COMMON QUARTERLY EXAMINATION - SEPTEMBER 2018

Standard 12

Reg. No 12AC31

PART - III - MATHEMATICS

Time Allowed: 2.30 Hours | Maximum Marks: 90

Instructions : 1. Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

2. Use Blue or Black ink to write and underlined and pencil to draw diagrams.

SECTION - I

Note : i) Answer all the questions. **20×1=20**

ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1) If I is the unit matrix of order n, where $k \neq 0$ is a constant, then adj(kI) =

a) k^n (adj I) b) k (adj I) c) k^2 (adj I) d) k^{n-1} (adj I)

2) Rank of the second order non-singular matrix is

a) 1 b) 2 c) 3 d) 4

3) Cramer's rule is applicable only (with three unknowns) when

a) $\Delta \neq 0$ b) $\Delta = 0$

c) $\Delta = 0, \Delta_x \neq 0$ d) $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

4) If $\bar{a}$ and $\bar{b}$ include an angle 120° and their magnitude are 2 and $\sqrt{3}$ then $\bar{a}$, $\bar{b}$ is equal to

a) $\sqrt{3}$ b) $-\sqrt{3}$ c) 2 d) $-\frac{\sqrt{3}}{2}$

5) If $\bar{a} = 3\bar{i} - 2\bar{j} - 6\bar{k}$ and $\bar{b} = 4\bar{i} - \bar{j} + 8\bar{k}$, then the angle between $2\bar{a}$ and $3\bar{b}$ is

a) $\cos^{-1}\left(\frac{34}{63}\right)$ b) $\sin^{-1}\left(-\frac{34}{63}\right)$ c) $\sin^{-1}\left(\frac{34}{63}\right)$ d) $\cos^{-1}\left(-\frac{34}{63}\right)$

6) The equation of the plane passing through the point (2, 1, −1) and the line of intersection of the planes $\bar{r}.(\bar{i} + 3\bar{j} - \bar{k}) = 0$ and $\bar{r}.(\bar{j} + 2\bar{k}) = 0$ is

a) $x+4y-z = 0$ b) $x+9y+11z = 0$ c) $2x+y-z+5 = 0$ d) $2x-y+z = 0$

7) The unit normal vectors to the plane $2x-y+2z = 5$ are

a) $2\bar{i} - \bar{j} + 2\bar{k}$ b) $\frac{1}{3}(2\bar{i} - \bar{j} + 2\bar{k})$ c) $-\frac{1}{3}(2\bar{i} - \bar{j} + 2\bar{k})$ d) $\pm\frac{1}{3}(2\bar{i} - \bar{j} + 2\bar{k})$

8) $\frac{1 + e^{-i\theta}}{1 + e^{i\theta}} =$

a) $\cos\theta + i\sin\theta$ b) $\cos\theta - i\sin\theta$ c) $\sin\theta - i\cos\theta$ d) $\sin\theta + i\cos\theta$

9) If $-i+2$ is one root of the equation $ax^2-bx+c = 0$, then the other root is

a) $-i-2$ b) $i-2$ c) $2+i$ d) $2i+1$

10) The arguments of nth roots of a complex number differ by

a) $\frac{2\pi}{n}$ b) $\frac{\pi}{n}$ c) $\frac{3\pi}{n}$ d) $\frac{4\pi}{n}$

11) The directrix of the parabola $y^2 = x+4$ is

a) $x = \frac{15}{4}$ b) $x = -\frac{15}{4}$ c) $x = -\frac{17}{4}$ d) $x = \frac{17}{4}$

12) The vertices of the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ are

a) $(0, \pm 3)$ b) $(\pm 2, 0)$ c) $(\pm 3, 0)$ d) $(0, \pm 2)$

13) The foot of the perpendicular drawn from one of the focus of the hyperbola to its asymptotes lies

a) centre b) corresponding directrix

c) vertex d) latus rectum

14) The angle between the asymptotes of the hyperbola $24x^2-8y^2 = 27$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ c) $\frac{2\pi}{3}$ d) $-\frac{2\pi}{3}$

15) If a normal makes an angle θ with positive x-axis then the slope of the curve at the point where the normal is drawn is

a) $-\cot\theta$ b) $\tan\theta$ c) $-\tan\theta$ d) $\cot\theta$

16) Evaluate: $\lim_{x\to\infty}\frac{x^2}{e^x}$

a) 2 b) 0 c) ∞ d) 1

17) L 'Hopital's rule cannot be applied to $\frac{x+1}{x+3}$ as $x\to 0$ because $f(x) = x+1$ and $g(x) = x+3$ are

a) not continuous

b) not differentiable

c) not in the indeterminate form as $x\to 0$

d) in the determinate form as $x\to 0$

18) An asymptotes to the curve $y^2(a+2x) = x^2(3a-x)$ is

a) $x = 3a$ b) $x = -\frac{a}{2}$ c) $x = \frac{a}{2}$ d) $x = 0$

19) The curve $y^2 = x^2(1-x)$ is defined only for

a) $x \le 1$ b) $x \ge 1$ c) $x < 1$ d) $x > 1$

20) The value of $\sqrt[3]{65}$ is

a) 4.201 b) 4.021 c) 4.12 d) 4

SECTION - II

Answer any seven questions. Question Number 30 is compulsory: 7×2=14

21) Solve the given system of linear equations by determinant method:
$x-y = 2;\ 3y = 3x-7$

22) If $A=\begin{bmatrix}-2\\4\\5\end{bmatrix}$, $B=\begin{bmatrix}1 & 3 & -6\end{bmatrix}$ verify that $(AB)^T = B^TA^T$.

23) Show that the torque about the point A(3, −1, 3) of a force $4\vec{i}+2\vec{j}+\vec{k}$ through the point B(5, 2, 4) is $\vec{i}+2\vec{j}-8\vec{k}$.

24) Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{-2}$ and the plane $3x+4y+z+5 = 0$.

25) If $\omega^3 = 1$, then prove that $\frac{1}{1+2\omega} - \frac{1}{1+\omega} + \frac{1}{2+\omega} = 0$.

26) Simplify: $\frac{(\cos 2\theta + i\sin 2\theta)^3 (\cos 3\theta - i\sin 3\theta)^3}{(\cos 6\theta + i\sin 6\theta)^{-4} (\cos\theta + i\sin\theta)^8}$

27) Find the equation of the ellipse if the foci are $(\pm 3, 0)$ and the vertices are $(\pm 5, 0)$.

28) Verify Rolle's theorem for the function $f(x) = |x-1|$, $0 \le x \le 2$.

29) What is the maximum value of the function $\sin x + \cos x$?

30) Find the differential dy, if $y = \sqrt{1-x}$, $x = 0$, $dx = 0.02$.

SECTION - III

Answer any seven questions. Question Number 40 is compulsory: 7x3=21

31) Show that the adjoint of $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ is $3A^T$.

32) Find the rank of the matrix $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$.

33) State and prove Sine formula by Vector method.

34) If $\bar{a}$ is a unit vector and $(\bar{x} - \bar{a}).(\bar{x} - \bar{b}) = 35$, then find $|\bar{x}|$.

35) Solve the equation $x^4-8x^3+24x^2-32x+20 = 0$ if $3+i$ is a root.

36) Find the eccentricity, vertices and foci of the hyperbola $100y^2-44x^2 = 275$.

37) Find the angle between the asymptotes to the hyperbola $3x^2-5xy-2y^2+17x+y+14 = 0$

38) Show that $y = e^x$ and $y = e^{-x}$ cut orthogonally.

39) Using chain rule find $\frac{dw}{dt}$ for $w = xy+z$, where $x = \cos t$, $y = \sin t$, $z = t$.

40) If $u = \log(\tan x+\tan y+\tan z)$, prove that $\Sigma \sin 2x \frac{du}{dx} = 2$.

SECTION - IV

Answer all the questions: 7x5=35

41) a) Solve the following non-homogeneous system of linear equations by determinant method:

$$\frac{1}{x} + \frac{2}{y} - \frac{1}{z} = 1, \quad \frac{2}{x} + \frac{4}{y} + \frac{1}{z} = 5, \quad \frac{3}{x} - \frac{2}{y} - \frac{2}{z} = 0$$

(OR)

b) P represent a variable complex number z if $\arg\left(\frac{z-2}{z-6i}\right) = \frac{\pi}{2}$.

42) a) If $\bar{a} = \bar{i} + \bar{j} + \bar{k}$, $\bar{b} = 2\bar{i} + \bar{k}$, $\bar{c} = 2\bar{i} + \bar{j} + \bar{k}$, $\bar{d} = \bar{i} + \bar{j} + 2\bar{k}$ then verify that $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = [\bar{a}\ \bar{b}\ \bar{d}]\bar{c} - [\bar{a}\ \bar{b}\ \bar{c}]\bar{d}$.

(OR)

b) Find all the values of $\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and hence prove that the product of the values is 1.

43) a) If α and β are the roots of $x^2 - 2x + 2 = 0$ and $\cot\theta = y + 1$, then show that $\frac{(y+\alpha)^n - (y+\beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n \theta}$.

(OR)

b) Find the axis, vertex, focus, equation of directrix, latus rectum and length of the latus rectum of the parabola $x^2 - 6x - 12y - 3 = 0$ and hence sketch its graph.

44) a) Derive the equation of the plane in the intercept form.

(OR)

b) Two sides of a triangle are 4m and 5m in length and the angle between them is increasing at a rate of 0.06 rad/sec. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.

45) a) The orbit of the planet mercury around the sun is in elliptical shape with sun at a focus. The semi-major axis is of length 36 million miles and the eccentricity of the orbit is 0.206. Find (i) how close the mercury gets to sun (ii) the greatest possible distance between mercury and sun.

(OR)

b) If $u = \sin xy$, verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

46) a) Trace the curve $y = x^3$.

(OR)

b) Find the points of inflection and determine the intervals of convexity and concavity of the Gaussion curve $y = e^{-x^2}$.

47) a) Use differentials to find an approximate value of $y = \sqrt[3]{1.02} + \sqrt[4]{1.02}$.

(OR)

b) Investigate for what values of λ, μ the simultaneous equations $x+y+z = 6$, $x+2y+3z = 10$, $x+2y+\lambda z = \mu$ have (i) no solution (ii) a unique solution and (iii) an infinite number of solutions.
