (x+1)(x-2)(2x+5) \end{aligned}$$

(b) Given $QR = OP$ ...(i)

and $\angle ORP = 20°$

$\because$ $\angle ORQ = \angle ORP = 20°$

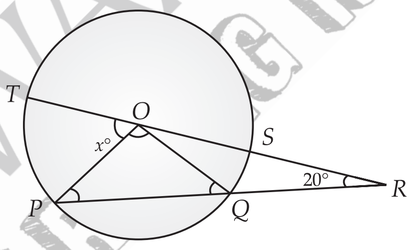
$\because$ $OP = OQ$ (Radius of the circle)

$\therefore$ $QR = OQ$ (From (i))

In ΔOQR :

$\because$ $OQ = QR$

$\therefore$ $\angle QOR = \angle ORQ = 20°$



Now $\angle OQR + \angle ORQ + \angle QOR = 180°$ (By angle sums property)

$$\begin{aligned} \angle OQR + 20° + 20° &= 180° \\ \angle OQR &= 180° - 40° \\ &= 140° \end{aligned}$$

Now $\angle OQR + \angle OQP = 180°$ (By linear pair)

$$\begin{aligned} 140° + \angle OQP &= 180° \\ \angle OQP &= 180° - 140° \\ &= 40° \end{aligned}$$

In ΔPOQ

$\because$ $OP = OQ$

$\therefore$ $\angle OPR = \angle OQP = 40°$

Now $\angle POQ + \angle OPR + \angle OQP = 180°$ (By angle sum property)

$$\begin{aligned} \angle POQ + 40° + 40° &= 180° \\ \angle POQ &= 180° - 80° \\ &= 100° \end{aligned}$$

Now

$\angle ROQ + \angle POQ + x = 180°$ (By Linear pair)

$$\begin{aligned} 20° + 100° + x &= 180° \\ x &= 180° - 120° \\ x &= 60° \end{aligned}$$

(c) Let the height of the tower PT be h and distance $PQ = xm$

In ΔPRQ

$$\tan 60^\circ = \frac{RQ}{PQ}$$

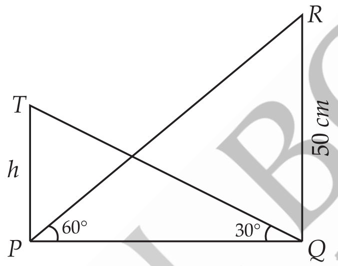
$$\sqrt{3} = \frac{50}{x}$$

or $$x = \frac{50}{\sqrt{3}}$$

In right ΔPTQ,

$$\tan 30^\circ = \frac{PT}{PQ} = \frac{h}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{50/\sqrt{3}}$$



$$3h = 50$$

$$h = \frac{50}{3} = 16.66 = 17 \text{ m}$$

11. (a) Let first term be a and common difference be d.

$$T_4 = a + 3d = 22, \quad \text{...(i)}$$

$$T_{15} = a + 14d = 66 \quad \text{...(ii)}$$

Subtracting (ii) from (i)

$$a + 14d - a - 3d = 66 - 22$$

$$11d = 44$$

$$d = 4$$

From (i)

$$a + 3 \times 4 = 22$$

$$a = 22 - 12$$

$$= 10$$

Thus, $$S_8 = \frac{8}{2}[2 \times 10 + (8-1)4]$$

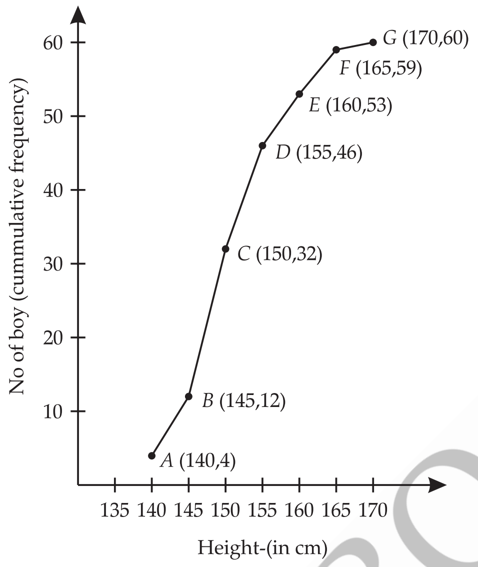
$$= 4[20 + 28]$$

$$= 4 \times 48$$

$$= 192$$

(b)

Height (in cm)	**Frequency (f)**	**Cumulative Frequency (C.F.)**
135-140	4	4
140-145	8	12
145-150	20	32
150-155	14	46
155-160	7	53
160-165	6	59
165-170	1	60
	$N = 60$	



Here, $n = 60$

(i) Median $= \frac{60}{2}$ th term = 30th term = 149.5

(ii) Lower quartile $= \frac{60}{4}$ th term = 15th term = 145.5

(iii) The number of boys in the class, where height is above 158 cm.

$= 60 - 50$

$= 10$