MATHEMATICS

1. Given, $A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$ & $f(x) = 1 + x + x^2 + \ldots + x^{16}$

$\Rightarrow$ $f(A) = I + A + A^2 + \ldots + A^{16}$

$$A^2 = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A^2 = A^3 = \ldots = A^{16} = 0$

$$\Rightarrow f(A) = I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$$

Ans. (B)

2.

Given that 5 balls cannot fit in small boxes.
So, we have only 4 balls to arrange in 3 small boxes.
Which can be arranged in ${}^4C_3 \times 3!$
Now we have 6 balls left, which can be arranged in 6! ways.
Hence the total number of ways will be
${}^4C_3 \times 3! \times 6! = 17280$

Ans. (c)

Note : This question can also be solved direct by SHORTCUT

3. Any function will be inverse of itself if f(f(x)) = x

From option (a) $f(x) = \dfrac{1-x}{1+x}$

then $f[f(x)] = f\left[\dfrac{1-x}{1+x}\right] = \dfrac{1-\left(\dfrac{1-x}{1+x}\right)}{1+\left(\dfrac{1-x}{1+x}\right)} = \dfrac{2x}{2} = x$

Ans. (a)

4. To select one president, one vice president, one secretary, one joint secretary & 2 executive committee members from 10 members;

First we will have to select 6 people from 10 which can be done in ${}^{10}C_6$ ways.

Now we have to appoint them in 6 posts & it can be done in 6! Ways, but there are 2 executive (repetition).

So, that no. of ways will be $\frac{6!}{2!}$

Hence, total ways in which we can appoint these members = ${}^{10}C_6 \times \frac{6!}{2!}$

= 75600

Ans. (b)

5. Let M, B & L be the set of students studying, Mathematics, Business & literature respectively

We have $n(M) = 32$, $n(B) = 38$, $n(L) = 30$

$n(M \cap L) = 7$, $n(M \cap B) = 10$, $n(B \cap L) = 8$

& $n(M \cap B \cap L) = 5$

Total no. of Students Studying Exactly one subject.

$= n(M) + n(B) + n(L) - 2\{n(m \cap B) + n(B \cap L) + n(M \cap L)\}$

$+ 3n(M \cap B \cap L)$

$= 32 + 38 + 30 - 2\{7 + 10 + 8\} + 3(5)$

$= 100 - 50 + 15$

$= 65$ **Ans. (b)**

Note : This question can also be solved direct by SHORTCUT

6. $6 + \log_{\frac{1}{4}} \frac{1}{\sqrt{2}} \left[\sqrt{1 - \frac{1}{\sqrt{2}} \sqrt{1 - \frac{1}{\sqrt{2}} \sqrt{1 - \frac{1}{\sqrt{2}} \ldots}}} \right]$ .....(1)

Let $y = \frac{1}{\sqrt{2}} \sqrt{1 - \frac{1}{\sqrt{2}} \sqrt{1 - \frac{1}{\sqrt{2}}} \ldots\ldots}$ $(y > 0)$

$2y^2 = 1 - y \Rightarrow 2y^2 + y - 1 = 0$

$$y = \frac{-1 \pm \sqrt{1+8}}{4} = -1, \frac{1}{2}$$

$y > 0 \Rightarrow y = ½$

So, putting the value of y in (1)

$$6 + \log_{\frac{1}{4}} \frac{1}{2} = 6 + \log_{\left(\frac{1}{2}\right)^2} \frac{1}{2}$$

$$= 6 + \frac{1}{2} = \frac{13}{2}$$

Ans. (b)

7. We have $f(x) = \log\left(x+\sqrt{x^2+1}\right)$

$\Rightarrow f(-x) = \log\left(-x+\sqrt{x^2+1}\right)$

$$= \log\left(\frac{\sqrt{x^2+1}-x}{1}\times\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\right) \text{ [on rationalizing]}$$

$$= \log\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}+x}\right) = \log\left(x+\sqrt{x^2+1}\right)^{-1}$$

$$= -\log\left(x+\sqrt{x^2+1}\right)$$

We have f(-x) = -f(x).
Hence, f(x) is an odd function.
Ans. (b)

Note : This question can also be solved direct by SHORTCUT

8. There are 60 minutes between 6 PM & 7 PM
Probability of Meeting longer than 20 minutes of 2 persons

$$= \left(\frac{40}{60}\right)^2 = \frac{4}{9}$$

So the probability of Meeting no longer then 20 minutes will be

$$1-\frac{4}{9} = \frac{5}{9}$$

Ans. (a)

Note : This question can also be solved direct by SHORTCUT

9. The no. of ways of choosing 3 numbers from the given set of numbers 1, 2, ……,99 is $^{99}C_3$.
Let us divide the given numbers into 3 groups G_1, G_2, G_3 as follows :
G_1 : 3, 6, 9, ……., 99 [33 elements]
G_2 : 1, 4, 7, ……., 97 [33 elements]
G_3 : 2, 5, 8, ……., 98 [33 elements]

We have $a^3 + b^3 + c^3 - 3abc$ to be divisible by 3.

If $a^3 + b^3 + c^3$ is divisible by 3, which is possible in the following cases

(i) All the numbers belong to the first group.

(ii) All the numbers belong to the second group.

(iii) All the numbers belong to the third group.

(iv) One no. belong to the first group, one belong to the IInd group & one belong to the third group.

Favourable cases are 3. $^{33}C_3 + (^{33}C_1)^3$

So required probability = $\dfrac{3.\ ^{33}C_3 + \left(^{33}C_1\right)^3}{^{99}C_3}$

Ans. (a)

10. A & B have to select a number from {1, 2, ...,25}
They will win if they choose the same no. i.e. then probability of choosing same number is $\frac{1}{25}$.

Hence, the probability that they

$$\text{Won't win} = 1-\frac{1}{25}=\frac{24}{25}$$ **Ans. (b)**

11. Given, $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}, P(A) = \frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad \frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3}$$

$$\Rightarrow \quad P(A) = \frac{5}{6} + \frac{1}{3} - \frac{1}{2}$$

$$\Rightarrow \quad P(A) = \frac{2}{3}$$

Now, $P(A) \cdot P(B) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3} = P(A \cap B)$

A and B are independent.

Ans. (b)

12. Using A.M. $\geq$ G.M. , we have

$$\frac{a_1 + a_2 + ... + a_{n-1} + 2a_n}{n} \geq (a_1.a_2.....2a_n)^{1/n}$$

$$\frac{a_1 + a_2 + ... + a_{n-1} + 2a_n}{n} \geq (2c)^{1/n} \quad [\because a_1, a_2, a_3.....a_n = C]$$

$$a_1 + a_2 + ... + a_{n-1} + 2a_n \geq n(2c)^{1/n}$$

Ans. (a)

13. **Ans. (d)**

14. $(1 - 2x + 3x^2 - 4x^3 + \ldots\ldots \text{ to } \infty) = (1 + x)^{-2}$ (By Binomial expansion)

$\therefore\ (1 - 2x + 3x^2 - 4x^3 + \ldots\ldots \text{ to } \infty)^{-n} = (1 + x)^{2n}$

Now, r^{th} term in the expansion of $(1 + x)^{2n}$ is

$= {}^{2n}C_r\ (1)^{2n-r}\ (x)^r$

For x^n put $r = n$

$\therefore$ coefficient of x^n is ${}^{2n}C_n$

$${}^{2n}C_n = \frac{2n!}{\lfloor 2n - n \rfloor \lfloor n}$$

$$= \frac{(2n!)}{(n!)^2}$$

Ans. (c)

15. α, β are the roots of eq. $x^2 - px + r = 0$

$\therefore\ \alpha + \beta = p$ ....(1) & $\alpha\beta = r$

Also, $\alpha/2 + 2\beta = q$

$\therefore\ \alpha + 4\beta = 2q$ ....(2)

Now, subtract (1) from 2)

(2) – (1)

$3\beta = 2q - p$

$$\Rightarrow \beta = \frac{2q - p}{3}$$

Put value of β in eq. (1)

$$\Rightarrow \alpha = \frac{2}{3}(2p - q)$$

As $r = \alpha.\beta$

$$\Rightarrow r = \frac{2}{9}(2p - q)(2q - p)$$

Ans. (d)

16. Given the roots are $\sin^2 18$ and $\cos^2 36$

Then sum of roots :

$$\sin^2 18^\circ + \cos^2 36^\circ$$

$$\Rightarrow \left(\frac{\sqrt{5}-1}{4}\right)^2 + \left(\frac{\sqrt{5}+1}{4}\right)^2$$

$$\Rightarrow \frac{5+1-2\sqrt{5}+5+1+2\sqrt{5}}{16}$$

$$\Rightarrow \frac{12}{16} = \frac{3}{4}$$

& product of roots

$(\sin^2 18^\circ) \times (\cos^2 36^\circ)$

$$\Rightarrow \left(\frac{\sqrt{5}-1}{4}\right)^2 \times \left(\frac{\sqrt{5}+1}{4}\right)^2 = \frac{1}{16}$$

Now, eq. will be :

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - \frac{3}{4}x + \frac{1}{16} = 0$$

$$\Rightarrow 16x^2 - 12x + 1 = 0$$ **Ans. (a)**

Note : This question can also be solved direct by SHORTCUT

17. Given : $\frac{a}{1-r} = 2(a + ar)$

$$\Rightarrow \frac{1}{1-r} = 2(1+r)$$

$$\Rightarrow (1 + r)(1 - r) = \frac{1}{2}$$

$$\Rightarrow 1 - r^2 = \frac{1}{2}$$

$$\Rightarrow r^2 = \frac{1}{2}$$

$$\Rightarrow r = \pm \frac{1}{\sqrt{2}}$$ **Ans. (a)**

18. Given m^{th} term of H.P.

$$\frac{1}{n} = \frac{1}{a} + (m-1)d \qquad \text{.....(1)}$$

and n^{th} term

$$\frac{1}{m} = \frac{1}{a} + (n-1)d \qquad \text{.....(2)}$$

$a = \frac{1}{mn}$, $d = \frac{1}{mn}$ by solving (1) and (2)

$$\therefore\ T_{m+n} = \frac{1}{\frac{1}{mn} + (m+n-1)\frac{1}{mn}}$$

$$T_{m+n} = \frac{mn}{m+n}$$

Ans. (b)

19. Standard Result

Ans. (b)

20. Given that

$$\frac{n(n+1)(2n+1)}{6n} = 11$$

$(n + 1)(2n + 1) = 66$

$2n^2 + 3n + 1 = 66$

$2n^2 + 3n - 65 = 0$

$$n = \frac{-3 \pm 23}{4}$$

$$\Rightarrow n = \frac{20}{4} = 5$$

$$\text{or } n = \frac{-26}{4}$$

but Q n cannot be negative or fraction.

$\therefore$ n = 5 **Ans. (c)**

Note : This question can also be solved direct by SHORTCUT

21. Nine digit number (less than 2×10^8) having only 1 and 2

$\Rightarrow 2^8$

Eight digit number (less than 2×10^8) having only digits 1 & 2 = 2^8

Similarly, Seven digit numbers = 2^7

Six digit no. = 2^6

five digit no. = 2^5

four digit no. = 2^4

three digit no. = 2^3

two digit no. = 2^2

one digit no. = 2

Number of all such natural numbers :

$2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^8 = 766$ **Ans. (c)**

22. Given :

$$\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$$

L.H.S. = using property

$$\Rightarrow e^{\lim_{x\to\infty}\left(\frac{a}{x}+\frac{b}{x^2}\right)^{2x}}$$

$$e^{\lim_{x\to\infty}\left(2a+\frac{2b}{x}\right)}$$

taking limit :

$e^{(2a+0)} = e^{2a}$

on comparing with R.H.S.

we get : $a = 1, b \in R$

Ans. (b)

Note : This question can also be solved direct by SHORTCUT

23. If $f(x) = \sin^5 x + \sin^3 x$

$f(-x) = \sin^5(-x) + \sin^3(-x)$

$= -\sin^5 x - \sin^3 x$

$f(-x) = -f(x)$

$f(x) + f(-x) = f(x) - f(x) = 0$

$$\int_0^{\pi/2} 0.[g(n)+g(-n)]dx = 0$$ **Ans. (a)**

24. $\frac{d^2x}{dy^2} =$

$$\frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}$$

$$\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}$$

$$\frac{d^2x}{dy^2} = \frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}\frac{dx}{dy}$$

$$\frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-2}\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{dy}$$

$$\frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-3}\left(\frac{d^2y}{dx^2}\right)$$

Ans. (d)

25. Let $u = \log_{10}^{x}$ and $v = \log_{x}^{10}$

$$u = \frac{\log_e^x}{\log_e^{10}},\ v = \frac{\log_e^{10}}{\log_e^x}$$

$$\frac{du}{dx} = \frac{1}{\log_e^{10}} \cdot \frac{1}{x} \qquad \frac{dv}{dx} = -\log_e^{10} \cdot \frac{1}{\left(\log_e^x\right)^2} \cdot \frac{1}{x}$$

$$\text{Now, } \frac{du}{dv} = \frac{du/dx}{dv/dx}$$

$$\frac{\frac{1}{\log_e^{10}} \cdot \frac{1}{x}}{\frac{-\log_e^{10}}{(\log_e^x)^2} \cdot \frac{1}{x}}$$

$$= \frac{-1}{\log_e^{10}} \cdot \frac{1}{x} \cdot \frac{x}{\log_e^{10}} \left(\log_e^x\right)^2$$

$$= -\frac{(\log x)^2}{(\log 10)^2}$$

Ans. (a)

26. We have,

$$f(x) = \begin{cases} \dfrac{x}{1-x} & \text{if } x < 0 \\ \dfrac{x}{1+x} & \text{if } x \geq 0 \end{cases}$$

The function f is differentiable at all points except possibly at x = 0

We have Lf'(0) = $\lim\limits_{x \to 0-} \dfrac{f(x)-f(0)}{x-0} = \lim\limits_{x \to 0} \dfrac{1}{1-x} = 1$

And Rf'(0) = $\lim\limits_{x \to 0+} \dfrac{f(x)-f(0)}{x-0}$

= $\lim\limits_{x \to 0+} \dfrac{1}{1+x} = 1$

As Lf'(0) = Rf'(0) = 1, we get f is differentiable at x = 0.

Thus, f is differentiable for all $x \in (-\infty, \infty)$

Ans. (b)

27. $\int_0^{\pi} xf(\sin x)dx =$

$\int_0^{\pi} (\pi - x)f\{\sin(\pi - x)\}dx$

$$\pi\int_0^{\pi} f(\sin x)dx - I$$

$$2I = \pi\int_0^{\pi/2} [f(\sin x) + f(\sin(\pi - x)]dx$$

$$= 2\pi\int_0^{\pi/2} f(\sin x)dx$$

$$I = \pi\int_0^{\pi/2} f\{\sin(\pi/2 - x)\}dx$$

$$= \pi\int_0^{\pi/2} (\cos x)dx$$

Ans. (c)

28. $$f(x)=\begin{cases} x+2 & ; \quad x<0 \\ -(x-2) & ; \quad 0\le x<2 \\ x-2 & ; \quad x\ge 2 \end{cases}$$

$$f(x)=\begin{cases} x+2 & ; \quad x<0 \\ 2-x & ; \quad 0\le x<2 \\ x-2 & ; \quad x\ge 2 \end{cases}$$

$$\int_{-2}^{0} f(x)dx + \int_{0}^{2} f(x)dx + \int_{2}^{3} f(x)dx$$

$$= \int_{-2}^{0} (x+2)dx + \int_{0}^{2} (2-x)dx + \int_{2}^{3} (x-2)dx$$

$$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{0} + \left[2x - \frac{x^2}{2}\right]_{0}^{2} + \left[\frac{x^2}{2} - 2x\right]_{2}^{3}$$

$$= \left[0 - \left(\frac{4}{2} - 4\right)\right] + \left[\left(4 - \frac{4}{2}\right) - 0\right] + \left[\left(\frac{9}{2} - 6\right) - \left(\frac{4}{2} - 4\right)\right]$$

$$= 2 + 2 - \frac{3}{2} + 2$$

$$= 6 - \frac{3}{2} = \frac{9}{2} = 4.5$$

Ans. (c)

Note : This question can also be solved direct by SHORTCUT

29. Here, $m_1 + m_2 = -\frac{2h}{b} = \frac{2}{(-2)} = -1$

$m_1 m_2 = \frac{a}{b} = \frac{6}{(-2)} = -3$

Now, $(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$

$(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$

$(m_1 - m_2)^2 = (-1)^2 - 4(-3) = 13$

$\therefore\ (m_1 - m_2)^2 = 13$

$m_1 - m_2 = \sqrt{13} \approx \frac{7}{2}$ **Ans. (b)**

30. $\frac{dr}{dt} = \frac{-2}{\pi} m/s$ (given)

We know,

Area of circle = πr^2

or

$A = \pi r^2$

$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

$\therefore \quad \frac{dA}{dt} = \pi(2r)\frac{dr}{dt}$

$= \quad 2\pi r\left(\frac{-2}{\pi}\right) m^2/s$

$= \quad \frac{-4\pi.10}{\pi} m^2/s$ (given r = 10)

$= \quad -40\ m^2/s$

Ans. (d)

31. $f(x) = x + |x|$

For continuity

L.H.L. = F(x) = f(a) = R.H.L.

$$\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$$

For L.H.L.

$= \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} (0-h) + |0-h|$

$= \lim_{h\to 0} -h + h$

$= 0$

At x = 0

f(0) = 0 + |0|

$= 0$

For R.H.L.

$\lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} (0+h) + |0+h|$

$= \lim_{h\to 0} h + h$

$= 0$

Here,

$$\lim_{x\to a^-} f(x) = f(a) = \lim_{x\to a^+} f(x) = 0$$

So, it is continuous $\forall x \in R$

Ans. (a)

32. Since $|a + b + c| = 0$

$$0 \leq |a|^2 + |b|^2 + |c|^2 + 2(a.b + b.c + c.a)$$

$$7 + 1 + 1 + 2(a.b + b.c + c.a) \geq 0$$

$$a.b + b.c + c.a \geq -3/2$$

now,

$|a - b|^2 + |b - c|^2 + |c - a|^2$

$= 2(|a|^2 + |b|^2 + |c|^2 - a.b - b.c - c.a)$

$\leq 2(1 + 1 + 1) + 3 = 9$

Ans. (b)

Note : This question can also be solved direct by SHORTCUT

33. Unit vector along angle bisector of b and c is

$$a = \frac{1}{2}\left(\frac{b}{|b|} + \frac{c}{|c|}\right)$$

$$= \frac{1}{\sqrt{2}}(i + 2j + k)$$

Now, $a = \lambda d$

$$= \alpha I + 2j + \beta k = \frac{\lambda}{2\sqrt{2}}(i + 2j + k)$$

Equating coefficients of j,

We get,

$\lambda = 2\sqrt{2}$

Thus, $\alpha = 1, \beta = 1$

Ans. (d)

34. Here,

$F_1 = 4i - 3j + 7k$

And $F_2 = -2i + 2j - 8k$

So, resultant force

$F = F_1 + F_2$

$F = 4i - 3j + 7k - 2i + 2j - 8k$

$F = 2i - j - k$

Now, given that

$d_1 = 5i + 7j + k$

$d_2 = 2i + 5j - 6k$

so, displacement

$d = d_2 - d_1$

$= (2i + 5j - 6k) - (5i + 7j + k)$

$= -3i - 2j - 7k$

Therefore

Work done = F.d

$= (2i - j - k).(-3i - 2j - 7k)$

$= -6 + 2 + 7$

$= 3$ (wrong)

None of the given option is correct.

Ans. (Wrong)

35. **Note : Direct by SHORTCUT** **Ans. (c)**

36. Here, circle

$S = x^2 + y^2 + 10x - 12y + 51 = 0$ .....(1)

And line

$L = x + 3y - 3 = 0$

or, $x = 3 - 3y$ .....(2)

put (2) into eq. (1)

$\therefore$ $(3 - 3y)^2 + y^2 + 10(3 - 3y) - 12y + 51 = 0$

$9 - 18y + 9y^2 + y^2 + 30 - 30y - 12y + 51 = 0$

or, $10y^2 - 60y + 90 = 0$

or, $10(y^2 - 60y + 90) = 0$

or, $y^2 - 6y + 9 = 0$

or, $y^2 - 3y - 3y + 9 = 0$

or, $y(y - 3) - 3(y - 3) = 0$

$y = 3, 3$

put $y = 3$ in eq. (1)

$\therefore$ $x = 3 - 3(3)$

$= 3 - 9$

$= -6$

So, point of intersection of circle and line is (-6, 3)

Ans.(a)

37. Here $\sin x + \sin 5x = \sin 3x$ [Q $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$]

$$2\sin\frac{x+5x}{2}\cos\frac{5x-x}{2} = \sin 3x$$

or, $2\sin 3x \cos 2x = \sin 3x$

or, $\sin 3x (2\cos 2x - 1) = 0$

$\sin 3x = 0$ or $\cos 2x = ½$

$3x = 0$ or $\cos 2x = ½$

$3x = 0, \pi, 2\pi, 3\pi$ or $2x = \pi/3, 5\pi/3$

$x = 0, \pi/3, 2\pi/3, \pi, \pi/6, 5\pi/6$

hence,

No. of solution = 6

Ans. (b)

38. The answer to maxima/minima about triangles usually occurs when the triangle is equilateral. For a triangle with each angle 60 degrees, sec 60 = 2,
so the minimum value of secA + secB + secC
= sec 60° + sec 60° + sec 60° = 2 + 2 + 2 = 6

Ans. (a)

39. $\theta \in P$

$\sin\theta - \cos\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2}+1)\cos\theta$

$\Rightarrow \sin\theta = \dfrac{1}{\sqrt{2}-1}\cos\theta$

$\Rightarrow \left(\sqrt{2}-1\right)\sin\theta = \cos\theta$

$\Rightarrow \sqrt{2}\sin\theta = \sin\theta + \cos\theta$

$\Rightarrow \theta \in Q$

$\therefore P = Q$

Ans. (d)

Note : This question can also be solved direct by SHORTCUT

40. $x + y + z = \pi$

$\tan x + \tan y + \tan z = \tan x \tan y \tan z$

$2k + 3k + 5k = 2k \times 3k \times 5k$

$K^2 = 1/3$

$\tan^2 x + \tan^2 y + \tan^2 z = \left(\dfrac{1}{3}\right)(4 + 9 + 25)$

$= \dfrac{38}{3}$

Ans. (a)

41. $\operatorname{cosec}^{-1}\left(\frac{5}{3}\right) = \cot^{-1}\left(\frac{4}{3}\right)$

So, $\cot\left[\cot^{-1}\left(\frac{4}{3}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right]$

$$\frac{\frac{4}{3}\times\frac{3}{2}-1}{\frac{4}{3}+\frac{3}{2}} = \frac{6}{17}$$

Ans. (a)

42. Here,

$\sin\theta = 3\sin(\theta + 2\alpha)$

$$\frac{\sin\theta}{\sin(\theta+2\alpha)} = \frac{3}{1}$$

Comp. and Div.

$$\frac{\sin\theta + \sin(\theta+2\alpha)}{\sin\theta - \sin(\theta+2\alpha)} = \frac{4}{-2}$$

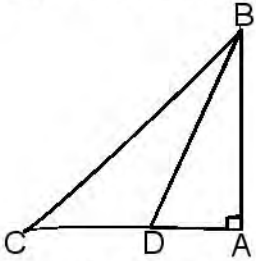
$$\frac{2\sin(\theta+2)\cos(\alpha)}{2\sin(\alpha)\cos(\theta+\alpha)} = -2$$

$$\frac{\tan(\theta+\alpha)}{\tan\alpha} = -2$$

$\tan(\theta + \alpha) + 2\tan\alpha = 0$

Ans. (d)

43. Given D is midpoint point of AC and angle A is equal to 90°



In $\triangle ABC$ by pythagorus theorem

$BC^2 = AB^2 + AC^2$ .....(1)

Again in $\triangle ABD$,

By pythagorus theorem

$BD^2 = AB^2 + AD^2$ .....(2)

Here,

$AC = 2AD$ .....(3)

(1) – (2)

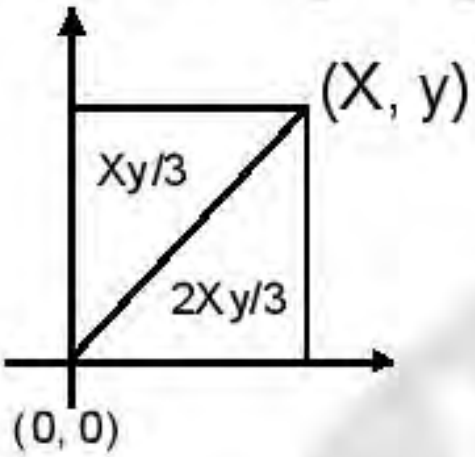
$BC^2 - BD^2 = AC^2 - AD^2$

$BC^2 - BD^2 = (2AD)^2 - AD^2$

$BC^2 - BD^2 = 3AD^2$

Ans. (c)

44. According to given question



$$\int_0^x y dx = \frac{2xy}{3}$$

$$y = \frac{2y}{3} + \frac{2x}{3}\frac{dy}{dx}$$

$$\frac{y}{3} = \frac{2x}{3}\frac{dy}{dx}$$

$$\frac{dx}{x} = \frac{2dy}{y}$$

$\log c + \log x = 2 \log y$

$\log cx = \log y^2$

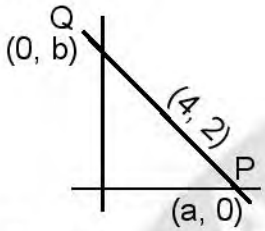
$y^2 = cx$

i.e. parabolas

Ans. (b)

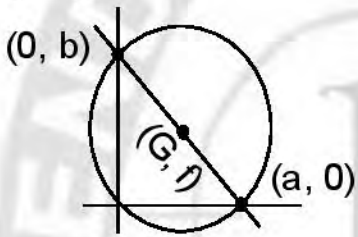
45. Let, the equation of line which makes intercepts a, b on the coordinate axis be

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \text{.....(1)}$$



Eq.(1) passes through (4, 2)

$$\frac{4}{a}+\frac{2}{b}=1 \qquad \text{.....(2)}$$



$g=\frac{a}{2}$

$f=\frac{b}{2}$

$a = 2g$

$b = 2f$

put value of a & b in eq. (2)

$$\frac{4}{2g}+\frac{2}{2f}=1$$

$$\frac{2}{g}+\frac{1}{f}=1$$

Replace g by x and f by y $\frac{2}{x}+\frac{1}{y}=1$ **Ans. (b)**

46. Centre of circle $x^2 + y^2 - 2ax + c^2 = 0$ .....(1)

is $C_1(a, 0)$

Radius = $r_1 = \sqrt{a^2 - c^2}$

Centre of circle $x^2 + y^2 - 2by + c^2 = 0$ .....(2)

is $C_2(0, b)$

Radius = $r_2 = \sqrt{b^2 - c^2}$

Both circles will touch each other externally if

$C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{a^2 - c^2} + \sqrt{b^2 - c^2}$

Squaring

$\Rightarrow a^2 + b^2 = a^2 - c^2 + b^2 - c^2 + 2\sqrt{a^2 - c^2}\sqrt{b^2 - c^2}$

$\Rightarrow 2c^2 = 2\sqrt{a^2 - c^2}\sqrt{b^2 - c^2}$

$\Rightarrow C^2 = \sqrt{a^2 - c^2}\sqrt{b^2 - c^2}$

Squaring again

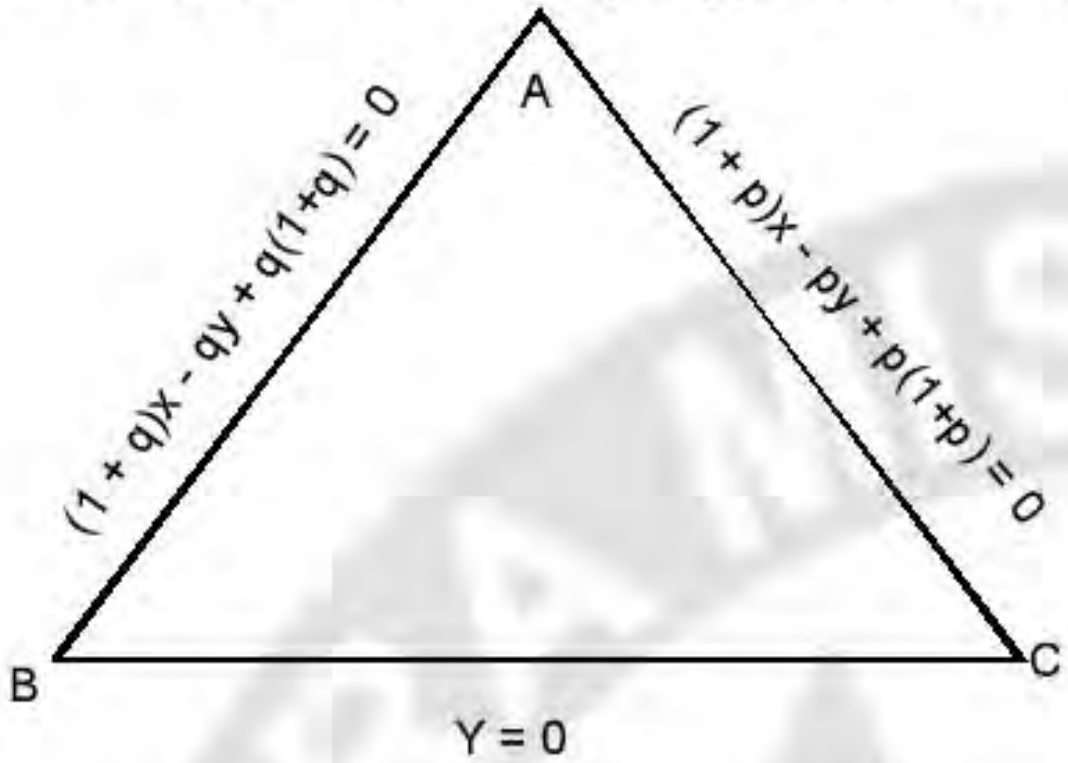
$\Rightarrow C^4 = (a^2 - c^2)(b^2 - c^2)$ $\Rightarrow C^4 = a^2b^2 - a^2c^2 - b^2c^2 + c^4$

$\Rightarrow a^2c^2 + b^2c^2 = a^2b^2$ .....(3)

Divide above equation (3) by $a^2b^2c^2$

$\Rightarrow \frac{1}{b^2} + \frac{1}{a^2} = \frac{1}{c^2}$ **Ans. (c)**

47. The orthocenter (x, y) lies on the lines



$$y = \frac{-P}{1+P}(x+q)$$

And $y = \frac{-q}{(1+q)}(x+p)$

$[(1 + p) - (1 + q)]y + [p - q]x = 0$

$(p - q)(x + y) = 0$

$x + y = 0$ as $p \neq q$

which is a straight line

Ans. (d)

48. Let the equation of line be $y = mx + c$ .....(1)

If eq. (1) is tangent to the ellipse $\frac{x^2}{9} - \frac{y^2}{4} = 1$

$\therefore \quad c = \pm\sqrt{a^2m^2 - b^2}$

Or, $c = \pm\sqrt{9m^2 - 4}$

$\therefore$ Eq. of line (1)

$y = mx \pm \sqrt{9m^2 - 4}$ .....(1)

It is also tangent to the circle $x^2 + y^2 - 8x = 0$

$\therefore \quad p = r$

$C = (4, 0), r = 4$

$\therefore \quad \frac{|4m \pm \sqrt{9m^2 - 4}|}{\sqrt{1+m^2}} = 4$

$\left(4m \pm \sqrt{9m^2 - 4}\right)^2 = 16(1+m^2)$

Or $m = \frac{2}{\sqrt{5}}$

Put $m = \frac{2}{\sqrt{5}}$ in eq. (1)

$y = \frac{2}{\sqrt{5}} + \sqrt{36/5 - 4}$

$\sqrt{5}y = 2x + 4$

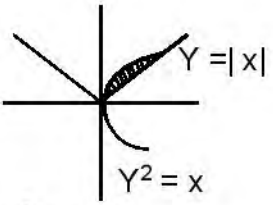
$2x - \sqrt{5}y + 4 = 0$

Ans. (b)

49. $y = |x|$ .....(1)

$y^2 = x$ .....(2)

from (1) & (2)



$x^2 = x$

or $x^2 = x$

$x(x - 1) = 0$

$x = 0$ or $x = 1$

required Area $= \int_0^1 (x^2 - x)x$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1$$

$$= \left|\frac{1}{3} - \frac{1}{2}\right|$$

$$= \left|\frac{2-3}{6}\right|$$

$$= \frac{1}{6}$$

Ans. (c)

50. here, $x - 2y = 1$ .....(1)

Eq. of line perpendicular to (1)

$2x + y = c$ .....(2)

Eq. (2) passes through (1, 1)

$\therefore$ $2(1) + (1) = c$

$C = 3$

$\therefore$ Eq. of line is

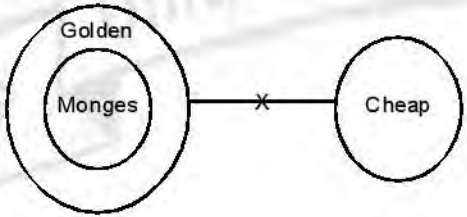
$2x + y = 3$

Or, $y = -2x + 3$

Ans. (d)

ANALYTICAL ABILITY AND LOGICAL REASONING

51.



Conclusion I = All mangoes are cheap.

Conclusion I do not hold bcoz mangoes are golden and not cheap.

So, Conclusion II follows only. **Ans. (b)**

52. Using statement I & II

Let market price be x

Using statement I = SP of article = $x - \frac{5x}{100} = \frac{95x}{100}$

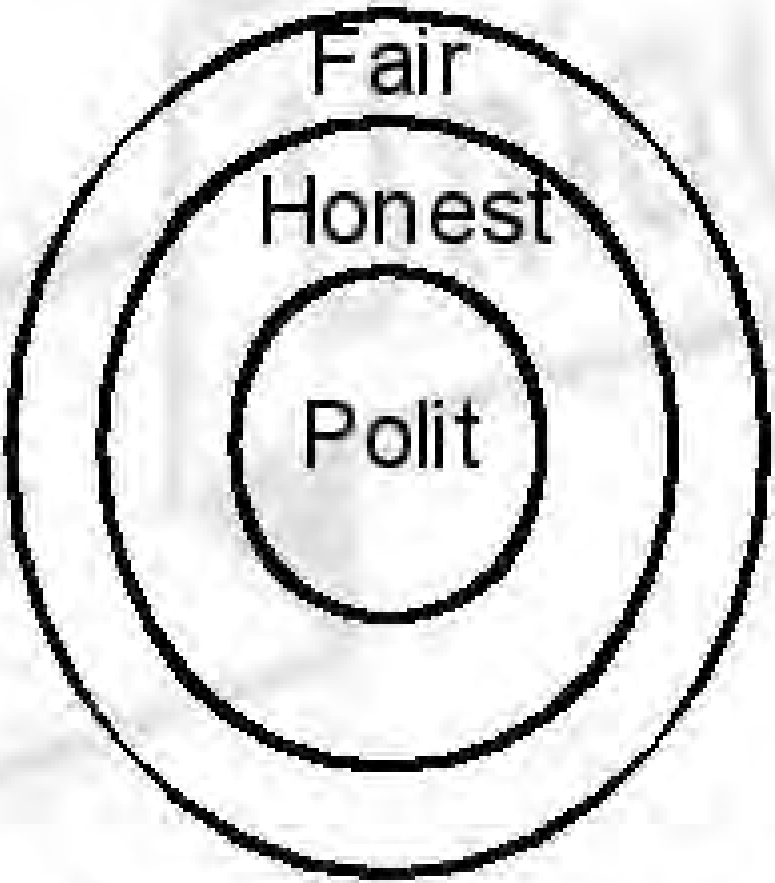
Using statement II = CP of article = $x - \frac{20x}{100} = \frac{80x}{100}$

Profit % = $\frac{\text{Profit}}{\text{CP}} \times 100\% = \frac{15x/100}{80x/100} \times 100\% = \frac{300}{16}\%$

So, I & II

Ans. (a)

53.



Clearly Option D

Only I & III follow

Ans. (d)

54. Let x students watched film 1 time only

So, (40 – x) students watched film 3 times

Total times films being watched = $x \times 1 + (40 - x)3$ .....(1)

According to Question – Films being watched = A + B + C

= 13 + 16 + 19 = 48 .....(2)

Equation (1) & (2) we get

$x + 120 - 3x = 48$

4 students watched 3 films

Ans. (c)

Note : This question can also be solved direct by SHORTCUT

55. Let fare from city A to B be Rs. x, and fare from city A to C be Rs. y

Then according to question,

$2x + 3y = 77$ .....(1)

And $3x + 2y = 73$ .....(2)

On solving equation (1) & (2), we get

X = 13, y = 17

Ans. (b)

56. Let total population be 100

Then no. of adults and children be 40 and 60
Respectively (because the given ratio be 2 : 3)
Given 40% of the adults are illiterate then
60% of the adults are literate then
No. of literate adults be 24.
Similarly no. of literate children be 51
Then present of the population literate is

$$\frac{(24+51)}{100}\times 100\%$$

= 75%

Ans. (d)

Note : This question can also be solved direct by SHORTCUT

57. Total students = 50
A = students speak English
B = Students speak Hindi
Given $A \cap B = 10$, $A = 21$
$A \cup B = A + B - A \cap B$
$50 = 21 + B - 10$
$B = 39$
Students speak Hindi = 39
Students speak Hindi only = $B - A \cap B$
= $39 - 10 = 29$
Students speak English only = $A - A \cap B$
= $21 - 10 = 11$
Ans. (d)

58. $81 \times 82 \times 83 \times 84 \times 85 \times 86 \times 87 \times 88 \times 89$

Any even no. multiplied by 5 gives 0 as unit digit 50

$84 \times 85 = 0$ and digit 0 further multiplied gives 0 as unit digit

Ans. (a)

59. Since placement ratio plays a key role to predict the future of any graduate course

So, option (A) is probable cause for given statement.

Option (B) & (C) do not hold

Option (A) is correct

Ans. (a)

60. If both the needles are coincident the angle between them is 0°

By using formula $\frac{2}{11}(H \times 30 \pm A)$

Put A = 0, H = 3

$= \frac{2}{11}(3 \times 30 \pm 0) = \frac{180}{11} = 16\frac{4}{11}$ min past 3

Ans. (d)

61. Assumption I is implicit as from the given statement it is concluded directly that Artificial Honey is also there in the market.

So, they promoting their brand by selling Pure and Natural Honey.
But nothing can be said about conclusion II.
Conclusion III is clearly wrong.
Ans. (a)

62. Conclusion I is wrong as some Indians scientists are working in America and since, all the scientists working in America are talented
Those Indian scientists are also talented.
Conclusion 2 is clearly right.
Conclusion 3 is incorrect as some scientists working in America does not belong to India.
Conclusion 4 is correct as those Indian scientists are working in America so they are clearing talented.
Hence, only conclusion 2 and 4 are correct.
Ans. (d)

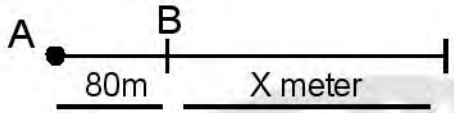
63. The line “As scholor point out, we Psychological insights into both male and female character are amazing even today” supports the statement (B) completely.

Ans. (b)

64. Let Speed of B be y m/s

Then speed of A = 5y/3 m/s

A gives B a start of 80 m means starting point of B is 80m ahead of A i.e.



Given Time of A = Time of B

$$\frac{D_A}{S_A} = \frac{D_B}{S_B}$$

$$\frac{80 + x}{\frac{5}{3}y} = \frac{x}{y}$$

= 240 + 3x = 5x

x = 120

Total distance 120 + 80 = 200 m

Ans. (a)

65. Let speed of 1 man = 1/x

Speed of 1 boy = 1/y

So, ATQ = $10\left(\frac{2}{x}+\frac{3}{y}\right)=1$ .....(1)

$8\left(\frac{3}{x}+\frac{2}{y}\right)=1$ .....(2)

Put $\frac{1}{x}$ = u, $\frac{1}{y}$ = v then

$2u + 3v = \frac{1}{10}$ on solving equations

$3u + 2v = \frac{1}{8}$

We get $\frac{1}{x}=\frac{7}{200}$ & $\frac{1}{y}=\frac{1}{100}$

So, time taken by 2 men & 1 boy

$T\times\left(\frac{2}{x}+\frac{1}{y}\right)=1$

$T\times\left(\frac{14}{200}+\frac{1}{100}\right)=1$

T = 12.5 days **Ans. (a)**

Note : This question can also be solved direct by SHORTCUT

66. $2\log_3(2^x - 5) = \log_3 2 + \log_3(2^x - 7/2)$

Let $2^x = t$

$\log_3(t - 5)^2 = \log_3(2)\,(t - 7/2)$

$t^2 + 25 - 10t = 2t - 7$

$t^2 - 12t + 32 = 0$

$(t - 8)\,(t - 4) = 0$

$t = 4 \quad 2^x = 4 \Rightarrow x = 2$

$t = 8 \quad 2^x = 8 \Rightarrow x = 3$

$x \neq 2$ because $\log_3(2^x - 7/2)$ becomes negative

so, x = 3 **Ans. (c)**

Note : This question can also be solved direct by SHORTCUT

67. The no. divisible by 4 have last 2 digits divisible by 4

There are 8 cases for 5 digits no. 5 divisible by 4

= (12, 16, 24, 32, 36, 52, 56, 64)

No. of permutations are

1 ___ ___ ___ 1 2

4 × 3 × 2 - 24 cases

2 16 = ___ ___ ___ 1 6

4 × 3 × 2 - 24 cases

& so on

Therefore total cases are 24 × 8 = 192

Ans. (c)

Note : This question can also be solved direct by SHORTCUT

68. Integer between 100 & 200 that are divisible by 3.

66 – 33 = 33,

Integer divisible by 3 that are odd = $\frac{33-1}{2} = 16$

Integer divisible by 7 between 100 and 200

9 – 4 = 5

Integer that are odd = 5 – 2 = 3

Total integers in the set s = 16 – 3 = 13

Ans. (d)

69. Let B is turned off after T minutes then,

$$\frac{1}{37.5}+\frac{T}{45}=\frac{1}{30}$$

$$T=\left(\frac{1}{30}-\frac{1}{37.5}\right)45=9 \text{ minutes}$$

Ans. (b)

70.

Person	Shirt			Pant		
	Black	Blue	Orange	Green	Yellow	Orange
A	X	X	✓	X	✓	X
B	✓	X	X	X	X	✓
C	X	✓	X	✓	X	X

Ans. (b)

71. LCM of 2, 3, 4, 5 and 6 = 60

60, 120, 180 ....., 900, 960

$$n = \left(\frac{l - a}{d}\right) + 1$$

$$n = \left(\frac{960 - 60}{60}\right) + 1$$

n = 16 Ans. (a)

72. 1 + 3 + 6 + 10 + ... + n

To find n = ?

Find nth term

$1 + 3 + 6 + 10.....$

$$\frac{-(1+3+6.....)}{1+(2+3+4.....)}$$

$$T_n = 1 + \frac{n-1}{2}(4 + (n-2)1)$$

$$= \frac{2}{2} + (n-1)(n+2)$$

$$= \frac{2 + n^2 + 2n - n - 2}{2} = \frac{n^2 + n}{2}$$

$$S_n = \sum_{n}^{1} \frac{n^2 + n}{2} = \frac{1}{2}\left[\frac{(n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$8436 \times 2 = \frac{n(n+1)(n+2)}{3}$$

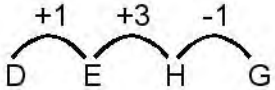
$n = 36$

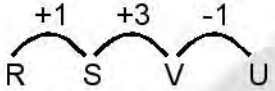
Ans (c)

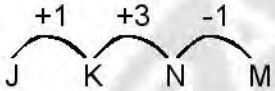
73. Average Speed $= \frac{2 \times 50 \times 100}{100 + 50} = \frac{200}{3} \frac{\text{km}}{\text{h}}$

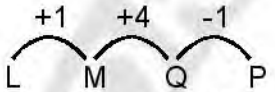
Average velocity = $\frac{\text{Total Displacement}}{\text{Total time taken}} = \frac{0}{100+50} = 0$ **Ans. (d)**

74.









So, LMQP will be odd **Ans. (d)**

Note : This question can also be solved direct by SHORTCUT

75. $15 \times 1 + 1 = 16$
$16 \times 2 + 2 = 34$
$34 \times 3 + 3 = 105$
$105 \times 4 + 4 = 424$
$424 \times 5 + 5 = 2125$
$2125 \times 6 + 6 = 12756$
So, wrong term will be 2124. **Ans. (d)**

76. Area of parallelogram
= Base × height

208 = 13 × height

Height = 16 cm

Ans. (c)

77. DISTRIBUTION

After rearrangement

STDIBURIONTI

Left

Seventh letter from left will be R.

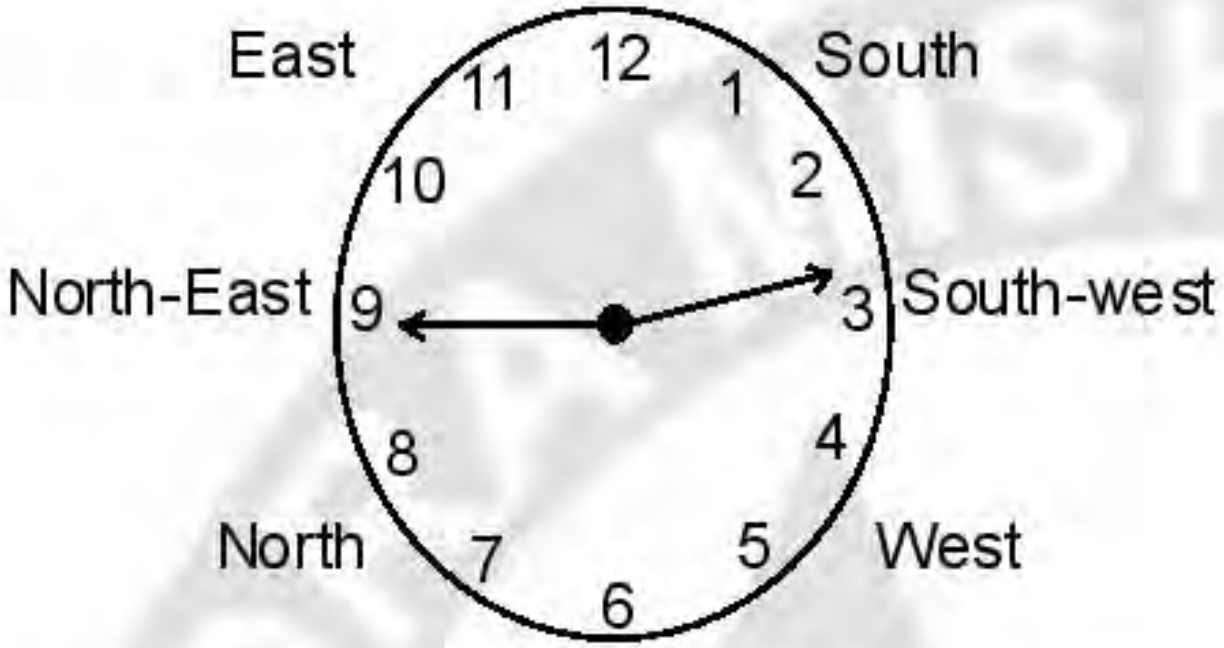
Ans. (b)

78. J + 1 = K
A + 1 = B
K + 1 = L
Similarly
M + 1 = N
D + 1 = E
N + 1 = 0
N E O **Ans. (b)**

79. Each day it crawls up 5 inches & each night it slides down 4 inches then resultant of 1 day will be 1 inches.

Then, resultant of 70 days will be 70 inches.
But on the end of 71 days it reach top of the pole.
Ans. (b)

80.



Ans. (b)

81. let my brother's age is x

Then, my mother's age = 2x

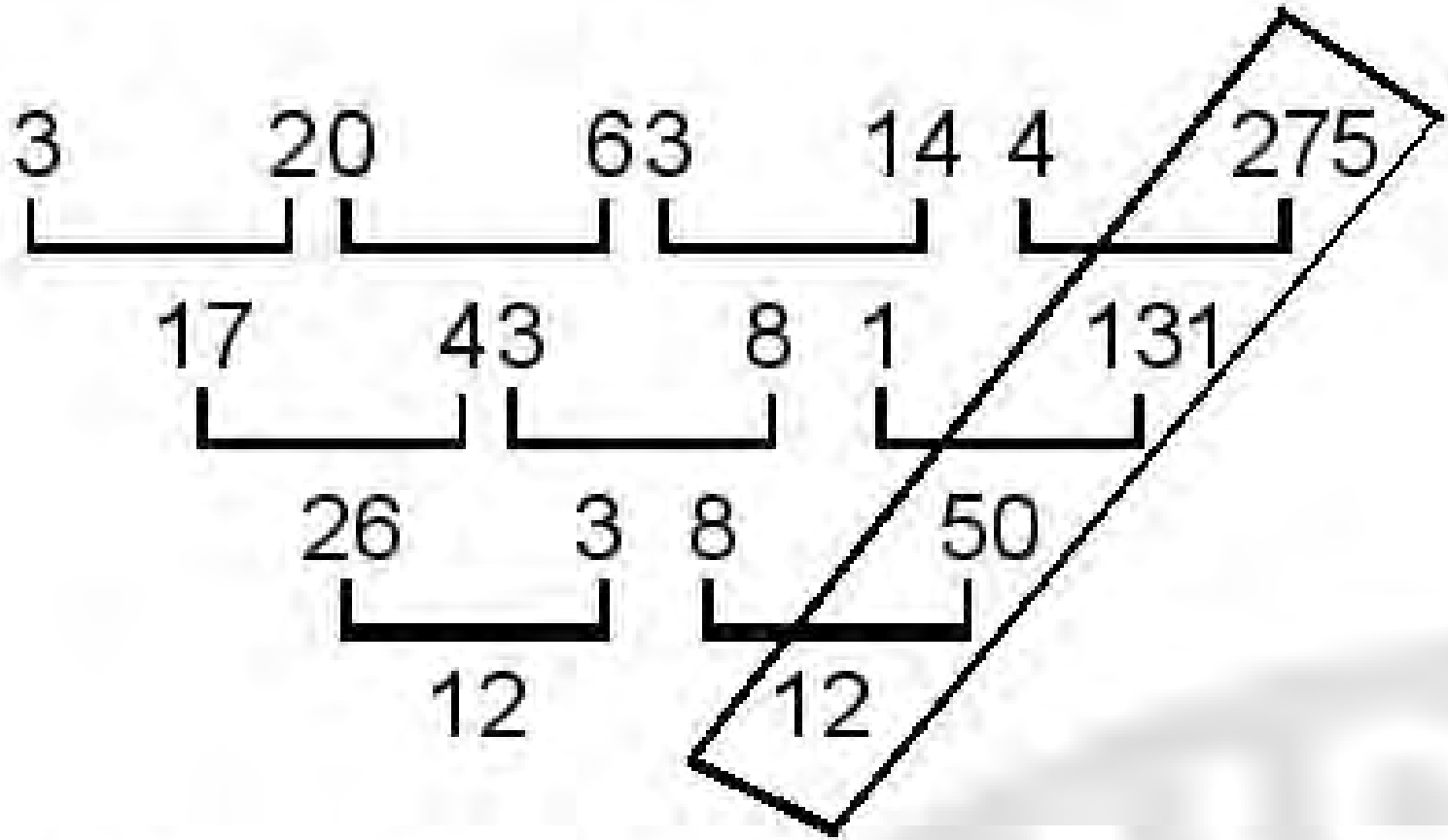
$\therefore$ My age = x – 5

Then, my sister's age = x – 8

Given : x – 8 = 12

X = 20

Mother's age = 40 years. **Ans. (c)**



∴ Missing term

275 + 131 + 50 + 12 = 468

Ans. (b)

For Q. 83 to 85

Given committee A has 1 member more than committee B

$n(A) = n(B) + 1$

Also, $(z, y, x) \notin A$, $(w, v, u) \notin B$, $(T, S, R) \notin C$

83. If T and Z are in committee B i.e. 2 members

So, committee A has 3 members

$\therefore$ committee C has $9-(3+2)=4$

Ans. (B)

84. According to questions

$(T, S, X) \in B$ so, $(w, v, u, R) \in A$

$\therefore$ $(z, y) \in C$

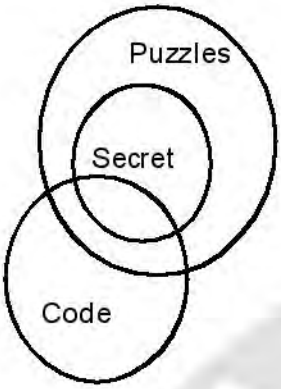
Ans. (A)

85. Given R $\in$ B so A must have 2 members
(T and S) $\notin$ C so
T and S must belongs to A
$\therefore$ T and S will serve on A
Ans. (d)

Note : Question no. 83 – 85 can also be solved direct by **SHORTCUT**

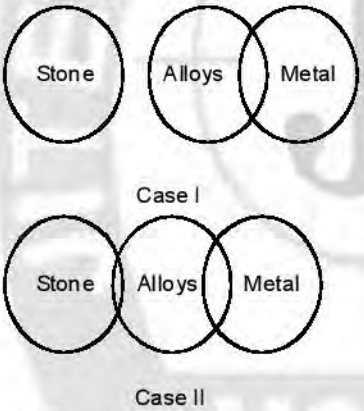
86. **Ans. (b)**

87. From the picture
T-shirt and shoes belong to apparels
So, Ans. Must be sofa
Because sofa and Almirah belongs to furniture.
Ans. (b)



From the diagram
All secrete being code is a
Possibility and clearing some puzzle and also
Ans. (d)

89.



Only conclusion II is true.
Ans. (b)

90. No. of triangle = 14
[ABC, BCD, CDI, BCI, FGM, FGJ, EGJ, EFG, EFH, CFG, BCG, BJG, CFJ, ACG]
Ans. (c)

91. **Ans. (c)** Logical Error

92. $(43)_x = (y3)_8$

From this equation it is clear x should be greater than 4 and y should be less than 8.

x>=5, and y<=7

Now convert this equation in decimal

=> 4 × x + 3 = y × 8 + 3

4x = 8y

ie x = 2y

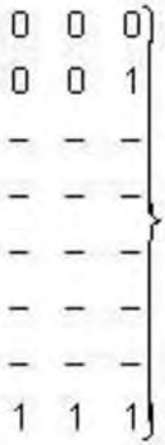
With given constraint (x> = 5 && y<=7)

possible values of x and y are (6,3), (8,4),(10,5),(12,6), (14,7)

So possible number of solution is 5.

Ans. (b)

93. Direct by shortcut



Put A = 0, B = 1, C = 1

Hence Ans. (c) , B(C + A) is correct answer.

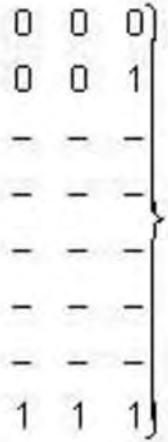
Ans. (c)

Note : This question can also be solved direct by SHORTCUT

94. $\overline{x} \oplus \overline{y}$

Ans. (d)

93. Direct by shortcut



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Ans. (d)

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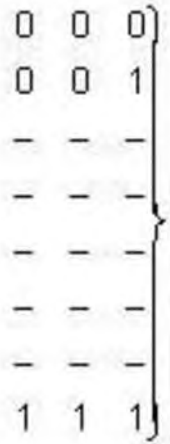

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Ans. (c)

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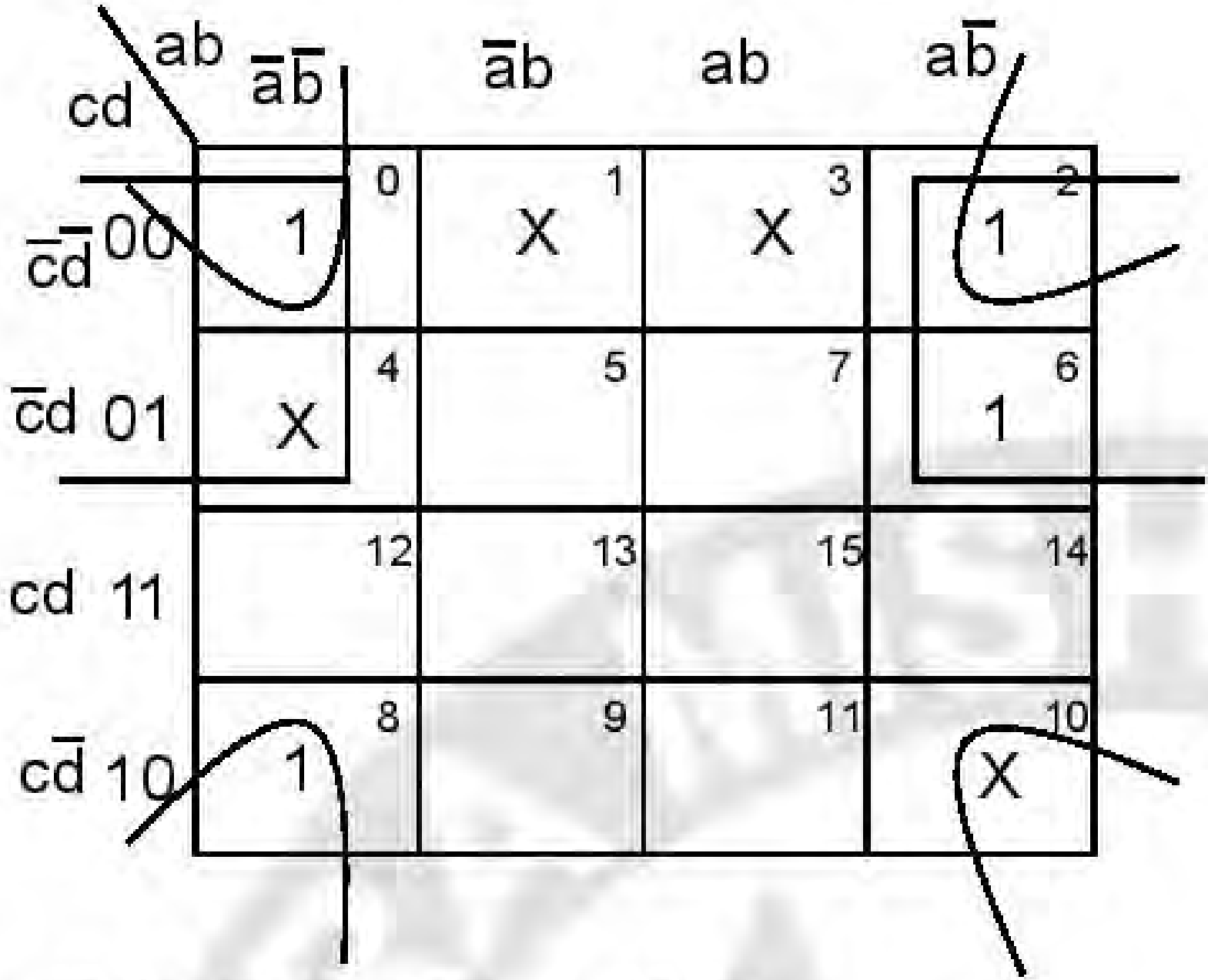
94. $\overline{x} \oplus \overline{y}$

Ans. (d)



$$\bar{a}\bar{b}a\bar{b}\bar{c}\bar{d}c\bar{d} + \bar{c}\bar{d}\bar{c}d\bar{a}\bar{b}a\bar{b}$$

$$\bar{b}\bar{d} + \bar{b}\bar{c}$$

Ans. (B)

96. The following figure shows the layout for single (32-bit) precision floating-point values:

Floating Point Components			
	Sign	Exponent	Fraction
Single Precision	1 [31]	8 [30–23]	23 [22–00]

From the above table Ans. is (8, 23)

Ans. (d)

97. In all option only NAND gate show such property

NAND gate

A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0

Ans. (b)

Note : This question can also be solved direct by SHORTCUT

98. $\sqrt{(224)_r} = (13)_r$

$2 \times r^2 + 2r + 4 = (r + 3)^2$

$2r^2 + 2r + 4 = r^2 + 9 + 6r$

$r^2 - 4r - 5 = 0$

$r^2 - 5r + r - 5$

$r(r - 5) + 1 (r - 5) = 0$

$r = -1$ or $r = 5$

Ans. (c)

99. Effective Memory Access Time (EAT) = (Hit Ratio) × (Cache memory time + Main Memory Access Time) + (1 – hit) × (Cache memory time + 2 Main Memory Access Time)

$$= \frac{15}{100}\left(\frac{10}{1000}+10\right)+\left(1-\frac{15}{100}\right)\left(\frac{10}{1000}+2\times 10\right)$$

= 1.85 milliseconds

Ans. (d)

Note : This question can also be solved direct by SHORTCUT

100. $2 \times R_1^2 + 3R_1 + 5 = 565$

$R_2^3 + 0 \times R_2^2 + 6 \times R_2 + 5$

$R_1 = 16$

$R_2 = 8$

Ans. (b)

Note : This question can also be solved direct by SHORTCUT

GENERAL ENGLISH

101. Cope up **Ans.(B)**

102. Meagre **Ans. (A)**

103. is **Ans. (A)**

104. I started to feel nostalgic as I stood in the old neighbourhood that echoed with my childhood
Ans. (B)

105. He warned her calmly that he would shoot her if she didn't keep quite
Ans. (C)

106. Ceiling **Ans. (C)**

107. Self reliant : Buoyant **Ans. (B)**

108. Jester **Ans. (D)**

109. The synonym that is most nearly similary in meaning to the word : DEBACLE is Catastrophe **Ans. (A)**

110. faff about **Ans.(B)**

111. Injudicious **Ans.(D)**

112. no correction required **Ans.(D)**

113. over **Ans.(C)**

114. Hasn't he ? **Ans.(B)**

115. Most nearly same as meaning of the word **Epitome** is **Essence**

102. Meagre **Ans. (A)**

103. is **Ans. (A)**

104. I started to feel nostalgic as I stood in the old neighbourhood that echoed with my childhood
Ans. (B)

105. He warned her calmly that he would shoot her if she didn't keep quite
Ans. (C)

106. Ceiling **Ans. (C)**

107. Self reliant : Buoyant **Ans. (B)**

108. Jester **Ans. (D)**

109. The synonym that is most nearly similary in meaning to the word : DEBACLE is Catastrophe **Ans. (A)**

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112. no correction required **Ans.(D)**

113. over **Ans.(C)**

114. Hasn't he ? **Ans.(B)**

115. Most nearly same as meaning of the word **Epitome** is **Essence**

Ans.(C)

116. fully **Ans.(B)**

117. QTRPS **Ans.(D)**

118. Corporatism **Ans.(A)**

119. Transition **Ans.(D)**

120. Conflicting interests of the board of directors **Ans.(D)**