1. Since (1, 1), (2, 2), (3, 3) $\in$ R, therefore R is reflexive (1, 2) $\in$ R but (2, 1) $\notin$ R, therefore R is not symmetric .
It can be easily seen that R is transitive.
Ans. (a)

2. **Ans. (b)**

3. Since R is reflxive relation on A, therefore
(a, a) $\in$ R for all a $\in$ A
$\Rightarrow$ the minimum number of ordered pairs in R is n.
Hence, $m \geq n$
Ans. (a)

4. $\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50} = 3^{25}(x+iy)$

$\Rightarrow (i\sqrt{30})^{50}\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^{50} = 3^{25}(x+iy)$

$\Rightarrow -(3)^{25}\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^{50} = 3^{25}(x+iy)$

$\Rightarrow -\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^{50} = (x+iy)$

$\Rightarrow -(-\omega)^{50} = x + iy \qquad \left[\because \omega = -\frac{1}{2}+\frac{i\sqrt{3}}{2}\right]$

$\Rightarrow -\omega^{50} = x + iy$
$\Rightarrow -\omega^{2} = x + iy$

$\Rightarrow \frac{1}{2}+\frac{i\sqrt{3}}{2} = x+iy \qquad \left[\because \omega^2 = -\frac{1}{2}-\frac{i\sqrt{3}}{2}\right]$

$\Rightarrow x = \frac{1}{2}, y = \frac{\sqrt{3}}{2} \Rightarrow (x,y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Ans. (b)

Note : This question can also be solved direct by SHORTCUT.

5. $|z^2 + 2z\cos\alpha| \le |z^2| + |2z\cos\alpha|$
$= |z|^2 + 2|z|\,|\cos\alpha|$
$\le |z|^2 + 2|z|$
< 1
Ans. (a)

6. Let $\omega = -1 + 4z$. Then,
$\omega + 1 = 4z \Rightarrow |\omega + 1| = 4|z| = 12$
Thus, ω lies on a circle with centre at -1 and radius equal to 12.
Ans. (b)

7. Since $\log_x a$, $a^{x/2}$ and $\log_b x$ are in G.P. Therefore,
$(a^{x/2})^2 = \log_x a, \log_b x$
$\Rightarrow a^x = \log_b a \Rightarrow x = \log_a(\log_b a)$
Ans. (a)

8. We have, $x = \frac{1}{1-a}, y = \frac{1}{1-b}, z = \frac{1}{1-c}$
Now, a, b, c are in A.P. $\Rightarrow 1 - a, 1 - b, 1 - c$ are in A.P.
$\Rightarrow \frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in H.P.
$\Rightarrow x, y, z$ are in H.P.
Ans. (c)

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9. Let d be the common difference of the AP, Then,

$$a_{10} = 3 \Rightarrow a_1 + 9d = 3 \Rightarrow 2 + 9d = 3 \Rightarrow d = \frac{1}{9}$$

$$\therefore \quad a_4 = a_1 + 3d = 2 + \frac{1}{3} = \frac{7}{3}.$$

Let D be the common difference of $\frac{1}{h_1}, \frac{1}{h_2} \cdots\cdots \frac{1}{h_{10}}$. Then,

$$h_{10} = 3 \Rightarrow \frac{1}{h_{10}} = \frac{1}{3} \Rightarrow \frac{1}{h_1} + 9D = \frac{1}{3} \Rightarrow \frac{1}{2} + 9D = \frac{1}{3} \Rightarrow 9D = -\frac{1}{6} \Rightarrow D = -\frac{1}{54}$$

$$\therefore \quad \frac{1}{h_7} = \frac{1}{h_1} + 6D = \frac{1}{2} - \frac{1}{9} = \frac{7}{18}, \Rightarrow h_7 = \frac{18}{7}$$

$$\therefore \quad a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6$$

Ans. (d)

Note : This question can also be solved direct by SHORTCUT.

10. $ax^2 + 2bx + c = 0$

$\Rightarrow \quad ax^2 + 2\sqrt{ac}\,x + c = 0$

$\Rightarrow \quad \left(\sqrt{a}x + \sqrt{c}\right)^2 = 0 \Rightarrow x = -\frac{\sqrt{c}}{\sqrt{a}}$

This satisfies $dx^2 + 2ex + f = 0$

$\Rightarrow \quad d\left(\frac{c}{a}\right) + 2e\left(\frac{-\sqrt{c}}{\sqrt{a}}\right) + f = 0 \quad \Rightarrow \left(\frac{dc}{a} + f\right) = 2e\frac{\sqrt{c}}{a}$

$\Rightarrow \quad \left(\frac{d}{a} + \frac{f}{c}\right) = 2e\frac{\sqrt{1}}{ac} \quad \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$

$\Rightarrow \quad \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

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11. Product of roots = 31

$\Rightarrow \quad 2e^{2\log k - 1} = 31$

$\Rightarrow \quad 2k^2 - 1 = 31 \Rightarrow 2k^2 = 32 \Rightarrow k^2 = 16 \Rightarrow k = \pm 4$

But $k > 0$. Therefore, $k = 4$

Now, Disc $= 8k^2 - 8e^{2\log k} + 4 = 8k^2 - 8k^2 + 4 = 4 > 0$ for all k,

Hence, $k = 4$

Ans. (d)

12. We have,

$$a-\sqrt{b}=\frac{(a-\sqrt{b})(a+\sqrt{b})}{a+\sqrt{b}}=\frac{a^2-b}{a+\sqrt{b}}=\frac{1}{a+\sqrt{b}}[\because a^2-b=1]$$

by putting $(a-\sqrt{b})^{x^2-15}=y$, the given equation becomes

$$y+\frac{1}{y}=2a \Rightarrow y^2-2ay+1=0$$

$\Rightarrow (y-a)^2=a^2-1 \Rightarrow y-1=\pm\sqrt{a^2-1}$

$\Rightarrow y-a=\pm\sqrt{b}$

$\Rightarrow y=a\pm\sqrt{b} \Rightarrow \left(a+\sqrt{b}\right)^{x^2-15}=a+\sqrt{b}, a-\sqrt{b}$

$\Rightarrow x^2-15=1$ or $x2-15=-1 \Rightarrow x=\pm4, x=\pm\sqrt{14}$

Ans. (b)

Note : This question can also be solved direct by SHORTCUT.

13. Let α, β be the roots of the given equation.
Then $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$.
Required equation is

$$x^2-x\left(\frac{\alpha+\beta}{2}+\frac{2\alpha\beta}{\alpha+\beta}\right)\left(\frac{\alpha+\beta}{2}\right)\left(\frac{2\alpha\beta}{\alpha+\beta}\right)=0$$

$\Rightarrow 2ax^2+(b^2+4ac)x+2bc=0$ **Ans. (b)**

14. Let $x=\sqrt{8+2\sqrt{8+2\sqrt{8+2\sqrt{8+2\sqrt{8}}}}}$. Then,

$x=\sqrt{8+2x} \Rightarrow x^2=8+2x \Rightarrow x^2-2x-8=0$

$\Rightarrow x=4. \quad [\because 8>0]$

Ans. (d)

15. $x^2-2x\cos\theta+1=0 \Rightarrow x=\cos\theta\pm i\sin\theta$
$\Rightarrow x^{2n}=\cos 2n\theta\pm i\sin 2n\theta$ and $x^n=\cos n\theta\pm i\sin n\theta$
$\Rightarrow x^{2n}-2xn\cos n\theta+1$
$=\cos 2n\theta\pm i\sin 2n\theta-2\cos^2 n\theta\pm 2i\sin n\theta\cos n\theta+1=0$
Ans. (c)

16. The Discriminant of the given quadratic is
$D = 9b^2 - 32\,ac$
$= 9(-a-c)^2 - 32\,ac \qquad [\because a + b + c = 0]$
$= 9a^2 + 9c^2 - 14ac = c^2\,[9(a/c)^2 - 14a/c + 9]$
Since the discriminant of $9(a/c)^2 - 14(a/c) + 9$ is negative
therefore the sign of the expression $9(a/c)^2 - 14(a/c) + 9$ is always positive.
Hence, the roots of the given equation are real.
Ans. (c)

Note : This question can also be solved direct by SHORTCUT.

17. Each letter can be posted in any one of the 2 letter
So, required number of ways $= 2 \times 2 \times 2 \times 2 \times 2 = 2^5$
Ans. (d)

18. We have : $\sum_{r=1}^{n} r^2 \cdot {}^nC_r = n(n-1)2^{n-2} + n \cdot 2^{n-1}$

and $\sum_{r=1}^{n} (-1)^{r-1} r^2\ {}^nC_r = 0$

Adding these two, we get
$2[1^2C_1 + 3^2C_3 + 5^2C_5 + \ldots.] = n(n-1)\,2^{n-2} + n \cdot 2^{n-1}$
$\Rightarrow \quad 1^2C_1 + 3^2C_3 + 5^2C_5 + \ldots. = n(n-1)\,2^{n-3} + n \cdot 2^{n-2}$
Ans. (d)

Note : This question can also be solved direct by SHORTCUT.

19. $(1+x)^m (1-x)^n = ({}^mC_0 + {}^mC_1x + {}^mC_2x_2 + \ldots.. + {}^mC_mx^m)$
$\times ({}^nC_0 - {}^nC_1\,x + {}^nC_2x^2 \ldots + (-1)^n\,{}^nC_nx^n)$
$= {}^mC_0 \cdot {}^nC_0 - ({}^mC_0\,{}^nC_1 - {}^nC_0\,{}^mC_1)\,x + ({}^mC_0\,{}^nC_2 + {}^nC_0\,{}^mC_2 - {}^mC_1\,{}^nC_1)x^2 + \ldots.$
It is given that the coefficients of x and x^2 in the expression of $(1+x)^m (1-x)^n$ are 3 and –6 repestively.
Therefore,
$-({}^mC_0 \cdot {}^nC_1 - {}^nC_0 \cdot {}^mC_1) = 3$
and ${}^mC_0\,{}^nC_2 + {}^nC_0\,{}^mC_2 - {}^mC_1\,{}^nC_1 = -6$
$\Rightarrow \quad m - n = 3$ and $n(n-1) + m(m-1) - 2mn = -12$
$\Rightarrow \quad m - n = 3$ and $(m-n)^2 - (m+n) = -12$
$\Rightarrow \quad m - n = 3$ and $m + n = 21 \Rightarrow m = 12,\ n = 9$
Ans. (c)

Note : This question can also be solved direct by SHORTCUT.

20. $f(g(x)) = (\sin\sqrt{x})^2 \Rightarrow g(x) = \sqrt{x}$ and $f(x) = (\sin x)^2$

Ans. (a)

21. We have,

$$\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots. \text{ to n terms}$$

$$= \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+\ldots\ldots \text{ to n terms}$$

$$= n-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots.+\frac{1}{2^n}\right) = n-\frac{1}{2}\frac{(1-1/2^n)}{(1-1/2)}$$

$= n - 1 + 2^{-n}$.

Ans. (c)

Note : This question can also be solved direct by SHORTCUT.

22. Let α be a common roots of $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$

Then, $\alpha^2 + p\alpha + q = 0$ and $\alpha^2 + p'\alpha + q' = 0$

$$\Rightarrow \quad \alpha = -\left(\frac{q-q'}{p-p'}\right)$$

Ans. (c)

23. Each object can be put either in box B_1 (say) or in box B_2(say). So, there are two choices for each of the n objects.

Therefore the number of choices for n distinct objects is $\underbrace{2\times 2\times\ldots.\times 2}_{n-\text{times}} = 2^n$.

The of these choices correspond to either the first or the second box being empty.

Thus, there are $2^n - 2$ ways in which neither box is empty.

Ans. (c)

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24. The general term in the expansion of $(1 + x^2 - x^3)^8$ is

$$\frac{8!}{r!s!t!}(1)^r(x^2)^s(-x^3)^t$$

$$= \frac{8!}{r!s!t!}(-1)^t x^{2s+3t}, \text{ where } r + s + t = 8,$$

For the coefficient of x^6, we must have $2x + 3t = 6$.

Now, $r + s + t = 8$ and $2s + 3t = 6$

$\Rightarrow \quad r = \frac{10+t}{2}, s = \frac{6-3t}{2}$, where $0 \le t \le 8$.

Fot t, = 0, r = 5, s = 3 For t = 2, r = 6, s = 0

$\therefore$ Coefficient of $x^8 = \frac{8!}{5!3!0!}(-1)^0 + \frac{8!}{6!0!2!}(-1)^2$

$= \frac{8!}{5!3!} + \frac{8!}{6!2!} = 56 + 28 = 84$

Ans. (b)

Note : This question can also be solved direct by SHORTCUT.

25. **Ans. (c)**

26. Four any square matix X, we have

$X (adj X) = |x| I_n$

Taking X = adj A, we get

$(adj A) (adj (adj A)) = |adj A| I_n$

$\Rightarrow$ $adj A (adj (adj A)) = |A|^{n-1} I^n$

$\Rightarrow$ $(A adj A) (adj (adj A)) = |A|^{n-1} A$

$\Rightarrow$ $(|A| I_n) (adj (adj A)) = |A|^{n-1} A$

$\Rightarrow$ $adj (adj A) = |A|^{n-2} A$

Ans. (c)

Note : This question can also be solved direct by SHORTCUT.

27. $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow X^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$, Clearly for n = 2, then matrices in (a), (b), (c) do not tally with $\begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

Ans. (d)

28. **Ans. (c)**

29. **Ans. (c)**

30. We have AB = B and BA = A. Therefore,
$A^2 + B^2 = AA + BB = A(BA) + B(AB) = (AB)A + (BA)B$
$= BA + AB = A + B.$ [$\because$ AB = B and BA = A]
Ans. (c)

31. The two circle are
$x^2 + y^2 - 2ax + c^2 = 0$ and $x^2 + y^2 - 2by + c^2 = 0$
Centres : $C_1(a, 0)$, $C_2(0, b)$
radii : $r_1 = \sqrt{a^2 - c^2}$, $r_2 = \sqrt{b^2 - c^2}$
Since the two circle touch each other externally, therefore
$C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{a^2 - c^2} + \sqrt{b^2 - c^2}$

$\Rightarrow a^2 + b^2 = a^2 - c^2 + b^2 - c^2 + 2\sqrt{a^2 - c^2}\sqrt{b^2 - c^2}$

$\Rightarrow c^4 = a^2b^2 - c^2(a^2 + b^2) + c^4$

$\Rightarrow a^2b^2 = c^2(a^2 + b^2)$

$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

Ans. (c)

Note : This question can also be solved direct by SHORTCUT.

32. The coordinates of centres C_1 and C_2 of two circles are (1, 0) and (2, 3) respectively. Let r_1 and r_2 be the radii of two circles. Then $r_1 = 2$, and $r_2 = \sqrt{21}$. Clearly $r_1 - r_2 < c_1c_2 < r_1 + r_2$.
Hence the two circle intersect each other.
Ans. (b)

33. Equation of normal to the parabola $y^2 = 8x$ at (x_1, y_1)

$$y - y_1 = -\left(\frac{dx}{dy}\right)(x - x_1)$$

it is given that $-\left(\frac{dx}{dy}\right) = 1$

$y^2 = 8x$ then $\frac{dy}{dx} = \frac{4}{y}$

$y_1 = -4$
then $x_1 = 2$
then equation of normal will be $y + 4 = x - 2$
$x - y = 6$
Ans. (b)

34. **Ans. (a)**

35. The maximum area corresponds to when P is at either of the minor axis and hence area for such a position of P is $\frac{1}{2}(2a)(b) = ab$

Ans. (a)

36. Given $2a = 6$, $2b = 4$. Therefore,

$$e = \sqrt{1-\frac{b^2}{a^2}} \Rightarrow e = \sqrt{\frac{5}{3}},$$

So, Distance between foci = $2ae = 6\sqrt{\frac{5}{3}} = 2\sqrt{5}$

and, Length of the string = $2a + 2ae = 6 + 2\sqrt{5}$

Ans. (d)

37. Eccentricity of rectangular hyperbola is $\sqrt{2}$

$e_1^2 + e_2^2 = 2 + 2 = 4$

Ans. (b)

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38. Let the equation of asymptotes be

$$2x^2 + 5xy + 2y^2 + 4x + 5y = 0 \quad \ldots\ldots(i)$$

This equation represents a pair of straight line. Therefore,

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

Here, $a = 2$, $b = 2$, $h = 5/2$, $g = 2$, $f = 5/2$ and $c = \lambda$

$$\therefore \quad 4\lambda + 25 - \frac{25}{2} - 8 - \frac{25}{4}\lambda = 0 \Rightarrow -\frac{9\lambda}{4} + \frac{9}{2} = 0 \Rightarrow \lambda = 2$$

Putting the value of λ in (i), we get

$$2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$$

This is the equation of the asymptotes.

Ans. (a)

39. Clearly, $f : R \to R$ is a one-one onto function. So, it is invertible.

Let $f(x) = y$. Then, $3x - 5 = y \Rightarrow x = \frac{y+5}{3} \Rightarrow f^{-1}(y) = \frac{y+5}{3}$

Hence, $f^{-1}(x) = \frac{x+5}{3}$

Ans. (b)

40. Since f(x) is continuous at x = 0, therefore

$$\lim_{x\to 0} f(x) = f(0) = 0 \Rightarrow \lim_{x\to 0} x^n \sin\left(\frac{1}{x}\right) = 0 \Rightarrow n > 0$$

f(x) is differentiable at x = 0 if

$$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0} \text{ exists finitely}$$

$$\Rightarrow \lim_{x\to 0} \frac{x^n \sin\frac{1}{x} - 0}{x} \text{ exists finitely}$$

$$\Rightarrow \lim_{x\to 0} x^{n-1} \sin\left(\frac{1}{x}\right) \text{ exists finitely.}$$

$\Rightarrow$ n – 1 > 0
$\Rightarrow$ n > 1.

If $n \le 1$, then $\lim_{x\to 0} x^{n-1} \sin\left(\frac{1}{x}\right)$ does not exist and hence f(x) is not differentiable at x = 0

Hence, f(x) is continuous but not differentiable at x = 0 for $0 < n \le 1$ i.e. $n \in (0, 1]$.
Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

41. Since g(x) is the inverse f(x), therefore
f(x) = y $\Leftrightarrow$ g(y) = x

$$g'(f(x)) = \frac{1}{f'(x)}, \forall x$$

$\Rightarrow$ $g'(f(x)) = 1 + x^3, \forall x$
$\Rightarrow$ $g'(y) = 1 + \{g(y))\}^3$ [Using f(x) = y x = $\Leftrightarrow$ x = g(y)]
$\Rightarrow$ $g'(y) = 1 + \{g(x))\}^3$ [replacing y by x]
Ans. (c)

42. $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$

$$\Rightarrow f'(x) = \frac{1}{1+\left(\frac{x^x - x^{-x}}{2}\right)^2} \cdot \frac{d}{dx}\left(\frac{x^x - x^{-x}}{2}\right)$$

$$\Rightarrow f'(x) = \frac{-2}{4+(x^x - x^{-x})^2} \cdot \frac{d}{dx}(x^x - x^{-x})$$

$$\Rightarrow f'(x) = \frac{-2}{4+(x^x - x^{-x})^2} \cdot \frac{d}{dx}\left(e^{x\log x} - e^{-x\log x}\right)$$

$$\Rightarrow f'(x) = \frac{-2}{(x^x - x^{-x})^2} \times \left(x^x(1+\log x) - x^{-x}(1+\log x)\right)$$

$$\Rightarrow f'(x) = \frac{-2(1+\log x)}{(x^x - x^{-x})^2} \cdot (x^x + x^{-x}) = \frac{-2(1+\log x)}{x+x^{-x}}$$

$$\Rightarrow f'(1) = \frac{-2}{(1+1)} = -1$$

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

43. $f(x) = \log_x\{\ln(x)\} = \frac{\ln(\ln(x))}{\ln(x)}$

$$\therefore f(x) = \frac{\ln(x) \cdot \frac{1}{\ln(x)} \cdot \frac{1}{x} - \ln\{\ln(x)\}\frac{1}{x}}{(\ln(x))^2} = \frac{1-\ln\{\ln(x)\}}{x\{\ln(x)\}^2}$$

$$\Rightarrow f'(e) = \frac{1-\ln\{\ln(e)\}}{x\{\ln(e)\}^2} = \frac{1-\ln(1)}{e} = \frac{1}{e}$$

Ans. (d)

Note : This question can also be solved direct by SHORTCUT.

44. We have $F(x) = \frac{1}{x^2}\int_4^x (4t^2 - 2F'(t))dt$. Therefore,

$$x^2F(x) = \int_4^x (4t^2 - 2F'(t)dt$$

Differentiating both sides with respect to x, we get

$2x\,F(x) + x^2\,F'(x) = 4x^2 - 2F'(x)$

Putting $x = 4$, we get

$8F(4) + 16F'(4) = 64 - 2F'(4) \Rightarrow 18\,F'(4) = 64$

$$\Rightarrow \quad F'(4) = \frac{32}{9}$$

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

45. We have $x^p y^q = (x + y)^{p+q}$

$\Rightarrow$ $p \log x + q \log y = (p + q) \log (x + y)$

Diff. w.r.t. x, we get $\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow$ $\frac{dy}{dx}\left(\frac{q}{y} - \frac{p+q}{x+y}\right) = \frac{p+q}{x+y} - \frac{p}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$

Ans. (a)

46. We have, $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$= \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right) = \frac{\pi}{2}$

$\therefore$ $\frac{dy}{dx} = 0$

Ans. (c)

47. $\int_{-1}^{1}(x - |x|)dx = \int_{-1}^{0}(x - |x|)dx + \int_{0}^{1}(x - |x|)dx$

$= \int_{-1}^{0}(x + 1)dx + \int_{0}^{1}(x - 0)dx$

$= \left[\frac{(x+1)^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1} = \frac{1}{2} + \frac{1}{2} = 1$

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

48. Putting $\log x = t$ i.e. $x = e^t$ in I_1, we get

$I_1 = \int_1^2 \frac{e^t}{t}dt = \int_1^2 \frac{e^x}{x}dx = I_2$

Ans. (a)

49. $\int_0^{\pi/2} \log|\tan x + \cot x|\,dx = \int_0^{\pi/2} \log\left|\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right| dx$

$= \int_0^{\pi/2} \log\left(\frac{1}{\sin x \cos x}\right) dx$

$= -\int_0^{\pi/2} \log \sin x\,dx - \int_0^{\pi/2} \log \cos x\,dx$

$= -(-\pi/2 \log 2) - (-\pi/2 \log 2) = \pi \log 2$

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

50. We have $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$

$\Rightarrow f(x) = -\frac{1}{x^2} f\left(\frac{1}{x}\right)$

$\therefore \int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)dx = \int_{\sin\theta}^{\operatorname{cosec}\theta} -\frac{1}{x^2} f\left(\frac{1}{x}\right)dx = \int_{\operatorname{cosec}\theta}^{\sin\theta} f(t)dx$, where $t = \frac{1}{x}$

$= -\int_{\sin\theta}^{\operatorname{cosec}\theta} f(t)dx = -I$

$\therefore 2I = 0 \Rightarrow I = 0$

Ans. (d)

Note : This question can also be solved direct by SHORTCUT.

51. The equation of any tangent to $x^2 = 4y$ is

$x = m\,y + \frac{1}{m}$; where m is an arbitrary constant.

Differentiating this w.r. to x, we get

$1 = m\frac{dy}{dx} \Rightarrow m = \frac{1}{\frac{dy}{dx}}$

Putting the value of m in $x = my + \frac{1}{m}$; we get

$$x = \frac{y}{\frac{dy}{dx}} + \frac{dy}{dx} \Rightarrow \left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$$

which is differential equation of order 1 and degree 2.

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

52. Clearly, $y = 2x - 4$ satisfies the given differential equation.

Ans. (c)

53. We have, $y^2 = 2c\left(x + \sqrt{c}\right)$ .....(i)

$\Rightarrow \quad 2yy_1 = 2x \Rightarrow yy_1 = c$ .....(ii)

Eliminating c from (i) and (ii), we get

$$y^2 = 2yy_1\left(x + \sqrt{yy_1}\right) \Rightarrow y - 2xy_1 = 2\sqrt{y}y_1^{3/2}$$

$$\Rightarrow (y - 2xy_1)^2 = yy_1^3$$

Clearly, it is a differential equation of order one and degree 3.

Ans. (d)

54. We have $\vec{\alpha} = x\left(\vec{a} \times \vec{b}\right) + y\left(\vec{b} \times \vec{c}\right) + z\left(\vec{c} \times \vec{a}\right)$

Taking dot products with $\vec{a}\ \vec{b}\ \vec{c}$, we get

$$\vec{\alpha}.\vec{a} = y[\vec{a}\ \vec{b}\ \vec{c}] \Rightarrow \vec{y} = 8(\vec{\alpha}.\vec{a})$$

$$\vec{\alpha}.\vec{b} = z((\vec{c} \times \vec{a}).\vec{b})$$

$$\Rightarrow \quad \vec{\alpha}.\vec{b} = z[\vec{a}\ \vec{b}\ \vec{c}] \Rightarrow \vec{z} = 8(\vec{\alpha}.\vec{b})$$

and $\vec{\alpha}.\vec{c} = x((\vec{a} \times \vec{b}).\vec{c}))$

$$\Rightarrow \quad \vec{\alpha}.\vec{c} = x[\vec{a}\ \vec{b}\ \vec{c}] \Rightarrow x = 8(\vec{\alpha}.\vec{c})$$

$$\therefore \quad x + y + z = 8\alpha(\vec{a} + \vec{b} + \vec{c})$$

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

55. Let $x\hat{i}+y\hat{j}+z\hat{k}$ be the unit vector along $\vec{c}$. Since, $-\hat{i}+\hat{j}-\hat{k}$ bisect the angle between $\vec{c}$ and $3\hat{i}+\hat{j}$. Therefore,

$$\lambda(\hat{i}+\hat{j}-\hat{k})=(x\hat{i}+y\hat{j}+z\hat{k})+\frac{3\hat{i}+4\hat{j}}{5}$$

$$\Rightarrow x+\frac{3}{5}=-\lambda,\ y+\frac{4}{5}=\lambda \text{ and } z=-\lambda$$

Now, $x^2+y^2+z^2=1$ $\quad$ [$x\hat{i}+y\hat{j}+z\hat{k}$ is a unit vector]

$$\left(-\lambda-\frac{3}{5}\right)^2+\left(\lambda-\frac{4}{5}\right)^2+\lambda^2=1\Rightarrow\lambda=0 \text{ or } \lambda=\frac{2}{15}$$

But $\lambda\neq 0$. Because $\lambda=0$ implies that the given vectors are parallel

$$\therefore \lambda=\frac{2}{15}\Rightarrow x=-\frac{11}{15}, y=\frac{-10}{15} \text{ and } z=\frac{-2}{15}$$

Hence, $x\hat{i}+y\hat{j}+z\hat{k}=-\frac{1}{15}(11\hat{i}+10\hat{j}+2\hat{k})$

Ans. (d)

Note : This question can also be solved direct by SHORTCUT.

56. We have, $(\vec{a}+3\vec{b})\perp(7\vec{a}-5\vec{b})=(\vec{a}+3\vec{b}).(7\vec{a}-5\vec{b})=0$

$$=7|\vec{a}|^2+16(\vec{a}.\vec{b})-15|\vec{b}|^2=0$$
$$=7+16\cos\theta-15=0$$

$$\Rightarrow\cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$$

Ans. (c)

57. Since each ball can be placed in any one of the 3 boxes, therefore there are 3 ways in which a ball can e placed in any one of the three boxes. Thus there are 3^{12} ways in which 12 balls can be placed in 3 boxes. The number of ways in which 3 balls out of 12 can be put in the first box is ${}^{12}C_3$. The remaining 9 balls can be placed in 2 boxes in 2^9 ways.

So, required probability = $\frac{{}^{12}C_3}{3^{12}}.2^9=\frac{110}{9}\left(\frac{2}{3}\right)^{10}$

Ans. (a)

58. Since $P(A/\bar{B})+P(\bar{A}/\bar{B})=1$, Therefore,

$P(\bar{A}/\bar{B})=1-P(A/\bar{B})$

Correct answer is $1-P(A/\bar{B})$ **which is not given in any of the four option.**

Ans. (wrong)

59. **Ans. (a)**

60. As $\angle A = 45°$, $\angle B = 75°$, we have
$\angle C = 180° - (45° - 75°) = 60°$

$\therefore \quad a+c\sqrt{2} = k(\sin A + \sqrt{2}\sin C)$

$= k(\sin 45° + \sqrt{2}\sin 60°) = k\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$ .....(i)

Now, $b = k\sin B \Rightarrow b = k\sin 75° = k\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

$\Rightarrow \quad 2b = k\frac{(\sqrt{3}+1)}{\sqrt{2}}$ .....(ii)

From (i) and (ii), $a+c\sqrt{2}=2b$

Ans. (c)

Note : This question can also be solved direct by SHORTCUT.

61. **Ans. (b)**

62. Let $A = 6+\sqrt{12}, b = \sqrt{48}, c = \sqrt{24}$.
Clearly c is the smallest side. Therefore the smallest angle C is given by

$$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{\sqrt{3}}{2} \Rightarrow C = \frac{\pi}{6}$$

Ans. (c)

63. $\int[f(x)g''(x)-f''(x)g(x)]dx$

$\int f(x)g''(x)dx-\int f''(x)g(x)dx$

$=\left(f(x)g'(x)-\int f'(x)g'(x)dx\right)-\left(g(x)f'(x)-\int g'(x)f'(x)dx\right)$

$=f(x)\,g'(x)-f'(x)\,g(x)$

Ans. (c)

64. $\int e^{3\log x}\left(x^4+1\right)^{-1}dx=\int e^{\log x^3}\frac{1}{x^4+1}dx$

$=\int\frac{x^3}{x^4+1}dx=\frac{1}{4}\log\left(x^4+1\right)+C$

Ans. (b)

65. Let $I=\int\frac{x+2}{(x^2+3x+3)\sqrt{x+1}}dx$

Putting $x+1=t^2$, $dx=2t$, we get

$I=2\int\frac{t^2+1}{t^4+t^2+1}dt=2\int\frac{1+(1/t)^2}{(t-\frac{1}{t})^2+3}dt$

$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{3}}\right)+C=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3(x+1)}}\right)+C$

Ans. (b)

66. $\lim_{x\to a}\frac{a^x-x^n}{x^x-a^a}=-1$

$\Rightarrow\lim_{x\to a}\frac{a^x\log a-ax^{n-1}}{x^x(1+\log x)}=-1$ [Using L-Hospital's Rule]

$\Rightarrow\frac{a^a\log_e a-a.a^{a-1}}{a^a(1+\log_e a)}=-1\Rightarrow\frac{\log_e a-1}{\log_e a+1}=-1$

Then is satisfied only when a = 1.

Ans. (a)

67. $\lim_{x\to 0}(\cos x)^{\cot x} = \lim_{x\to 0}(1+\cos x-1)^{\cot x}$

$$\lim_{x\to 0}\left(1-2\sin^2\left(\frac{x}{2}\right)\right)^{\cot x}$$

$$= e^{\lim_{x\to 0} -2\sin^2(x/2).\cot x}$$

$$= e^{\lim_{x\to 0} -2\frac{\sin^2(x/2).\cos x}{2\sin(x/2)\cos x/2}}$$

$$= e^{\lim_{x\to 0} -\tan(x/2).\cos x} = e^0 = 1$$

Ans. (b)

68. Given Question is incomplete it should have been

If $(1 + x)^n = (C_0 + C_1x + C_2x^2 + ... + C_nx^n)$ then $C_0^2 + C_1^2 + C_2^2 + ...C_n^2$ is equal to

We have,

$(1 + x)^n = (C_0 + C_1x + C_2x^2 + ... + C_nx^n)$ .....(i)

Also,

$(1 + x)^n = (C_0x^n + C_1x^{n-1} + ... + C_{n-1}x + C_n)$ .....(ii)

Multiplying (ii) and (iii), we get

$(C_0 + C_1x + C_2x^2 + C_3x^3 + ... + C_n x^n)$

$\times (C_0x^n + C_1 x^{n-1} + C_2x^{n-2} + C_n x^{n-2} + ... + C_{n-1}x + C_n) = (1 + x)^{2n}$ .....(iii)

Equating coefficient of x^n on both sides of (iii), we get

$$C_0^2 + C_1^2 + C_2^2 + ...C_n^2 = {}^{2n}C_n$$

$$= C_0^2 + C_1^2 + C_2^2 + ...C_n^2 = \frac{(2n)!}{n!n!}$$

Ans. (Given question is incomplete)

Note : This question can also be solved direct by SHORTCUT.

69. In ΔPQR, the radisu of the circumcircle is given by $\frac{PQ}{2\sin R} = \frac{QR}{2\sin P} = \frac{PR}{2\sin Q}$. But it the given the radius is

$$\therefore \quad PQ = PR = \frac{PQ}{2\sin R} = \frac{QR}{2\sin P} = \frac{PQ}{2\sin Q}$$

$$\Rightarrow \sin R = \sin Q = \frac{1}{2} \Rightarrow \angle R = \angle Q = \frac{\pi}{6}$$

$$\Rightarrow \angle P = \pi - \angle R - \angle Q = \frac{2\pi}{3}$$

Ans. (d)

70. We know that the equation second degree curve in
$ax^2 + by^2 + 2xy + 2gx + 2fy + c = 0$
for pair of straight line $\Delta = 0$
when $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ .....(i)
$a = -1, b = 1, c = -1, f = 0, g = a, h = 0$
Put in equation (i)
$-1 + 2 \times 0 \times a \times 0 - 1 \times 0 + 1 \times a^2 - 1 \times 0 = 0$
$-1 + a^2 = 0, \ \Delta \neq 0$
where a is variable
All the four option given are incorrect.
Ans. (wrong)

Note : This question can also be solved direct by SHORTCUT.

71. We know thatn $\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

Hence, $\tan^{-1}(1) + \frac{\pi}{2} \Rightarrow \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$ $\left\{\text{where } \tan^{-1}(1) = \frac{\pi}{4}\right\}$

Ans. (a)

72. We know that period of function $|\sin x| = \pi$
Hence, $\pi x = \pi$
$\Rightarrow \ x = 1$
Ans. (c)

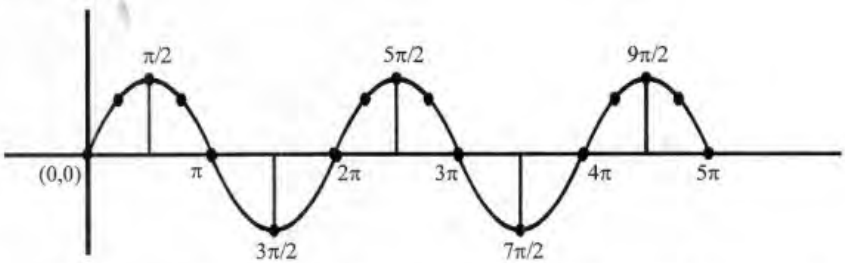
73. Given that,
$2^y + 2^x = 2$
$2^y = 2 - 2^x$
taking log on both side
$y \log 2 = \log (2 - 2^x)$
for $\log (2 - 2^x)$ the neccessary condition that $2 - 2^x > 0$
$2^x < 2$
Hence, $x < 1$
So, domain is $(-\infty, 1)$
Ans. (b)

74. Given $1! + 2! + 3! + 4! + ...$
Expand the given equation
$1 + 2 + 6 + 24 + 120 + 720 + ...$ {After 4! the digit at unit place is 0}
Add upto 4!
Hence digit at unit place 3
Ans. (a)

75. Given equation is $3\sin^2x - 7\sin x + 2 = 0$
By factoring
$3\sin^2x - 6\sin x - \sin x + 2 = 0$
$3\sin x(\sin x - 2) - 1(\sin x - 2) = 0$

$\sin x = \frac{1}{3}$ and $\sin x \neq 2$ (because $\sin x = [-1, 1]$)

Graph at sin x



From the above graph it is clear that in every $(0, \pi)$, $(2\pi, 3\pi)$, $(4\pi, 5\pi)$ there are two solution, hence total no. of solution is 6 from $[0, 5\pi]$
Hence there are 6 value of x which satisfy the given equation.
Ans. (b)

Note : This question can also be solved direct by SHORTCUT.

76. Given that
$1 - \cos\theta = \sin\theta/2 \cdot \sin\theta$ .....(1)
We know that $\cos\theta = 1 - 2\sin^2\theta/2$ put in eq. (1)
$1 - (1 - 2\sin^2\theta/2) = \sin\theta/2 \sin\theta$
$2\sin^2\theta/2 = \sin\theta/2 \sin\theta$
$\sin\theta/2 = 0$
$\sin\theta/2 = k\pi$ where $k \in I$
$\theta = 2k\pi$
Ans. (b)

77. Given that $\frac{dy}{dx} = e^{-2y}$
$e^{2y}dy = dx$
on integrating both side
$\int e^{2y}dy = \int dx$
$\frac{e^{2y}}{2} = x + c$

put $x = 5, y = 0$

$$\frac{1}{2} - 5 = c \;=\; c = \frac{-9}{2}$$

$$\frac{e^6}{2} = x - \frac{9}{2}$$

$$x = \frac{e^6 + 9}{2}$$

Ans. (c)

Note : This question can also be solved direct by SHORTCUT.

78. We have

$$F(x) = \int x \log\left(1 + \frac{1}{x}\right) dx$$

$$\int x \log\left(\frac{x+1}{x}\right) dx$$

Integrate by part $I = \log\left(\frac{x+1}{x}\right), II = x$

$$= I\int II\,dx - \int\left[\frac{d}{dx} I \int II\,dx\right] dx$$

$$= \log\left(\frac{x+1}{x}\right)\frac{x^2}{x} - \int \frac{x^2}{2} \cdot \frac{x}{x+1}\left(-\frac{1}{x^2}\right) dx$$

$$= \log\left(\frac{x+1}{x}\right)\frac{x^2}{x} + \frac{1}{2}\int \frac{x}{x+1} dx$$

$$= \log\left(\frac{x+1}{x}\right).\frac{x^2}{x}\log(x) + \frac{1}{2}x - \frac{1}{2}\log(x+1) + c$$

$$= \left(\frac{x^2-1}{2}\right)\log(x+1) - \frac{x^2}{2}\log x + \frac{x}{2} + c$$

Compare with $f(x) \log (x + 1) + g(x) x^2 + Lx + C$

$$f(x) = \frac{x^2-1}{2},\quad g(x) = -\frac{\log x}{2},\quad L = \frac{1}{2}$$

hence, none of these is correct options.

Ans (d)

Note : This question can also be solved direct by SHORTCUT.

79. We have $f(x) = |\sin 2x - \cos 2x|$

We know that the maximum and minimum value of function $a \sin x + b \cos x$ is

$\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$ respectively

Hence, $a = 1, b = 1$

the max. value of function is $\sqrt{2}$

and Minimum value is $-\sqrt{2}$

hence, the range will be $[-\sqrt{2}, \sqrt{2}]$ but it is not given in the option so approximately range is $[-1, 1]$

Ans. (d)

80. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$\Rightarrow abc - a^3 - b^3 + abc + abc - c^3$

$= -(a + b + c)(a + bk + ck^2)(a + bk^2 + ck)$

$\Rightarrow +(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a + bk + ck^2)(a + bk^2 + ck)$

$\Rightarrow (a + b + c)(a + b\omega + c\omega^2)(a + \omega^2 b + c\omega)$

$= (a + b + c)(a + bk + ck^2)(a + bk^2 + ck)$

Hence $k = \omega$ {where ω is cube root of unity $1 + \omega + \omega^2 = 0$}

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

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81. $I = \int_{\pi/4}^{3\pi/4} \frac{1}{1+\cos x} dx \Rightarrow \int_{\pi/4}^{3\pi/4} \frac{(1-\cos x)}{(1-\cos x)(1+\cos x)} dx$

$\Rightarrow \int_{\pi/4}^{3\pi/4} \frac{1-\cos x}{\sin^2 x} dx \Rightarrow \int_{\pi/4}^{3\pi/4} (\operatorname{cosec}^2 x - \cot x . \operatorname{cosec} x) dx$

$\Rightarrow [-\cot x + \operatorname{cosec} x]_{\pi/4}^{3\pi/4}$

$\Rightarrow 2$

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

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82. All expect Maths are branches of Maths
Ans. (a)

83. All expect Diagonal are terms associated with circle.
Ans. (c)

84. INFLUENZA is caused by VIRUS and
TYPHOID is caused by BACTERIA.
Ans. (d)

85. Angle traced by hour hand in 12 hour = 360°

Angle traced by it 5 hrs. 10 min. i.e. $\frac{31}{6}$ hours

$$= \left(\frac{360}{12} \times \frac{31}{6}\right)^\circ = 155^\circ$$

Angle traced by minute hand in 60 min = 360°

Angle traced by it in 10 min = $\frac{360}{60} \times 10 = 60^\circ$

$\therefore$ Required angle = $(155^\circ - 60^\circ) = 95^\circ$
Ans. (b)

Note : This question can also be solved direct by SHORTCUT.

86. Let the number of girls in the class be x
then, the number of boys in class = 3x
$\therefore$ Total no. of students = x + 3x = 4x
It means total no. of students must be a multiple of 4
$\therefore$ 42 cannot be total no. of students as it is not multiple of 4.
Ans. (c)

Note : This question can also be solved direct by SHORTCUT.

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87. First worker's one day's work = $\frac{1}{10}$

Second worker's one day's work = $\frac{1}{15}$

Both worker's one day's work = $\frac{1}{10}+\frac{1}{15}=\frac{5}{30}=\frac{1}{6}$

Therefore both the workers will finish the work in 6 days.

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

88. Clearly, the given sequence follows the pattern
+6, +12, +24, +48,.....
Thus, 7 + 6 = 13, 13+12 = 25, ...
So missing term = 49 + 48 = 97
Ans. (b)

89. MPUTER
MPUTERS
UT
UTER
TER
RS
there are 6 such pairs
Ans. (d)

90. **Note : This question solved direct by SHORTCUT.**
Ans. (c)

91. 55 min spaces are covered in 60 min

60 min spaces are covered in $\left(\frac{60}{55}\times 60\right)$ min

$= 65\frac{5}{11}$ min

Loss in 64 min = $\left(65\frac{5}{11}-64\right)=\frac{16}{11}$ min

Loss in 24 hrs. = $\left(\frac{16}{11}\times\frac{1}{64}\times 24\times 60\right)$ min

$= 32\frac{8}{11}$ min

Ans. (c)

Note : This question can also be solved direct by SHORTCUT.

92. $x = 2\sqrt{3}$ (given)
$xy = 1$ (given)
then $y = 2-\sqrt{3}$

$$\sqrt{x} = \frac{1+\sqrt{3}}{\sqrt{2}}$$

$$\sqrt{y} = \frac{1-\sqrt{3}}{\sqrt{2}}$$

Substituting the value of $\sqrt{x}, \sqrt{y}$, x and y in the expression, $\frac{x}{\sqrt{2}+\sqrt{x}} + \frac{y}{\sqrt{2}-\sqrt{y}}$

$$\Rightarrow \frac{\left(\sqrt{2}-\sqrt{y}\right)x + \left(\sqrt{2}+\sqrt{x}\right)y}{2+\sqrt{2}\sqrt{x}-\sqrt{2}\sqrt{y}-\sqrt{xy}}$$

$$\Rightarrow \frac{\left[\frac{2-(1-\sqrt{3})}{\sqrt{2}}\right](2+\sqrt{3}) + \left[\frac{2+1+\sqrt{3}}{\sqrt{2}}\right](2-\sqrt{3})}{2+1+\sqrt{3}-(1-\sqrt{3})-1}$$

$$\Rightarrow \frac{(1+\sqrt{3})(2+\sqrt{3}) + (4+\sqrt{3})(2-\sqrt{3})}{\sqrt{2}(1+2\sqrt{3})} \quad \frac{10+\sqrt{3}}{\sqrt{2}+2\sqrt{6}}$$

Ans. (d)

Note : This question can also be solved direct by SHORTCUT.

93. Let the train will meet after time t hours.
then, distance travelled by train before meet is equal to 90t

now, $90t = 80\left(t + \frac{30}{60}\right)$

$90t = 80t + 40$
$10t = 40$
$t = 4$ hrs.
Hence distance travelled = $90 \times 4 = 360$ km.
Ans. (c)

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94. Let the number of coins of each type be x.
the sum of money = Rs. 35 (given)
Hence, sum of money = (x × 1) + (x × 0.5) + (x × 0.25)
35 = x × (1 + 0.5 + 0.25)
35 = x × 1.75

$$x = \frac{35}{1.75} = 20$$

Ans. (a)

95. The fourth day after 6th January is Saturday (Given)
So, the day on 6th January will be Tuesday
Number of days between 1st december and 6th January are 36 days.
Number of odd days = 1 day
So, the day on 1st December will be 1 day before Tuesday i.e. Monday
Correct Ans. Monday which is not given any of the four option.
Ans. (wrong)

96. First is the product of the second.
Ans. (a)

97. Clearly the man initially faces in the direction SOUTH
On moving 135° anticlockwise he faces in the direction NORTH - EAST
Finally moving 180° clockwise he faces in the direction SOUTH-WEST.
Ans. (d)

98. $$\begin{vmatrix} \cos\theta & \sin\theta & \cos\theta \\ -\sin\theta & \cos\theta & \sin\theta \\ -\cos\theta & -\sin\theta & \cos\theta \end{vmatrix} = 0$$

$R_1 \to R_1 + R_3$

$$\begin{vmatrix} 0 & 0 & 2\cos\theta \\ -\sin\theta & \cos\theta & \sin\theta \\ -\cos\theta & -\sin\theta & \cos\theta \end{vmatrix} = 0$$

$2\cos\theta\,[\sin^2\theta + \cos^2\theta] = 0$
$2\cos\theta = 0$
$\cos\theta = 0$

$$\theta = 2n\pi \pm \frac{\pi}{2}$$

99. Circumference of the wheel = $4\frac{2}{7}m = \frac{30}{7}m$ (given)

It makes 7 revolutions in 4 second (given)

Distance travelled by wheel in 4 second = $\frac{30}{7} \times 7 = 30m$

Distance travelled in 1 second = $\frac{30}{4} = \frac{15}{2}m$

Distance travelled in 1 hr. (in km)

$$= \frac{15}{2} \times \frac{60 \times 60}{1000} = 27km$$

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

100. $\left(1+2\sqrt{x}\right)^{40} = {}^{40}C_0 + {}^{40}C_1(2\sqrt{x}) + {}^{40}C_2\left(2\sqrt{x}\right)^2 + ... + {}^{40}C_{40}(2\sqrt{x})^{40}$

$$\left(1+2\sqrt{x}\right)^{40} = \left({}^{40}C_0 + {}^{40}C_2 \times 2x + {}^{40}C_4 \times 2^2x^2 + ... + {}^{40}C_{20} \times 2^{10}x^{10}\right)$$

Putting $\sqrt{x} = 1$ and -1 respectively, we get

$$3^{40} = \left\{{}^{40}C_0 + {}^{40}C_2 \times 2 + {}^{40}C_4 \times 2^2 + ... + {}^{40}C_{20} \times 2^{10}\right\} + \left\{{}^{40}C_1 \times 2 + {}^{40}C_3 \times 2^3 + ... + {}^{40}C_{19}2^{19}\right\}$$

and,

$$1 = \left\{{}^{40}C_0 + {}^{40}C_2 \times 2 + {}^{40}C_4 \times 2^2 + ... + {}^{40}C_{20} \times 2^{10}\right\} - \left\{{}^{40}C_1 \times 2 + {}^{40}C_3 \times 2^3 + ... + {}^{40}C_{19}2^{19}\right\}$$

$$\therefore \quad 3^{40} + 1 = 2\left\{{}^{40}C_0 + {}^{40}C_2 \times 2 + {}^{40}C_4 \times 2^2 + ... + {}^{40}C_{20} \times 2^{10}\right\}$$

$$\Rightarrow \quad {}^{40}C_0 + {}^{40}C_2 \times 2 + {}^{40}C_4 \times 2^2 + ... + {}^{40}C_{20} \times 2^{10} = \frac{3^{40}+1}{2}$$

Ans. (d)

Note : This question can also be solved direct by SHORTCUT.

101. Putting $x^n + 1 = t$ and $n\,x^{n-1}\,dx = dt$, we get

$$I = \int \frac{1}{x(x^n+1)}dx = \frac{1}{n}\int \frac{1}{t(t-1)}dt = \frac{1}{n}\int\left(\frac{1}{t-1} - \frac{1}{t}\right)dt$$

$$\Rightarrow \quad I = \frac{1}{n}\log\left(\frac{t-1}{t}\right) + C = \frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + C$$

Ans. (a)

102. From Defination : **Equivalence Relation :** A relation R on a set A is said to be an equivalence relatio on A iff

(i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$

(ii) it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

(iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$

Ans. (d)

103. Here, function $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x \neq 0 \\ 0, & x = 0 \end{cases}$

For continuity of function, Check for Left Hand Limit

$$\lim_{h\to 0^-} -he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}$$

$$\lim_{h\to 0^-} -he^{\infty} = 0$$

For Right Hand Limit

$$\lim_{h\to 0^-} (0+h)e^{-\left(\frac{1}{|0+h|}+\frac{1}{(0+h)}\right)}$$

$$\lim_{h\to 0^-} he^{-2h} = 0$$

L.H.L. = R.H.L. = 0

Hence given function is continuous at x = 0. For differentiability

For $\lim_{h\to 0^+} \frac{f(0-h)-f(h)}{(0-h)}$

$$\lim_{x\to 0^-} \frac{(0-h)e^{-\left(\frac{1}{|0-h|}+\frac{1}{h}\right)} - he^{\left(\frac{1}{h}+\frac{1}{h}\right)}}{0-h}$$

$$\lim_{x\to 0^-} \frac{-he^{\infty}}{-h} = 1$$

For R.H.L.

$$\lim_{x\to 0^-} \frac{f(0+h)-f(h)}{0+h} \Rightarrow \frac{(o+h)e^{-\left(\frac{1}{h}+\frac{1}{h}\right)} - he^{-\left(\frac{1}{h}+\frac{1}{h}\right)}}{0+h}$$

$$\frac{he^{-\frac{2}{h}}}{h} = e^{-\frac{2}{0}} = 0$$

L.H.L ≠ R.H.L.

Hence given function is not differentiable at x = 0

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

104. In the case of each book we may take 0,1 ,2 3, ..., p copies;
that is, we may deal with each book in p + 1 ways and therefore with all the books in $(p + 1)^n$ ways. But, this includes the case where all the books are rejcted and no selection is made.

$\therefore$ Number of ways in which selection can be made
$= (p + 1)^n - 1$

Ans. (c)

105. $AA^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9I$$

$$\Rightarrow \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Equation $a + 4 + 2b = 0 \Rightarrow a + 2b = -4$ .....(1)
$2a + 2 - 2b = 0 \Rightarrow 2a - 2b = -2$ .....(2)
& $a^2 + 4 + b^2 = 0 \Rightarrow a^2 + b^2 = 5$ .....(3)
Solving $a = -2, b = -1, a + b = -3$

Ans. (a)

106. Check by option

(a) $\left(2 - \frac{1}{\sqrt{2}}\right)(1 - i)$ put in $|z - 2 + 2i| = 1$

$$\left|2 - \frac{1}{\sqrt{2}} - 2i + \frac{i}{\sqrt{2}} - 2 + 2i\right| = \left|-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right| = 1$$

Ans. (a)

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107. Set of natural number {1, 2,....., 120}
divisible of 5 are {5, 10, 15, ..., 120}
total divisible of 5 = 24
divisible of 15 are {15, 30, 45, ..., 120}
total divisible of 15 are 8
$(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) = \text{divisible of } 5 = \frac{24}{120}$

$P(B) = \text{divisible of } 15 = \frac{8}{120}$

$P(A \cap B) = \text{divisible by 5 and 15} = \frac{8}{120}$

put all above values in the above formula

$$P(A \cup B) = \frac{24}{120} + \frac{8}{120} - \frac{8}{120}$$

$$P(A \cup B) = \frac{1}{5}$$

Ans. (a)

Note : This question can also be solved direct by SHORTCUT.

108. for (a) $f(x) = \frac{1}{1-x}$

inverse of $f(x)$ $y = \frac{1}{1-x}$

$y - xy = 1$

$x = \frac{1-y}{y}$

Replace $x = y$

It is clear that inverse of $f(x)$ does not exist for $x = 0$

(b) $f(x) = x^2$ for $x \in R$

inverse of $f(x)$ $y = x^2$

$x = \sqrt{y}$

Replace $x = y$

It is clear that inverse of $f(x)$ does not exist for negative value of x

(c) $f(x) = x^2$ for all $x \geq 0$

inverse of $f(x)$ $y = x^2$

$x = \sqrt{y}$

Replace $x = y$

inverse of given function exist for all positive value of x

Hence Ans. is (c)

Ans. (c)

109. $\sec 4\theta - \sec 2\theta = 2$

$$\frac{1}{\cos 4\theta} - \frac{1}{\cos 2\theta} = 2$$

$$\frac{1}{2\cos^2 2\theta - 1} - \frac{1}{\cos 2\theta} = 2$$

$\cos 2\theta - 2\cos^2 2\theta + 1 = 2\cos 2\theta\,(2\cos^2 2\theta - 1)$

$-2\cos^2 2\theta + 2\cos 2\theta - \cos 2\theta + 1 = 2\cos 2\theta\,(2\cos^2 2\theta - 1)$

$-2\cos 2\theta\,(\cos 2\theta - 1) - (\cos 2\theta - 1) = 2\cos 2\theta\,(2\cos^2 2\theta - 1)$

$\cos\theta = 0$

$$\theta = 2n\pi \pm \frac{\pi}{2}$$

Ans.(c)

Note : This question can also be solved direct by SHORTCUT.

110. If $f(x) = x^3 + 3x^2 - 9x + c = (x-\alpha)^2(x-\beta)$, then $(x-\alpha)$ is a factor of order 2. So, $x-\alpha$ is a factor of order one of $f'(x)$ i.e. $3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x+3)(x-1)$

$\therefore \quad f'(x) = 0 \Rightarrow x = 1$ or $x = -3$

This show that either $\alpha = 1$ or $\alpha = -3$

If $\alpha = 1$, then as α is a root of $x^3 + 3x^2 - 9x + c = 0$.

Therefore,

$1 + 3 - 9 + c = 0 \Rightarrow c = 5$

If $\alpha = -3$, then as α is a root of $x^3 + 3x^2 - 9x + c = 0$.

Therefore, $-27 + 27 + 27 + c = 0 \Rightarrow c = -27$

Ans. (c)

111. Brother of woman's mother is maternal uncle of woman
hence, son of maternal uncle is cousin of the woman.
hence, answer is (b)
Ans. (b)

112. Daughter of Nilesh's wife is daughter of Nilesh also.
And Husband of his daughter is man, whom he is pointing
Hence, Nilesh is father-in-law of the man.
Ans. (b)

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113. Out of 9 socks, 2 can be drawn in ${}^9C_2$ ways

∴ Total number of elementary event = ${}^9C_2$

Two socks drawn from the drawer will match if either both are brown or both are blue.

∴ Favourable number of elementary events = ${}^5C_2 + {}^4C_2$

Hence, required probability = $\frac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \frac{4}{9}$

Ans. (a)

114. Given $y_2(x^2 + 1) = 2xy_1$

$\Rightarrow \frac{y_2}{y_1} = \frac{2x}{x^2+1}$

Integrating both sides, we get

$\log y_1 = \log (x^2 + 1) + \log c \Rightarrow y_1 = c(x^2 + 1)$

Given, $y_1 = 3$ at $x = 0 \Rightarrow c = 3$

∴ $y_1 = 3(x^2 + 1)$

Again, integrating we get = $y = \frac{3x^3}{3} + 3x + c_1$

This passes through (0, 1) ∴ $c_1 = 1$

Equation of the curve is $= x^3 + 3x + 1$

Correct ans. is $x^3 + 3x + 1$ which is not given in any of the four option

Ans. (Wrong)

115. We have, $\frac{x^2}{9} + \frac{y^2}{5} = 1$

Let e be the eccentricity of this ellipse, Then,

$e^2 = 1 - \frac{5}{9} \Rightarrow e = \frac{2}{3}$

The coordinates of the end-points of latusrecta are

L(2, 5/3), M(–2, 5/3), M' (–2, –5/3) and L' (2, –5/3)

The equation of tangents at these points are

$2x + 3y - 9 = 0$ .....(i)

$-2x + 3y - 9 = 0$ .....(ii)

$2x + 3y + 9 = 0$ .....(iii)

$-2x + 3y + 9 = 0$ .....(iv)

Clearly, these tangents form a parallelogram whose are is given by

$$A = \left| \frac{\{9-(-9)\} \times \{9(-9)}{\begin{vmatrix} 2 & 3 \\ -2 & 3 \end{vmatrix}} \right| = \frac{18 \times 18}{12} \text{ sq.units}$$

$\Rightarrow$ A = 27 sq. units

Ans. (d)

116. Consider the following events.

E = Student A solves the problem
F = Student B solves the problem
G = Student C solves the problem
H = Student D solves the problem

Clearly, E, F, G, H are independent events such that

$$P(E) = \frac{1}{2}, P(F) = \frac{2}{3}, P(G) = \frac{3}{4}, P(H) = \frac{2}{7}$$

Required probability is $P(E \cup F \cup G \cup H)$

$\Rightarrow$ Required probability $= 1 - P(\bar{E})P(\bar{F})P(\bar{G})P(\bar{H})$

$\Rightarrow$ Required probability $= 1 - \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{5}{7} = 1 - \frac{5}{168}$

$$= \frac{163}{168}$$

Ans. (c)

117. Equation of circle through origin and chord of contact is

$x^2 + y^2 + 2gx + 2fy + c + \lambda(gx + fy + c) = 0$

$\Rightarrow \lambda = -1$ [By x = 0, y = 0]

Therefore, equation is $x^2 + y^2 + gx + fy = 0$

Hence, circumcentre is $\left(-\frac{g}{2}, -\frac{f}{2}\right)$

Ans. (d)

Note : This question can also be solved direct by SHORTCUT.

118. Let E_1, E_2 and A be the event defined as follows reports that is a six

We have $P(E_1) = \frac{1}{6}, P(E_2) = \frac{5}{6}$

Now, $P(A/E_1)$ = Probability that the man reports that there is a six on the die given that six has occured on the die

= Probability that man speak truth = 3/4 and $P(A/E_2)$

= Probability that the man reports that there six on the die given that six has not occured on the die

= Probability that the man does not speak truth

$= 1 - \frac{3}{4} = \frac{1}{4}$

By Baye's Rule, we have

$$P(E_1 / A) = \frac{P(E_1)P(A / E_1)}{P(E_1)P(A / E_1) + P(E_2)P(A / E_2)} = \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}$$

Ans. (c)

Note : This question can also be solved direct by SHORTCUT.