HIGHER SECONDARY II YEAR

MATHEMATICS

Model Question Paper - 2

Time : 2.30 Hours Marks : 90

Part - I

All questions are compulsory **20 × 1 = 20**

Choose the correct answer

1. If the matrix $\begin{bmatrix} -1 & 3 & 2 \\ 1 & K & -3 \\ 1 & 4 & 5 \end{bmatrix}$ has an inverse, then the value of K

   a) K is any real number b) K = –4 c) K ≠ –4 d) K ≠ 4

2. The value of $\begin{vmatrix} \text{Cos } 15^0 & \text{Sin } 15^0 \\ \text{Cos } 45^0 & \text{Sin } 45^0 \end{vmatrix} \times \begin{vmatrix} \text{Cos } 45^0 & \text{Cos } 15^0 \\ \text{Sin } 45^0 & \text{Sin } 15^0 \end{vmatrix}$ is

   a) $\frac{1}{4}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{-\sqrt{3}}{4}$ d) $\frac{-1}{4}$

3. $\vec{r} = s\vec{i} + t\vec{j}$ is the equation of

   a) a straight line joining the points $\vec{i}$ and $\vec{j}$ b) xoy plane

   c) yoz plane d) zox plane

4. If $\vec{a} = \vec{i} + \vec{j} - \vec{k}$, $\vec{b} = \vec{i} - \vec{j} + \vec{k}$ $\vec{c} = \vec{i} - \vec{j} - \vec{k}$ then the value of $\vec{a} \times \left(\vec{b} \times \vec{c} \right)$ is

   a) $\vec{i} - \vec{j} + \vec{k}$ b) $2\vec{i} - 2\vec{j}$ c) $3\vec{i} - \vec{j} + \vec{k}$ d) $2\vec{i} + 2\vec{j} - \vec{k}$

5. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, The minimum value of $|z_1 - z_2|$ is

   a) 0 b) 2 c) 7 d) 17

6. If $-i + 3$ is a root of $x^2 - 6x + K = 0$ then the value of K is

   a) 5 b) $\sqrt{5}$ c) $\sqrt{10}$ d)10

7. The distance between the foci of the ellipse $9x^2 + 5y^2 = 180$ is

   a) 4 b) 6 c) 8 d) 2

8. If the foci of an ellipse are (3, 0), (–3, 0) and the eccentricity is 1/2 then the equation of the ellipse is

a) $\frac{x^2}{16}+\frac{y^2}{9}=1$ b) $\frac{x^2}{9}+\frac{y^2}{4}=1,$ c) $\frac{x^2}{36}+\frac{y^2}{27}=1$ d) $\frac{x^2}{25}+\frac{y^2}{9}=1$

9. A particle's velocity V at time t is given by $V = 2e^{2t}\cos\left(\pi t/3\right)$ what is the least value of t at which the acceleration becomes zero ?

a) 0 b) $\frac{3}{2}$ c) $\left(\frac{3}{\pi}\right)\tan^{-1}\left(\frac{6}{\pi}\right)$ d) $\frac{3}{\pi}\cot^{-1}\left(\frac{6}{\pi}\right)$

10. The 'c' of Lagrange's mean value theorem for the function $f(x) = x^2 + 2x - 1$, a = 0, b = 1 is

a) –1 b) 1 c) 0 d) 1/2

11. If $u = \log\left(\frac{x^2+y^2}{xy}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is

a) 0 b) u c) 2u d) u^{-1}

12. $\frac{\partial^2}{\partial x\,\partial y}(x^y) =$

a) $x^{y-1}(1 + y \log x)$ b) $y(y - 1)x^{y-2}$

c) $x^{y-1} + (y - 1)x^{y-2}$ d) $x^y(x - y \log x)$

13. The value of $\int_0^{\pi/2}\frac{\text{Sin x} - \text{Cos x}}{1+\text{Sin x Cos x}}$ is

1) $\pi/2$ 2) 0 3) $\pi/4$ 4) π

14. The plane region is enclosed by the line x + y – 2 = 0, x axis and y axis. The volume generated by this region when it is revolved about x - axis is

a) $\frac{\pi}{3}$ cu. unit b) $\frac{2\pi}{3}$ cu. units c) $\frac{4\pi}{3}$ cu. units

d) $\frac{8\pi}{3}$ cu. units

15. The solution of the equation $\frac{dy}{dx}=\frac{y}{x}+\tan\left(\frac{y}{x}\right)$ is

a) $cx = \tan^{-1}\left(\frac{y}{x}\right)$ b) $cy = \cos\left(\frac{x}{y}\right)$ c) $cx = \text{Sin}\left(\frac{y}{x}\right)$ d) $cy = \text{Sin}\left(\frac{x}{y}\right)$

16. The differential equation $\left(\frac{dx}{dy}\right)^2 + 5y^{1/3} = x$ is

a) of order 2 and degree 1 b) of order 1 and degree 2

c) of order 1 and degree 6 d) of order 1 and degree 3

17. If P is T and q is F, then which of the following have the truth value T ?

(i) $p \vee q$ (ii) $\sim p \vee q$ (iii) $p \vee \sim q$ (iv) $P \wedge \sim q$

a) (i), (ii), (iii) b) (i), (ii), (iv) c) (i), (iii), (iv) d) (ii), (iii), (iv)

18. The set of all nth roots of unity form an abelian group under multiplication. The inverse of the element $\cos(n-1)\frac{2\pi}{n} + i \sin(n-1)\frac{2\pi}{n}$ is

a) $\cos(n-1)\pi + i \sin(n-1)\pi$ b) $\cos n\pi + i \sin n\pi$ c) $\cos\frac{2\pi}{n} + i \sin\frac{2\pi}{n}$

d) $\cos 2\pi + i \operatorname{Sin} 2\pi$

19. The probability that any number between 1 and 20 be divisible either by 3 or by 7 is

a) $\frac{2}{5}$ b) $\frac{1}{3}$ c) $\frac{4}{9}$ d) $\frac{5}{10}$

20. If f(x) is a p.d.f of a normal variate X and $X \sim N(\mu, \sigma^2)$ then $\int_{-\infty}^{\mu} f(x)\,dx$

a) undefined b)1 c) 0.5 d) –0.5

Part - II

Answer any Seven questions Question 30 is Compulsory **7 × 2 = 14**

21. Find the inverse of A, where $A = \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}$

22. For any two vectors $\vec{a}$ and $\vec{b}$ prove that $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$

23. Compute the square roots of $Z = -1 - i$

24. Compute real and imaginary parts of $Z = \frac{i-4}{2i-3}$

25. Find the equation of the parabola, if the curve is open rightward, vertex is (2, 1) and passing through the point (6, 5).

26. Prove that the function $f(x) = \sin x + \cos 2x$ is not monotonic on the interval $\left[0, \frac{\pi}{4}\right]$

27. Estimate $\sqrt{4.001}$ by approximate value using differentials.

28. Find the value of $\int_0^{\pi/2} e^{ax} \cos bx\,dx$,.

29. Find the degree and order of the equation $x\frac{dy}{dx} = y + \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2}$.

30. Out of 13 applications for a job, there are 8 men and 5 women. It is decided to select 2 persons for the job. Find the probability that atleast one of the selected person will be a woman.

Part - III

Answer any SEVEN questions **7 × 3 = 21**

Question No.40 is compulsory.

31. Find the rank of the matrix.

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$$

32. A particle is acted upon by constant forces $4\vec{i} + \vec{j} - 3\vec{k}$ and $3\vec{i} + \vec{j} - \vec{k}$ which displace it from a point $\vec{i} + 2\vec{j} + 3\vec{k}$ to the point $5\vec{i} + 4\vec{j} + \vec{k}$. What is the work done in standard units by the force.

33. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$,then find (a, b).

34. Find the equation of a point which moves so that the sum of its distances from (–4, 0) and (4, 0) is 10.

35. Find the points of inflection of y = tan x in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

36. If u = log (tan x + tan y + tan z), prove that $\sum \text{Sin } 2x \frac{\partial u}{\partial x} = 2$.

37. Find the area between the curves $y = e^x$ and $y = e^{-x}$ from x = –1 to x = 2.

38. Use truth table to determine whether the statement $((\sim p) \vee q) \vee (p \wedge (\sim q))$ is a tautology.

39. Solve $2y \cot x \frac{dy}{dx} = 1 + y^2$

40. A random variable X has the probability mass function as follows :

X	–2	3	1
P (X = x)	$\frac{\lambda}{6}$	$\frac{\lambda}{4}$	$\frac{\lambda}{12}$

Then find E(x).

Part - IV

Answer all the questions **7 × 5 = 35**

41. (a) Investigate for what values of λ, μ the simultaneous equations x + y + z = 6, x + 2y + 3z = 10, x + 2y + λz = μ have (i) no solution (ii) a unique solutions (iii) an infinite number of solutions

or

(b) Altitudes of a triangle are concurrent prove by vector method.

42. (a) P represents the variable complex number z, Find locus of P if arg $\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$

or

(b) Find the axes, centre, focus and directrix of the parabola $y^2 - 6y - 8x + 25 = 0$.

43. (a) The orbit of the earth around the sun is elliptical in shape with sun at a focus. The semi major axis is of length 92.9 million miles and eccentricity is 0.017. Find how close the earth gets to sun and the greatest possible distance between the earth and the sun.

(or)

(b) Show that the value of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$

44. (a) Evaluate $\lim_{x\to\frac{\pi}{2}^-}(\tan x)^{\cos x}$

or

(b) If $[\vec{b}, \vec{c}, \vec{d}] = 24$ and $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})+(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})+K\vec{a}=0$ then find the value of K/16.

45. (a) Trace the curve $y^2 = 2x^3$

or

(b) The curves y = sinx and y = tanx touch each other at x = 0. What is the area bounded by them and the line $x=\frac{\pi}{3}$?

46. (a) Find the length of the curve $\left(x/a\right)^{2/3}+\left(y/a\right)^{2/3}=1$

or

(b) Let $L(y)=\frac{d^2y}{dx^2}+7\frac{dy}{dx}+3y$. If $y_1(x)$ and $y_2(x)$ are solutions of $L(y)=Sinx$ and $L(y)=Cosx$ respectively then find the differential equation having $y_1(x) + y_2(x)$ as a solution.

47. (a) Show that $(z_n, +_n)$ forms group.

or

(b) The probability distribution of a discrete random variable x is given by

X	–2	2	5
P (x=x)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

then find $4E(x^2) – Var(2x)$.