QUARTERLY EXAMINATION – 2018

MATHEMATICS – KEY ANSWER

Marks : 90

Std XII

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I. One marks $20 \times 1 = 20$

1). d). $k^{n-1}(\text{adj } I)$

2). b) 2

3) a) $\Delta \neq 0$

4). b) $-\sqrt{3}$

5) d) $\cos^{-1}\left(\frac{-34}{\sqrt{63}}\right)$

6). b) $x+9y+11z=0$

7) d) $\pm\frac{1}{3}(2\vec{i}-\vec{j}+2\vec{k})$

8). b) $\cos\theta - i\sin n\theta$

9). c) $2+i$

10). a) $\frac{2\pi}{n}$

11). c) $x = -17/4$

12). a) $(0, \pm 3)$

13). b). Corresponding directrix

14). c) $\frac{2\pi}{3}$

15) d). $-\cot\theta$

16). b) 0

17). c). not in the indeterminate form as $x \to 0$
d). in the determinate form as $x \to 0$

18). b). $x = -9/2$

19). a). $x \leq 1$

20) b). 4.021

II Two marks: $7 \times 2 = 14$

21). $\Delta = 0 \quad \Delta_x \neq 0$
No Solution

22). $AB = \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & 24 \\ 5 & 15 & -30 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & 24 & -30 \end{bmatrix} = B^T A^T$

23). $\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{r} = 2\vec{i} + 3\vec{j} + \vec{k}$

$m = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 1 \\ 4 & 2 & 1 \end{vmatrix}$

$\vec{m} = \vec{i} + 2\vec{j} - 8\vec{k}$

25). $\frac{1}{1+2\omega} + \frac{1}{2+\omega} = -\frac{\omega^2}{\omega}$

$\frac{1}{1+2\omega} + \frac{1}{2+\omega} - \frac{1}{1+\omega} = -\omega - \left(\frac{\omega^3}{-\omega^2}\right)$
$= 0$

26). $(e^{i\theta})^{6-9+24-8} = e^{i13\theta}$
$= \cos 13\theta + i\sin 13\theta$

27). $a = 5 \quad e = 3/5 \quad b^2 = 25(1 - 9/25)$
$b^2 = 16$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

28). $f(x) = |x-1|$
$f(x)$ is continuous on $[0, 2]$
not differentiable at $x = 1$
$f(0) = 1 = f(2)$
Rolle's theorem fails.

29). $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

$f'(x) = 0$

$\boxed{x = \pi/4}$

30). $y = \sqrt{1-x}$ $\quad x = 0 \quad dx = 0.02$

$dy = \frac{1}{2}(1-x)^{-1/2} \cdot (-1)\, dx$

$dy = -\frac{1}{2\cdot(1-x)^{1/2}}\ 0.02$

$\boxed{dy = -0.01}$

Three marks:

31) $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

$3A^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} = \text{adj} A$

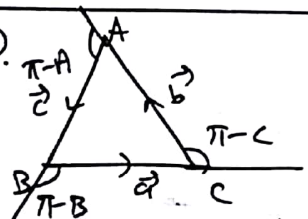
32). $C_1 \leftrightarrow C_3$

$A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 4 & 3 & 6 & 7 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 2$

33).



$\frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}|\vec{b}\times\vec{c}| = \frac{1}{2}|\vec{c}\times\vec{a}|$

$ab\sin C = bc\sin A = ca\sin B$

$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$

$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

34). Given Question wrong.

ie. $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 35$ $\quad \vec{b}$

$\vec{x}^2 - \vec{a}^2 = 35 \qquad |\vec{a}| = 1$

$\vec{x}^2 - 1 = 35$

$\vec{x}^2 = 36$

$\boxed{|\vec{x}| = 6}$

35). Given root $3+i$

other root $3-i$ $\qquad$ Sum $= 6$

product $= 10$

factor $x^2 - 6x + 10$.

other factor $x^2 - 2x + 2$

$x \Rightarrow 2+i,\ 2-i$

$\therefore$ The roots are $\cancel{1} \pm i,\ 3 \pm i$

36). $e = \sqrt{1 + b^2/a^2}$ $\qquad \frac{y^2}{11/4} - \frac{x^2}{275/44}$

$e = \frac{6}{\sqrt{11}}$ $\qquad$ Focus: $(0, \pm 3)$

Vertices: $(0, \pm \frac{\sqrt{11}}{2})$

37).

$3x^2 - 5xy - 2y^2 + 17x + y + 14$

$= (3x + y + l)(x - 2y + m)$

$3x + y + l = 0 \qquad m_1 = -3$

$x - 2y + m = 0 \qquad m_2 = 1/2$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \tan\theta = 7$

$\theta = \tan^{-1}(7)$

38). $y = e^x \qquad y = e^{-x}$

$m_1 = e^x \qquad m_2 = -e^{-x}$

$m_1 \times m_2 = -1 \Rightarrow \cancel{e^x} \times -\frac{1}{\cancel{e^x}} = -1$

$\therefore$ Curves cuts orthogonal.

39). $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$

$= y(-\sin t) + x(\cos t) + 1$

$= -\sin^2 t + \cos^2 t + 1$

$= 2\cos^2 t$ (or) $2 - 2\sin^2 t$ (or) $\cos 2t + 1$

40). $\frac{\partial u}{\partial x} = \frac{\sec^2 x}{\tan x + \tan y + \tan z}$

$\sin 2x \frac{\partial u}{\partial x} = \frac{2\tan x}{\tan x + \tan y + \tan z}$

$|||^{ly}$ $\sin 2y \frac{\partial u}{\partial y} = \frac{2\tan y}{\tan x + \tan y + \tan z}$

$\sin 2z \frac{\partial u}{\partial z} = \frac{2\tan z}{\tan x + \tan y + \tan z}$

$\therefore \sum \sin 2x \frac{\partial u}{\partial x} = 2$

5 marks

$1/x = a,\ 1/y = b,\ 1/z = c$

41) a). $a + 2b - c = 1 \rightarrow (1)$

$2a + 4b + c = 5 \rightarrow (2)$

$3a - 2b - 2c = 0 \rightarrow (3)$

$\Delta = 24 \quad \Delta a = 24 \quad \Delta b = 12 \quad \Delta c = 24$

$a = 1, \quad b = 1/2 \quad c = 1$

$\Rightarrow x = 1, \quad y = 2, \quad c = 1$

b). $z = x + iy$

$\arg(z-2) - \arg(z - 6i) = \pi/2$

$\arg(x+iy-2) - \arg(x+iy-6i) = \pi/2$

$\tan^{-1}\frac{y}{x-2} - \tan^{-1}\frac{y-6}{x} = \pi/2$

$$\frac{\frac{y}{x-2} - \frac{y-6}{x}}{1 + \frac{y}{x-2}\cdot\frac{y-6}{x}} = \tan \pi/2$$

$0 = 1 + \frac{y^2 - 6y}{x^2 - 2x}$

$\Rightarrow \boxed{x^2 - 2x + y^2 - 6y = 0}$

42). $\vec{a} \times \vec{b} = \vec{i} + \vec{j} - 2\vec{k}$

a). $\vec{c} \times \vec{d} = \vec{i} - 3\vec{j} + \vec{k}$.

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = -5\vec{i} - 3\vec{j} - 4\vec{k} \rightarrow (1)$

$[\vec{a}\ \vec{b}\ \vec{c}] = 1 \quad [\vec{a}\ \vec{b}\ \vec{d}] = -2$

$\therefore [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$

$= -5\vec{i} - 3\vec{j} - 4\vec{k} \rightarrow (2)$

from (1) & (2) proved.

42). b). let $\frac{1}{2} - \frac{i\sqrt{3}}{2} = r(\cos\theta + i\sin\theta)$

$r = 1 \quad \cos\theta = 1/2 \quad \sin\theta = -\sqrt{3}/2 \quad \theta = -\pi/3$

$\therefore \left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)^{3/4} = 1\left[\cos(-\pi/3) + i\sin(-\pi/3)\right]^{3/4}$

$= \cos\left(\frac{2k-1}{4}\right)\pi + i\sin\left(\frac{2k-1}{4}\right)\pi$

$k = 0, 1, 2, 3$

Product : $\text{cis}\left[-\pi/4 + \pi/4 + 3\pi/4 + 5\pi/4\right]$

$= \text{cis}\ 8\pi/4 = \text{cis}\ 2\pi = 1$

Values: $\text{cis}(-\pi/4),\ \text{cis}\ \pi/4,\ \text{cis}\ 3\pi/4,\ \text{cis}\ 5\pi/4$

43). a). $\alpha = 1+i \quad \beta = 1-i$

$(y+\alpha)^n = (\cot\theta + i)^n = \frac{\cos n\theta + i\sin n\theta}{\sin^n\theta}$

$(y+\beta)^n = \frac{\cos n\theta - i\sin n\theta}{\sin^n\theta}$

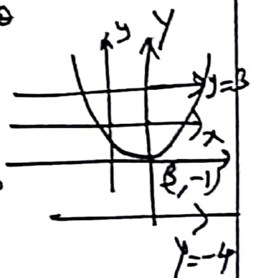
$(y+\alpha)^n - (y+\beta)^n = \frac{2i\sin n\theta}{\sin^n\theta}$

$\therefore \frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n\theta}$

b). $(x-3)^2 = 12(y+1)$

$X^2 = 12Y$

$a = 3$



Axis Y axis $X=0 \quad x-3=0$

Vertex $V(0,0) \quad V(3,-1)$

Focus $F(0,a) \quad F(3,2)$

$F(0,3)$

Direct $Y = -a \quad y+4=0$

Lat Rect $Y = a \quad y-2=0$

Length of LR = 12

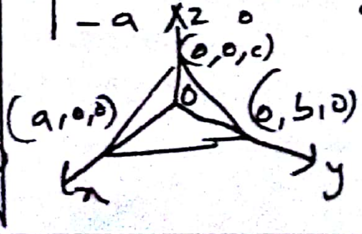
44) a). Vector form: $\vec{a} = a\vec{i},\ \vec{b} = b\vec{j},\ \vec{c} = c\vec{k}$

$\vec{r} = (1-s-t)\vec{a} + s\vec{b} + t\vec{c}$

$x\vec{i} + y\vec{j} + z\vec{k} = (1-s-t)a\vec{i} + sb\vec{j} + tc\vec{k}$

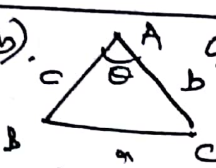
$\frac{x}{a} + y/b + z/c = 1$

CF:
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$$



$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

44) b).



Sin b = 5 cm   a = 4 cm

$\frac{d\theta}{dt} = 0.06$ rad/sec

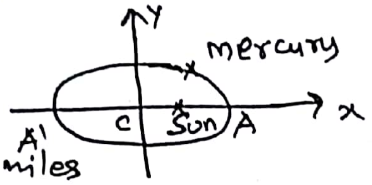
To find $\frac{d\Delta}{dt} = ?$ $\theta = \pi/3$

$\Delta = \frac{1}{2} bc \sin\theta$ $\frac{d\Delta}{dt} = \frac{1}{2} bc \cos\theta \frac{d\theta}{dt}$

$\frac{d\Delta}{dt} = \frac{1}{2} \times 5 \times 4 \times \cos\frac{\pi}{3} \times 0.06$

$\frac{d\Delta}{dt} = 0.3\ m^2/sec$

45). a).



$a = 36$ million miles

$e = 0.206$

CD :: $F_1 A$: $CA - CF = a(1-e)$

$= a - ae = 36 \times 0.794$

$CD = 28.584$ million miles

ii) Farthest position: $F_1 A' = F_1 C + CA'$

$= ae + e = a(e+1)$

$= 36(1 + 0.206) = 1.206 \times 36$

$= 43.416$ million miles

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b). $u = \sin xy$

$\frac{\partial u}{\partial x} = y \cos xy$ $\frac{\partial u}{\partial y} = x \cos xy$

$\frac{\partial^2 u}{\partial y \partial x} = \cos xy - xy \sin xy$ $\frac{\partial^2 u}{\partial x \partial y} = \cos xy - xy \sin xy$

$\therefore \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$

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46). a) Domain: $\forall x \in R$

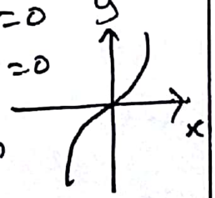
Extent: HE: $-\infty < x < \infty$

VE: $-\infty < y < \infty$

Intercepts: x intercept $y = 0$

y intercept $x = 0$

origin: yes



b) Symmetry: about origin

c). Asymptotes: No

d). Monotonicity: Increasing $(-\infty, \infty)$

e). Special pts: Concave upward $(0, \infty)$

Concave downward $(-\infty, 0)$

$\therefore (0,0)$ is the pt of Inflection.

$y = x^3$ $y' = 3x^2$ $y'' = 6x$

$y'' = 0$ $6x = 0$ $x = 0$

46) b). $y = e^{-x^2}$ $y' = -2x e^{-x^2}$

$y'' = 2e^{-x^2}(2x^2 - 1)$ $y'' = 0$

$x = -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

$(-\infty, -\frac{1}{\sqrt{2}})$ — Concave upward

$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ — Concave downward

$(\frac{1}{\sqrt{2}}, \infty)$ — Concave upward

pt of Infln: $(-\frac{1}{\sqrt{2}}, e^{-1/2}), (\frac{1}{\sqrt{2}}, e^{-1/2})$

47). a). $y = x^{1/3} + x^{1/4}$

$dy = \left(\frac{1}{3} x^{-2/3} + \frac{1}{4} x^{-3/4}\right) dx$

$x = 1$, $dx = \Delta x = 0.02$

$f(x + \Delta x) \approx y + dy = 2 + 0.0116$

$= 2.011$

(or)

b). $[A|B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$ $R_2 \to R_2 - R_1$

Case i). $\lambda - 3 = 0$ $\mu - 10 \neq 0$

$\rho(A) = 2$ $\rho(A|B) = 3$

$\rho(A) \neq \rho(A|B)$ — Inconsistent & No Soln

Case (ii) $\lambda - 3 \neq 0$, $\lambda \neq 3$

$\rho(A) = \rho(A|B) = 3$ Consistent & Unique soln

Case (iii) $\lambda - 3 = 0$ $\mu - 10 = 0$

$\rho(A) = \rho(A|B) = 2 <$ (No of unknowns)

Consistent & Infinitely many Solutions

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