XII Std
MODEL QUESTION PAPER 3
Time : 2.30 Hours   BUSINESS MATHEMATICS   Marks : 90
Part - A (20 × 1 = 20)

I. Answer all questions
Choose the correct answer from the given alternative

1. Rank of a 3 × 3 unit matrix is
   a) 0   b) 1   c) 2   d) 3

2. If A is a square matrix of order 2 then |Adj A| is
   a) $|A|^2$ b) $|A|$ c) $|A|^3$ d) $|A|^4$

3. If the foci of an ellipse are (3, 0), (–3, 0) and eccentricity is $\frac{1}{2}$. Then the equation of the ellipse is
   a) $\frac{x^2}{16} + \frac{y^2}{9} = 1$ b) $\frac{x^2}{9} + \frac{y^2}{4} = 1$ c) $\frac{x^2}{36} + \frac{y^2}{27} = 1$ d) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

4. If a is the length of the semi transverse axis of rectangular hyperbola $xy = c^2$ then the value of c^2 is
   a) a^2 b) $2a^2$ c) $\frac{a^2}{2}$ d) $\frac{a^2}{4}$

5. The slope of the curve $y = 2x^2 + 3x$ at $x = 0$
   a) 3   b) –3   c) 2   d) –2

6. If 20 units of some product cost Rs.2500 and 50 units cost Rs.3,400 to produce, then the linear cost function is
   a) $y = 30x + 1900$ b) $y = 20x + 5900$ c) $y = 50x + 3400$
   d) $y = 10x + 900$

7. A point on a curve $y = f(x)$, where concavity changes from upto down or vice versa is called
   a) a point of inflection   b) absolute maximum   c) concavity
   d) elasticity of demand

8. If $q_1 = 240 – p_2 + 8p_1$ then $\frac{\partial q_1}{\partial p_1}$ is
   a) 8   b) –1   c) 240   d) 0

9. The area under the curve $y = f(x)$, x axis and the ordinates at $x = a$ and $x = b$ is
   a) $\int_a^b y\,dx$ b) $\int_a^b y\,dy$ c) $\int_a^b x\,dy$ d) $\int_a^b x\,dx$

10. The demand and supply functions under pure competition are $P_d = 16 - x^2$, $P_s = 2x^2 + 14$. The value of x at the market equilibrium is
    a) 2   b) 4   c) 6   d) 12

11. When $y = vx$ the differential equation $x\frac{dy}{dx} = y + \sqrt{x^2 + y^2}$ reduces to
    a) $\frac{dv}{\sqrt{V^2 - 1}} = \frac{dx}{x}$ b) $\frac{Vdv}{\sqrt{V^2 + 1}} = \frac{dx}{x}$ c) $\frac{dv}{\sqrt{V^2 + 1}} = \frac{dx}{x}$ d) $\frac{Vdv}{\sqrt{1 - V^2}} = \frac{dx}{x}$

12. The order of the differential equation $1+\left(\dfrac{dy}{dx}\right)^5 = \dfrac{d^3y}{dx^3}$ is

    a) 5   b) 3   c) 1   d) 0

13. The normal equations for the line of best fit of the form $y = a + bx$ are

    a) $b\Sigma x + a\Sigma x^2 = y$, $a\Sigma x + b\Sigma x^2 = \Sigma xy$ b) $na + b\Sigma x = \Sigma y$, $b\Sigma x + a\Sigma x^2 = \Sigma xy$

    c) $na + b\Sigma y = \Sigma x$, $a\Sigma x + b\Sigma x^2 = \Sigma xy$ d) $na + b\Sigma x = \Sigma y$, $a\Sigma x + b\Sigma x^2 = \Sigma xy$

14. When $h = 1$ then $\Delta\,(x^2) =$

    a) $2x$ b) $2x - 1$ c) $2x + 1$ d) 1

15. A random variable X has the following probability distribution

X	−2	3	1
P (X=x)	$K/6$	$K/4$	$K/12$

Then the value of K is

    a) 2   b) 3   c) 5   d) 6

16. If $X \sim N\,(\mu, \sigma^2)$, the maximum probability at the point of inflection of normal distribution is

    a) $\dfrac{1}{\sqrt{2\pi}}\,e^{1/2}$ b) $\dfrac{1}{\sqrt{2\pi}}\,e^{-1/2}$ c) $\dfrac{1}{\sigma\sqrt{2\pi}}$ d) $\dfrac{1}{\sqrt{2\pi}}$

17. The acceptance region for Z at 5% level is

    a) $|Z| \le 1.96$ b) $|Z| \ge 2.58$ c) $|Z| < 1.96$ d) $|Z| > 2.58$

18. Which of the following statements is true ?

    a) Point estimate gives a range of values

    b) Sampling is done only to estimate a statistic

    c) Sampling is done to estimate the population parameter

    d) Sampling is not possible for an infinite population

19. Time series consists of

    a) Two components   b) Three components   c) Four components

    d) Five components

20. Laspeye's index formula uses the weights of the

    a) base year quantities   b) current year prices

    c) average of the weights of number of years   d) none of these

<div align="center">PART - B (7 × 2 = 14)</div>

Answer any seven questions Question No. 30 is compulsory.

21. Two square matrices of order 3 are given below.

$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ Verify that one is the inverse of each other.

22. A point moves in such a way that the sum of its distances from (−4, 0) and (4, 0) is 10. Find the length of the semi major axis.

23. If the area of a square increases at a uniform rate, then prove that the rate of increase of the perimeter varies inversely as the side of the square.

24. Find the stationary points of the function $f(x) = x^3 - 27x + 108$.

25. Evaluate : $\displaystyle\int_{-10}^{10} (x^3 - x)\, dx$

26. Solve : $\cot x \dfrac{dy}{dx} = 1 + y^2$

27. Find the mean for the following probability distribution $f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

28. What one Type I error and Type II error ?

29. Calculate the correlation coefficient from the following data.
    $N = 25$, $\Sigma x = 125$, $\Sigma y = 100$, $\Sigma x^2 = 650$, $\Sigma y^2 = 436$, $\Sigma xy = 520$

30. In Newton's Gregory Interpolation find u if
    $h = 5$, $x_0 = 75$, $x_0 = 82$

PART - C ($7 \times 3 = 21$)

Answer any seven questions :
Question no.40 is compulsory :

31. A machine sells at Rs. p and the demand for x (in hundreds) machines per year is given by
    $x = \dfrac{90}{p+5} - 6$. What type of demand curve corresponds to the above demand's law ? and of
    what price the demand reaches the value zero.

32. Show that the elasticity of demand function $p = \dfrac{100}{q}$ is unity for every value of q.

33. For the production function $P = C(L)\alpha\ (K)\beta$ where C is a positive constant and if $\alpha\ +$
    $\beta\ = 1$ Show that $\dfrac{K\, \partial P}{\partial K} + \dfrac{L\, \partial P}{\partial L} = P$

34. If the marginal revenue for a commodity is $MR = \dfrac{e^{-2x}}{50}$ Find the revenue function.

35. A curve passes through the origin and its gradient at any point (x, y) on it is $1 - \dfrac{x^2}{3}$. Form
    the differential equation.

36. Find the missing term from the following data.

x	1	2	3	4
f(x)	100	-	126	157

37. $F(x) = \dfrac{1}{\pi}\left(\dfrac{\pi}{2} + \tan^{-1} x\right)$, $-\infty < x < \infty$ is a distribution function of a continuous random variable. Then find $P\ (0 \leq x \leq 1)$

38. A random sample of marks in mathematics secured by 50 students out of 200 students showed a mean of 75 and a standard deviation of 10. Find the 95% confidence limits for the estimate of their mean marks.

39. Below are given figures of production (in thousand tonnes) of sugar factory. Obtain the trend values by 3 year moving average.

Year	1980	1981	1982	1983	1984	1985	1986
Production	80	90	92	83	94	99	92

40. For what values of K the equations $x + y + z = 1$, $x + 2y + 4z = k$, $x + 4y + 10z = k^2$ are consistent.

Part - D ($7 \times 5 = 35$)

Anser all the questions.

41. Two newspapers A & B are published in a city. Their present market shares are 15% for A and 85% for B, of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B, of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after two years.

(or)

Find the equations of the asymptotes of the hyperbola $6x^2 + 5xy - 6y^2 + 12x + 5y + 3 = 0$.

42. The total cost function y for x units is given by $y = 3x\left(\dfrac{x+7}{x+5}\right) + 5$. Prove that the marginal cost falls continuously as the output x increases.

(or)

Find the area of one loop of the curve $y^2 = x^2(a^2 - x^2)$ between $x = 0$ and $x = a$.

43. The annual demand for an item is 3200 units. The unit cost is Rs.6 and inventory carrying charges 25% per annum. If the cost of one procurement is Rs.150. Determine (i) Economic order quantity (ii) Time between two consecutive orders (iii) Number of orders per year (iv) Minimum average yearly cost.

(or)

Solve : $(16D^2 - 24D + 9)\,y = 13\,e^{3x/4}$

44. If $f(0) = 5$, $f(1) = 6$, $f(3) = 50$, $f(4) = 105$ find $f(2)$ by using Lagrange's formula.

(or)

Solve the equations by Cramer's method
$6x + 2y - 2z = 6$, $-2x + 2y + 2z = 2$, $2x + 2y + 2z = 6$.

45. The number of accidents in a year attributed to taxi drivers in a city follows a poisson distribution with mean 3. Out of 1000 taxi drivers, find the approximate number of drivers with (i) no accident in a year (ii) more than 3 accidents in a year.

(or)

What is the maximum slope of the tangent to the curve. $y = -x^3 + 3x^2 + 9x - 27$ and at what point is it ?

46. The income distribution of the population of a village has a mean of Rs.6,000 and variance of Rs.32,400. Could a sample of 64 persons with a mean income of Rs.5,950 belong to this population (Test at 5% level of significance)

(or)

Solve the linear programming problem. Maximize $Z = 5x_1 + 6x_2$ subject to
$x_1 + x_2 \le 10$
$x_1 - x_2 \ge 13$
$5x_1 + 4x_2 \le 35$ $x_1, x_2 \ge 0$

47. The net profit and quantity x satisfy the differential equation $\dfrac{dp}{dx} = \dfrac{2p^3 - x^3}{3xp^2}$. Find the relationship between the net profit and demand given that $p = 20$ when $x = 10$.

(or)

Using the following data, construct Fisher's Ideal index and show that it Satisfies Factor Reversal Test and Time Reversal Test.

Commodity	Price		Quantity	
	Base year	Current year	Base year	Current year
A	6	10	50	56
B	2	2	100	120
C	4	6	60	60
D	10	12	30	24
E	8	12	40	36