STATISTICS

HSE (II)

Qn. No.	Answer Key/Value points	Score	Total
1	$Y = a(x + k) + 6$ $\qquad k = 2007 - 2005 = 2$	½ + ½	
	$y = 2.9\ (x + 2) + 136.25$		
	$= 2.9\ x + 142.05$	1	2
2	$f(x) = x\,(4x - 3) = 4x^2 - 3x$	½	
	$\int f(x)dx = \int (4x^2 - 3x)dx$	½	
	$= 4\frac{x^3}{3} - 3\frac{x^2}{2} + C$	1	2
3	$n = 20,\ \overline{p} = 0.05$ $\qquad \overline{p} = 0.95$	½	
	$CL = n\overline{p} = 20 \times 0.05 = 1$	½	
	$LCL = n\overline{p} - 3\sqrt{n\overline{p}\overline{q}}$		
	$= 20 \times 0.05 - 3\sqrt{20 \times 0.05 \times 0.95}$	½	2
	$= 1 - 2.924 = -1.924 = 0$		
	$UCL = 1 + 2.924 = 3.924$	½	
4	(i) - (d), (ii) - (a), (iii) - (b), (iv) - (c)	4 x ½	2
5	i) Critical value	1	
	ii) Explanation with diagram	2	3
6	$\mu = 12400,\ \overline{x} = 12345,\ n = 20,\ s = 140$	½	
	To test to : $\mu = 12400$ $\qquad H_{1:}\ \mu \neq 12400$	½	
	$t = \frac{\overline{x} - \mu}{s/\sqrt{n-1}}$	½	
	$t = \frac{\sqrt{19}(12345 - 12400)}{140}$		3
	$= -1.71$		
	$\lvert t \rvert = 1.71$	½	
	$\alpha = 0.05$, table value of t at 19 df = 2.093		
	Accept H_0	½	
	Directors claim is correct at 5% level.	½	
7.	i) $1 - \alpha$	1	
	ii) Moment estimate for population mean $\mu = \overline{x}$		
	$= \frac{\sum x}{n}$	1	
	$= \frac{66 + 65 + 69 + 70 + 69 + 71 + 70 + 63 + 64 + 68}{10}$	½	3
	$= \frac{675}{10} = 67.5$	½	

8	i)	estimate	1	
	ii)	$E(X_1) = \mu$, $E(X_2) = \mu$, $E(X_3) = \mu$,	½	
		Given $E(T_1) = \mu$		
		$E(T_1) = E(kX_1 + 2X_2 + X_3) = \mu$	½	3
		$k\mu + 2\mu + \mu = \mu$		
		$k\mu = -2\mu$		
		$k = -2$	1	
9	i)	V(2X + 9) = 24	1	
	ii)	$E(x) = \int x.f(x)dx = \int_0^1 x\frac{3x^2}{8}dx$	½ + ½	
		$= \frac{3}{8}\int_0^2 x^3dx = \frac{3}{8}\left(\frac{x^4}{4}\right)_0^2$	½	3
		$= \frac{3}{8}\left(\frac{16}{4} - 0\right) = \frac{6}{4} = 1.5$	½	
10	i)	$b_{yx} = \frac{-1}{3}$	1	
	ii)	2x + 6y = 45 2x + 2y = 30 4y = 15		
		$y = \frac{15}{4}$	1	3
		$x = 15 - \frac{15}{4} = \frac{45}{4}$	1	
11	i)	0.42	1	
	ii)	$r = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2} \times \sqrt{n\sum y^2 - \left(\sum y\right)^2}}$	1	
		$= \frac{7 \times 209 - 43 \times 28}{\sqrt{7 \times 391 - (43)^2} \times \sqrt{7 \times 146 - (28)^2}}$	½	3
		= 0.563	½	
12	i)	2.52		
	ii)	$\lambda = 3$	1	
		$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$, x = 0, 1, 2, ...	1	
		P (more than 2 two students participate)		
		$= P(x > 2)$		
		$= 1 - P(x \leq 2)$		
		$= 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$	1	4
		$= 1 - \left[\frac{e^{-3}.3^0}{0} + \frac{e^{-3}.3^1}{1} + \frac{e^{-3}.3^2}{2}\right]$	½	
		= 1- 0.4233 = 0.5767	½	

13	i) 6 ii) X - Monthly sales = μ 8000, σ = 240 a) P (X < 7600) = P $\left(Z < \frac{7600-8000}{240}\right)$ = P (Z< - 1.67) = 0.5 - 0.4515 = 0.0485 No. of firms whose monthly sales less than 7600= 1000 × 0.0485 = 48.5 ≅ 49 firms b. P (7750 < X < 8250) = P (-1.04 < Z <1.04) = 2 × P (0 < Z <1.04) = 2 × 0.3508 = 0.7016 No. of firms whose monthly sales between 7750 and 8250 = 1000 ×0.7016 = 701.6 ≅ 702 firms	1 ½ ½ ½ ½ ½ ½ ½	 4
14	E (X_1) = E (X_2) = E (X_3) = μ V (X_1) = V (X_2) = V (X_3) = σ^2 a) E (T_1) = E ($X_1 + X_2 - X_3$) = $\mu + \mu - \mu$ = μ T_1 is unbiased for μ. E (T_2) = E ($4X_2 - X_2 - 2X_3$) = $4\mu - \mu - 2\mu$ = μ T_2 is unbiased for μ b) V (T_1) = $\sigma^2 + \sigma^2 + \sigma^2 = 3\sigma^2$ V (T_2) = $16\sigma^2 + \sigma^2 + 4\sigma^2 = 21\sigma^2$ V(T_1) < V (T_2) T_1 is more efficient than T_2.	 ½ ½ ½ ½ ½ ½ ½ ½	 4
15	$n_1 = 100$, $\bar{x} = 200$, $S_1 = 20$, $n_2 = 150$, $\bar{x}_2 = 205$, $S_2 = 22$ To test H_0: $\mu_1 = \mu_2$ against H_0: $\mu_1 \neq \mu_2$ Z = $\frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ = $\frac{200-205}{\sqrt{\frac{20^2}{100} + \frac{22^2}{100}}}$ = -1.86	 1 2	 4

<table>
<tr><td></td><td>$|z|$ cal= 1.86
$z_{\alpha/2}$ = 2.576
$|z|_{cal} < z_{\alpha/2}$ ∴ We accept H_0
ie, Mean production of samples are equal.</td><td>

1</td><td></td></tr>
<tr><td>16</td><td>
<table>
<tr><th>Source</th><th>df</th><th>SS</th><th>MSS</th><th>F</th></tr>
<tr><td>Between</td><td>3</td><td><u>48</u></td><td>16</td><td><u>1.23</u></td></tr>
<tr><td>Within</td><td><u>16</u></td><td><u>208</u></td><td><u>13</u></td><td></td></tr>
<tr><td>total</td><td>19</td><td><u>256</u></td><td></td><td></td></tr>
</table>
Degrees of freedom (3, 16)
H_0 : the means are equal
H_1 : the means are not equal
$F_{1,16}(0.05)$ = 3.24
F_{cal} = 1.23 F_{α} = 3.24
$F_{cal} < F_{\alpha}$
Accept H_0. ie, the means are equal</td><td>4 x ½=2

½
½

½
½</td><td>4</td></tr>
<tr><td>17</td><td>G = 72
C.F = $\frac{G^2}{N} = \frac{72^2}{12} = 432$
SST = 462 - 432 = 30
SSB = $\frac{20^2}{4} + \frac{24^2}{4} + \frac{28^2}{4} - 432$
= 440 - 432 = 8
SSW= SST - SSB = 30 - 8 = 22
<table>
<tr><th>Source</th><th>df</th><th>SS</th><th>MSS</th><th>F</th></tr>
<tr><td>Between</td><td>2</td><td>8</td><td>4</td><td>1.63</td></tr>
<tr><td>Within</td><td>9</td><td>22</td><td>2.44</td><td></td></tr>
<tr><td>to fal</td><td>11</td><td>30</td><td></td><td></td></tr>
</table>
F_{cal} = 1.63
$F_{2,9}(0.01)$ = 8.02
$F_{cal} < F_{\alpha}$ Accept H_0
There is no significant difference of the sales between territories.</td><td>
½
½

½
½
1
½

½</td><td>4</td></tr>
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</table>

18

i) $\sum x = 0,\ \sum y = 180,\ \sum x^2 = 10,\ \sum xy = 69$

Normal equations are

$\sum y = a\sum x + nb$ 1

$\sum xy = a\sum x^2 + b\sum x$

180 = 5b

b = $\frac{180}{5}$ = 36 ½ 4

69 = 10a

a = $\frac{69}{10}$ = 6.9 ½

Trend equation is y = 6.9x + 36

y = 6.9 (t - 2013) + 36 1

ii) For the year 2017

y = 6.9 (2017 - 2013) + 36 1

= 63.6

19

Sample	Sample value				Sample mean	Range
1	10	28	14	13	16.25	18
2	24	37	36	25	30.5	13
3	16	35	32	37	30	21
4	53	51	36	27	41.74	26
5	34	16	37	26	28.75	99
					146.75	99

2

$\bar{\bar{x}} = \frac{\sum(\bar{x})}{n} = \frac{146.75}{5} = 29.35$ ½

$\bar{R} = \frac{\sum R}{n} = \frac{99}{5} = 19.8$ ½ 6

LCL = $\bar{\bar{x}} - A_2\bar{R}$ ½

= 29.35 - 0.729 x 19.8= 29.35 - 14.4342 ½

= 14.91

UCL = $\bar{\bar{x}} + A_2\bar{R} = 43.78$

Proper construction of control chart and interpretation.
(Process is under control) 2

<table>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>20</td><td>
N = 5, n = 2
No. of samples = $NC_n = 5C_2 = 10$
<table>
<tr><th>Sample No</th><th>Sample Value</th><th>Sample Mean</th></tr>
<tr><td>1</td><td>8, 9</td><td>8.5</td></tr>
<tr><td>2</td><td>8, 13</td><td>10.5</td></tr>
<tr><td>3</td><td>8, 15</td><td>11.5</td></tr>
<tr><td>4</td><td>8, 16</td><td>12</td></tr>
<tr><td>5</td><td>9, 13</td><td>11</td></tr>
<tr><td>6</td><td>9, 15</td><td>12</td></tr>
<tr><td>7</td><td>9, 16</td><td>12.5</td></tr>
<tr><td>8</td><td>13, 15</td><td>14</td></tr>
<tr><td>9</td><td>13, 16</td><td>14.5</td></tr>
<tr><td>10</td><td>15, 16</td><td>$\underline{15.5}$</td></tr>
<tr><td></td><td></td><td>122</td></tr>
</table>
i) Mean of population = $\frac{8+9+13+15+16}{5} = 12.2$
ii) Standard deviation of population
= $\sqrt{(8-12.2)^2+(9-12.2)^2+(3-12.2)^2+(15-12.2)^2+(16-12.2)^2}$
= $\sqrt{\frac{50.56}{5}} = \sqrt{10.11}$
= 3.17
iii) Mean of sample mean = $\frac{122}{10} = 12.2$
iv) Standard error of sample mean = $\sqrt{\frac{N-n}{N-1}}\ \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{5-2}{5-1}}\ \frac{3.17}{\sqrt{2}}$
$= \frac{0.866\times3.17}{1.41} = 1.94$
</td><td>2
1
1
1
1</td><td>6</td></tr>
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</table>

21	Let X be the income of the group, μ and σ are the mean and SD of the income.		
	Given X follows N (μ , σ)		
	Given P (X > 6680) = 0.95	½	
	$P\left(Z > \frac{6680-\mu}{\sigma} \right) = 0.95$	½	
	P (Z > a) = 0.95		
	P (0 < Z < a) = 0.45	1	
	a = -1.645		
	Also P (X > 8320) = 0.05	½	
	$P\left(Z > \frac{8320-\mu}{\sigma} \right) = 0.05$		
	P (Z > b) = 0.05	½	
	P (0 < Z < b) = 0.45		
	b = 1.645	1	6
	$\frac{6680-\mu}{\sigma} = -1.645$		
	μ -1.645 σ = 6680 .... (1)	½	
	$\frac{8320-\mu}{\sigma}$ = 1.645		
	μ + 1.645 σ = 8320 ....(2)	½	
	Solving (1) and (2)		
	2 μ = 15000		
	μ = 7500	½	
	7500 - 1.645 σ = 6680		
	σ = 498	½	
	Note : a can be -1.64 or -1.65		
	b can be 1.64 or 1.65		