MATHEMATICS (Commerce)

HSE II

Scoring Indicators

Maximum Score: 80

Qn. No.	Answer Key/Value Points	Sub score	Total score
1.	i) $a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$	1	
	$a * (b * c) = a * \frac{bc}{4} = \frac{abc}{16}$		
	$(a * b) * c = \frac{ab}{4} * c = \frac{abc}{16}$	1	
	ii) $a * e = a$ $\quad e * a = a$		
	$\frac{ae}{4} = a$ $\quad \frac{ea}{4} = a$		
	$e = 4$ $\quad e = 4$	1	3
2.	i) $7x - 1 = 5x + 3$		
	$2x = 4$	1	
	$x = 2$		
	ii) $\lvert A \rvert = \begin{vmatrix} 4 & 13 \\ 13 & 5 \end{vmatrix}$	1	
	$= 20 - 169 = -149$	1	3
3.	i) $\frac{dy}{dx} = 2x + 2$; slope $= 2(1) + 2 = 4$	1	
	ii) $y = (1)^2 + 2(1) + 3 = 6$		
	point is $(1, 6)$	1	
	equation of tangent is		
	$y - 6 = 4(x - 1)$		
	$4x - y + 2 = 0$	1	3
4.	$I = \int \frac{1}{\sqrt{x^2+2x+1-1+2}}\, dx$	1	
	$= \int \frac{1}{\sqrt{(x+1)^2 + (1)^2}}$	1	
	$= \log \lvert (x+1) + \sqrt{x^2+2x+2} \rvert + c$	1	3
5.	i) $\int_a^b f(x)dx$	1	
	ii) Area $= 2\int_1^4 \sqrt{y}\, dy$	1	
	$= \frac{4}{3}\left[(y)^{3/2}\right]_1^4$		
	$= \frac{4}{3}[8 - 1]$		
	$= \frac{28}{3}$	1	3

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6.		Equation of family of lines passing through the origin is $y = mx$	1	
		$y = mx$ .... (1)		
		$\frac{dy}{dx} = m$ .... (2)	1	
		Eliminating 'm' using (1) and (2)		
		$y = \frac{dy}{dx} x$	1	3
7.	i)	Vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$	½	
	i)	$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 1 & -1 & 4 \end{vmatrix}$		
		$= 5i - 7j - 3k$	½	
	ii)	Unit vector = $\frac{\vec{a} \times \vec{b}}{\lvert\vec{a} \times \vec{b}\rvert} = \frac{5i - 7j - 3k}{\sqrt{83}}$	1	
		Vector of magniture 5 units = $5\left(\frac{5i - 7j - 3k}{\sqrt{83}}\right)$	1	3
8.				
	i)	$f\left(0, \frac{\pi}{2}\right) \to (0, 1)$	1	
		$f\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \to (-1, 1)$	1	
	ii)	$\sin^{-1} \sin\left(\frac{2\pi}{3}\right) = \sin^{-1} \sin\left(\frac{3\pi - \pi}{3}\right)$	1	
		$= \sin^{-1} \sin\left(\pi - \frac{\pi}{3}\right)$		
		$= \sin^{-1} \sin\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$	1	4
9.		$A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 2 \\ 0 & 1 & 5 \end{bmatrix}$, $A^T = \begin{bmatrix} 3 & 4 & 0 \\ -1 & 3 & 1 \\ 2 & 2 & 5 \end{bmatrix}$	1	
		$\frac{1}{2}(A + A^T) = \frac{1}{2}\begin{bmatrix} 6 & 3 & 2 \\ 3 & 6 & 3 \\ 2 & 3 & 10 \end{bmatrix}$ symmetric matrix	1	
		$\frac{1}{2}(A - A^T) = \frac{1}{2}\begin{bmatrix} 0 & -5 & 2 \\ 5 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$ skew symmetric matrix	1	
		$\frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 2 \\ 0 & 1 & 5 \end{bmatrix} = A$	1	4

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10.	i) 57; since $\lvert A \rvert = \lvert A^T \rvert$	1	
	ii) $\begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 4 \end{vmatrix}$	1	
	$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 4 \end{vmatrix}$	1	
	$= (1+x+x^2)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x & x(1-x) \\ 0 & x(x-1) & (1-x^2) \end{vmatrix}$	1	4
	$= (1+x+x^2)(1-x)^2\begin{vmatrix} 1 & x \\ -x & 1+x \end{vmatrix}$		
	$= (1-x)^2\ (1+x+x^2)(1+x+x^2)$		
	$= (1-x^3)^2$ = RHS		
11.	$\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1} = \lim\limits_{x\to 1}\dfrac{(x-1)(x+1)}{(x-1)}$	1	
	$= \lim\limits_{x\to 1}(x+1) = 2$	1	
	$f(1) = 2$	1	
	$\Rightarrow \lim\limits_{x\to 1}\dfrac{x^2-1}{x-1} = f(1) = 2$		
	$\therefore$ continuous at $x = 1$	1	4
12.	i) $f(x)$ is strictly decreasing in the intervals $[a, c_1)$ and $(c_2, b]$	1	
	$f'(x)$ is strictly increasing in the intervals (c_1, c_2)	1	
	ii) $f'(x) = 4x - 4 = 0$		
	$\Rightarrow x = 1$		
	In $(-\infty, 1) \Rightarrow f'(0) = -4 < 0$	1	
	$\therefore$ strictly decreasing		
	In $(1, \infty) \Rightarrow f'(2) = 4 > 0$	1	4
	$\therefore$ strictly increasing		
13.	i) $\dfrac{x^2}{2} + c$	1	
	ii) $I = \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$	1	
	$= \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$	1	
	$2I = \displaystyle\int_0^{\frac{\pi}{2}} dx = (x)_0^{\frac{\pi}{2}} = \dfrac{\pi}{2}$	½	
	$I = \dfrac{\pi}{4}$	½	4

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14.	i) Area $= \int_0^{\frac{\pi}{2}} \sin x \, dx$	1	
	$= -(\cos x)_0^{\frac{\pi}{2}}$	1	
	$= -(\cos \frac{\pi}{2} - \cos 0) = 1$	1	
	ii) The area bounded by $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{2}$ is same as the area bounded by the curve $y = \sin^{-1} x$ between $y = 0$ and $y = \frac{\pi}{2}$		
	Therefore area = 1	1	4
15.	$\int \frac{y}{y+2} dy = \frac{x+2}{x} dx$	1	
	$\int \frac{y+2}{y+2} dy - 2\int \frac{1}{y+2} dy = \int \left(1+\frac{2}{x}\right) dx$	1	
	$y - 2 \log \lvert y+2 \rvert = x + 2 \log \lvert x \rvert + c$		
	$y - x - c = \log \left(\frac{x^2}{(y+2)^2}\right)$ ________ (1)	1	
	$-1 - 1 - c = \log 1 \Rightarrow c = -2$	½	
	(1) $\Rightarrow y - x + 2 = \log \left(\frac{x^2}{(y+2)^2}\right)$	½	4
16.	i) $\overrightarrow{OP} = \frac{l(\overrightarrow{OB}) + m(\overrightarrow{OA})}{l+m}$	1	
	$= \frac{2(-i+j+k) + 1(i+2j-k)}{2+1}$		
	$= \frac{-3}{2} c + \frac{4}{3} j + \frac{1}{3} k$	1	
	ii) $\overrightarrow{AP} = \left(\frac{-1}{3}i + \frac{4}{3}j + \frac{1}{3}k\right) - (i + 2j - k)$	1	
	$= \frac{-4}{3}i - \frac{2}{3}j + \frac{4}{3}k$	1	4
17.	i) Unit vector $= \frac{1}{2} i + \frac{\sqrt{3}}{2} j$	1	
	ii) a) $\cos^2 45 + \cos^2 60 + \cos^2 \theta = 1$	1	
	$\frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$		
	$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60^{o}$	1	
	b) unit vector $= \frac{1}{\sqrt{2}} i + \frac{1}{2} j + \frac{1}{2} k$	1	4

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18.	$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1	
	$X = A^{-1} B$	1	
	$\lvert A \rvert = -17 \neq 0$	1	
	$A^{-1} = -\frac{1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	2	
	$X = -\frac{1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	1	6
	$\Rightarrow x = 1; y = 2; z = 3$		
19.	i) $A = \pi r^2$		
	$\frac{dA}{dr} = 2\pi r$	1	
	where $r = 10 \Rightarrow \frac{dA}{dr} = 2\pi\,(10) = 20\pi$	1	
	ii) Let $p = 2x + 2y \Rightarrow y = \frac{P}{2} - x$	1	
	Area $= xy \Rightarrow A = x\left(\frac{P}{2} - x\right)$		
	$\Rightarrow A = \frac{P}{2}x - x^2$	1	
	$\frac{dA}{dx} = \frac{P}{2} - 2x$	1	
	For turing points		
	$\frac{P}{2} - 2x = 0$		
	$x = \frac{P}{4}$		
	$\Rightarrow y = \frac{P}{4}$	1	6
	$\therefore$ rectangle is a square		
20.	i) $\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$	1	
	$\Rightarrow x = A\,(x-2) + B\,(x-1)$		
	$\Rightarrow A = -1; B = 2$	1	
	$\therefore \text{I} = \int \frac{-1}{x-1}dx + \int \frac{2}{x-2}dx$		
	$= -\log \lvert x-1 \rvert + 2\log \lvert x-2 \rvert$		
	$= \log \left\lvert \frac{(x-2)^2}{(x-1)} \right\rvert + c$	1	

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	ii) $\int_2^8 \lvert x-5\rvert\, dx = \int_2^5 -x+5\, dx + \int_5^8 x-5\, dx$	1	
	$= \left(\frac{-x^2}{2}+5x\right)_2^5 + \left(\frac{+x^2}{2}-5x\right)_5^8$	1	
	$= \frac{-5^2}{2} + 5\times 5 + \frac{2^2}{2} - 5\times 2 + \frac{8^2}{2} - 5\times 8 - \frac{5^2}{5} + 5\times 5$		
	$= 9$	1	6
21.	i) $x^2 + 3x^2 = 16$		
	$4x^2 = 16 \quad x = \pm 2$	1	
	$\therefore A$ is $(2, 2\sqrt{3})$; B is $(4, 0)$	1	
	ii) Area $= \int_0^2 y\, dx + \int_2^4 \sqrt{16-x^2}\, dx$	1	
	$= \int_0^2 \sqrt{3}\, x\, dx + \int_2^4 \sqrt{16-x^2}\, dx$		
	$= \sqrt{3}\left(\frac{x^2}{2}\right)_0^2 + \left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$	1	
	$= 2\sqrt{3} + \left(8\frac{\pi}{2} - 2\sqrt{3} - 8\frac{\pi}{6}\right)$	1	
	$= \frac{8\pi}{3}$	1	6
22.	i) $y = ae^x + be^{2x}$ ________ (1)		
	$\frac{dy}{dx} = ae^x + 2be^{2x}$ ________ (2)		
	$\frac{d^2y}{dx^2} = ae^x + 4be^{2x}$ ________ (3)	1	
	(3) –3 × (2) + 2 (1)		
	$\Rightarrow \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$	1	
	ii) order: 2; degree: 1	1	
	iii) $P = 1 \quad Q = \sin x$		
	I, $F = e^{\int p dx} = e^{\int dx} = e^x$	1	
	Solution is		
	$y \cdot e^x = \int \sin x . e^x\, dx$ ________ (1)		
	$I = \int \sin x\, e^x\, dx = e^x \sin x - \cos x\, e^x - I$		
	$2I = e^x(\sin x - \cos x) \Rightarrow I = \frac{e^x}{2}(\sin x - \cos x)$	1	
	$(1) \Rightarrow ye^x = \frac{e^x}{2}(\sin x - \cos x) + c$	1	6

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23.	i) $\lvert\bar{a}\rvert = 1; \lvert\bar{b}\rvert = 1; \lvert\bar{a}+\bar{b}\rvert = 1$		
	$\lvert\bar{a}+\bar{b}\rvert^2 = (\bar{a}+\bar{b}).(\bar{a}+\bar{b})$	1	
	$\lvert\bar{a}+\bar{b}\rvert^2 = a^2 + b^2 + 2ab\cos\theta$		
	$1 = 1 + 1 + 2 . \cos\theta$	1	
	$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$	1	
	ii) $2m + 3 + 3 = 0$		
	$2m = -6 \Rightarrow m = -3$	1	
	iii) $\bar{a} = 2i + j - 3k;\ \bar{b} = -3i + 3j - k$		
	$\bar{a}\times\bar{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -3 \\ -3 & 3 & -1 \end{vmatrix} = 8i + 11j + 9k$	1	
	Area $= \lvert\bar{a}\times\bar{b}\rvert = \sqrt{266}$	1	6
24.	i) $\bar{r} = i + j + \lambda\,(2i - j + k)$	2	
	ii) $\bar{b}_1 \times \bar{b}_2 = 3i - j - 7k$	1	
	$\lvert\bar{b}_1 \times \bar{b}_2\rvert = \sqrt{59};\ \bar{a}_2 - \bar{a}_1 = i - k$	1	
	$(\bar{b}_1 \times \bar{b}_2) . (\bar{a}_2 - \bar{a}_1) = 10$	1	
	$d = \left\lvert \frac{(\bar{b}_1 \times \bar{b}_2).(\bar{a}_1 - \bar{a}_2)}{\lvert\bar{b}_1 \times \bar{b}_2\rvert} \right\rvert = \frac{10}{\sqrt{59}}$	1	6