MATHEMATICS
Scoring Indicators (Science)

HSE II

Maximum Score: 80

Qn. No.	Answer Key /Value Points	Sub score	Total score
1.	Show $(a, b) * (c, d) = (c, d) * (a, b)$ identity element = (1, 1) invertible element = (1, 1)	1 1 1	 3
2.	Prove	3	3
3.	$A = \begin{bmatrix} 0 & -2 & -4 \\ 2 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$ Show $A^T = -A$	2 1	 3
4.	Prove	3	3
5.	length $= x$ width $= y$ then $\frac{dx}{dt} = -5\text{cm/m}$ $\frac{du}{dt} = 4\text{cm/m}$ A = xy $\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$ $= 8 \times 4 + 6 \times -5$ $= 32 - 30 = 2\ \text{cm}^2/\text{m}$	 1 1 1	 3
6.	$\overline{p} = 2i - j + k$ let $\overline{q} = 2i + 2j + xk$ (two components can be chosen randomly) $\overline{p} \cdot \overline{q} = 0 \Rightarrow 4 - 2 + x = 0$ $x = -2$ $\overline{q} = 2i + 2j - 2k$, find $\overline{r} = \overline{p} \times \overline{q}$ (give full score for any correct answer)	 1 1 1	 3
7.	a) $\lvert \overline{a} \rvert = \sqrt{9+1+4} = \sqrt{14}$ b) (ii) 6i + 2j + 4k ; [$\lvert \overline{a} \rvert$ = projection at $\overline{a}$ on $\overline{b} \Rightarrow \overline{a}$ and $\overline{b}$ are parallel] c) projection of $\overline{a} = \lvert \overline{a} \rvert \cos 60$ $= \sqrt{14} \times \frac{1}{2} = \frac{\sqrt{14}}{2}$	1 1 1	 3

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8.	$\|A\| = 40$	1	
	$\text{adj } A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$		
	$A^{-1} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$	2	
	$= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$	1	4
9.	$f(x) = 3x - 2$		
	a) prove	1	
	b) $(fof)\ x = f(3x-2)$ $= 3(3x-2) - 2$ $= 9x - 6 - 2$ $= 9x - 8$	1	
	c) Let $g = \frac{x+2}{3}$	1	
	$(fog)\ x = x$ $(gof)\ x = x$		
	$f^{-1}(x) = \frac{x+2}{3}$	1	4
10.	a) $\underset{x\to c^+}{\text{Lt}} f(x) = \underset{x\to c^-}{\text{Lt}} f(x) = f(c)$		
	$\sin c = \cos c$	1	
	$\Rightarrow c = \frac{\pi}{4}$	1	
	b) Left derivative at $\frac{\pi}{4}$ = -sin $\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$		
	Right derivative at $\frac{\pi}{4}$ = cos $\frac{\pi}{4} = \frac{1}{\sqrt{2}}$	1	
	Left derivative ≠ right derivative	1	4
	So $f(x)$ is not differentiable at $\frac{\pi}{4}$ [For illustrating the same with the help of graphs of sin x and cos x, give full score]		
11.	a) $\frac{dx}{d\theta} = 2\cos\theta$	1	
	$\frac{dy}{d\theta} = -3\sin\theta$	1	
	$\frac{dy}{dx} = \frac{-3}{2}\tan\theta$	1	
	b) cos | x | [cos x is an even function so it treats x and -x in the same way]	1	4

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12.		Evaluate	4	
13.	a)	$\frac{dy}{dx} = 3x^2 - 10$	1	
		$\left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 2$		
		$3x^2 - 10 = 2$		
		$3x^2 = 12$		
		$x = \pm 2$	1	
		$x = 2 \Rightarrow y = -4$		
		$x = -2 \Rightarrow y = 20$		
		Points are (2, –4) and (–2, 20)	1	
	b)	No. (2, –4) and (–2, 20) do not satisfy the equation $y = 2x + 1$	1	4
14.	a)	$\overrightarrow{AB} = 2i + j + 2k$	1	
	b)	$\lvert \overrightarrow{AB} \rvert = \sqrt{4+1+4} = 3$	1	
		direction cosines of $\overrightarrow{AB} = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$	1	
	c)	angle $\alpha = \cos^{-1}\left(\frac{2}{3}\right)$	1	4
15.	a)	$\cos\theta = \left\lvert \frac{b_1 . b_2}{\lvert b_1\rvert \lvert b_2\rvert} \right\rvert$	1	
		$= \frac{1.0 + 2.2 + 0 \times -1}{\sqrt{1+4} . \sqrt{1+4}}$		
		$= \frac{4}{5}$	1	
		$\theta = \cos^{-1}\left(\frac{4}{5}\right)$		
	b)	perpendicular vector = $b_1 \times b_2$		
		$\begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 0 & 2 & -1 \end{vmatrix}$		
		$= i(-2) - j(-1) + k(2)$	1	
		$= -2i + j + 2k$		
	c)	Equation of line, $(i + 2j - k) + \lambda(-2i + j + 2k)$	1	4
16.	a)	$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-1}{3}$	1	
	b)	$\bar{r} = (1 + 2k) + \lambda(i + 2j + 3k)$	1	
	c)	(2, 2, 5) and (0, –2, –1)	2	4
		(by giving $\lambda = +a$ and $-a$, $(a \in R)$ we can get different points); for example $\lambda = 1$ and - 1		
		[give full score for any correct answer]		

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17.	Equation: $x^2 + (y-a)^2 = a^2$	1	
	Expanding and differentiate		
	$2x + 2y \frac{dy}{dx} = 2a \frac{dy}{dx}$	1	
	$a = \dfrac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}$	1	
	Substituting and simplifying		
	$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$	1	4
18.	a) $f(x) = x^3 + 3x^2 - 9x + 4$		
	$f'(x) = 3x^2 + 6x - 9$	1	
	$f'(x) = 0 \Rightarrow x = 1$ or -3	1	
	In $(-\infty, -3)$ and $(1, \infty)$ function is increasing and in $(-3, 1)$ function is decreasing	1	
	b) $f''(x) = 6x + 6$	1	
	$f''(1) = 12 > 0$	½	
	$f''(-3) = -12 < 0$	½	
	Max at $x = -3$ and Min at $x = 1$		
	c) Answer (1)	1	6
19.	a) Base area $\overline{OA} \times \overline{OB}$	1	
	$= \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & -2 & 4 \end{vmatrix}$		
	$= i(14) - j(1) + k(-4)$	1	
	$= 14i - j - 4k$		
	$\overline{OA} \times \overline{OB} = \sqrt{196+1+16} = \sqrt{213}$	1	
	b) Volume of paralleloppied		
	$= [\overline{a}, \overline{b}, \overline{c}]$	1	
	$= \overline{OC}\left(\overline{OA} \times \overline{OB}\right)$		
	$= (2i + 2j + k) . (14i - j - 4k)$		
	$= 28 - 3 - 4 = 21$ units	1	
	c) height $= \frac{\text{volume}}{\text{base area}} = \frac{21}{\sqrt{213}}$ units	1	6

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20.	a) $\vec{r} = 2i + j + \lambda(\vec{i} + \vec{j} - \vec{k})$	1	
	b) $\vec{r} = 2i + j + \lambda(\vec{i} + \vec{j} - \vec{k})$		
	$\vec{r} = \vec{i} - \vec{j} + 2\vec{k} + \lambda(2\vec{i} + \vec{j} - 3\vec{k})$	1	
	Shortest distance $= \left\lvert \dfrac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 - \vec{b}_2)}{\lvert \vec{b}_1 - \vec{b}_2 \rvert} \right\rvert$		
	$\vec{a}_2 - \vec{a}_1 = -\vec{i} - 2\vec{j} + 2\vec{k}$	1	
	$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & 1 & -3 \end{vmatrix} = i(-3+1) - \vec{j}(-3+2) + \vec{k}(1-2)$	1	
	$= -2\vec{i} + \vec{j} - \vec{k}$	1	
	$\lvert \vec{b}_1 \times \vec{b}_2 \rvert = \sqrt{4+1+1} = \sqrt{6}$	½	
	$(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 2 - 2 - 2 = -2$		
	Shortest distance $= \left\lvert \dfrac{-2}{\sqrt{6}} \right\rvert = \dfrac{2}{\sqrt{6}}$ units	½	6
21.	a) $\int \dfrac{\sec^2 x}{\operatorname{cosec}^2 x} dx = \int \dfrac{\sin^2 x}{\cos^2 x} dx$	1	
	$= \int \tan^2 dx = \tan x - x + c$	1	
	b) $\int \dfrac{dx}{x^2 - 6x + 13} = \int \dfrac{1}{(x-3)^2 + (2)^2} dx$	1	
	$= \dfrac{1}{2} \tan^{-1} \dfrac{(x-3)}{2} + c$	1	
	c) $\int e^x \sin x\, dx = e^x(-\cos x) + \int e^x \cos x\, dx$	1	
	$= -e^x \cos x + I_1$ (say)		
	$I_1 = -e^x \sin x - \int e^x \sin x\, dx$		
	Substituting		
	$\int e^x \sin x\, dx = \dfrac{e^x}{2} (\sin x - \cos x)\, c$	1	6
22.	a) $\int_0^2 x^2 dx = \dfrac{8}{3}$ [evaluate by the method limit of a sum]	4	
	b) $\int_{-2}^{2} x^2 dx = \int_{-2}^{0} x^2 dx + \int_0^2 x^2 dx$	1	
	c) $\int_{-2}^{0} f(x) dx = -5$	1	6

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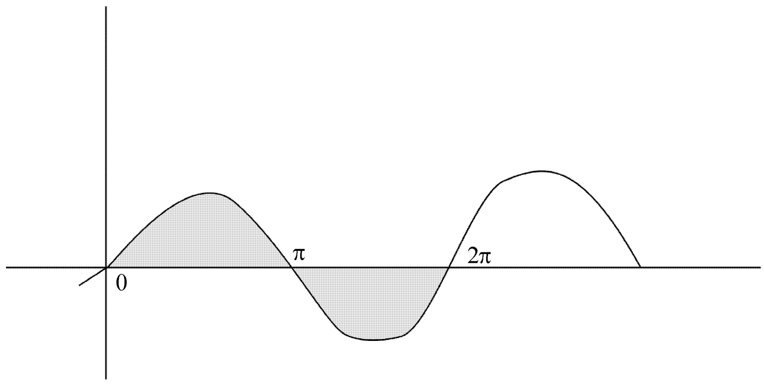
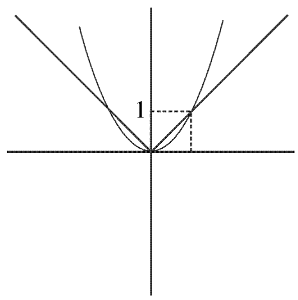
23.	a) put $y = vx$ and prove (or any other method)	1	
	b) solution: $\log\lvert x^2 + xy + y^2 \rvert = 2\sqrt{3}\ \tan^{-1}\left(\frac{x+2y}{\sqrt{3}x}\right) + c$	5	6
24.		1	
	a) $\int_0^{\pi} \sin dx + \left\lvert \int_{\pi}^{2\pi} \sin x\, dx \right\rvert = [\cos x]_o^{\pi} + [\cos x]_{\pi}^{2\pi}$	1	
	$= 2 + 2 = 4$		
	b) $\int_0^1 \lvert x \rvert\, dx - \int_0^1 x^2 dx$ 	1	
	$= \left(\frac{x^2}{2}\right)_0^1 - \left(\frac{x^3}{3}\right)_0^1$	1	
	$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ sq. units	1	
	Required area $= 2 \times \frac{1}{6} = \frac{1}{3}$	1	6