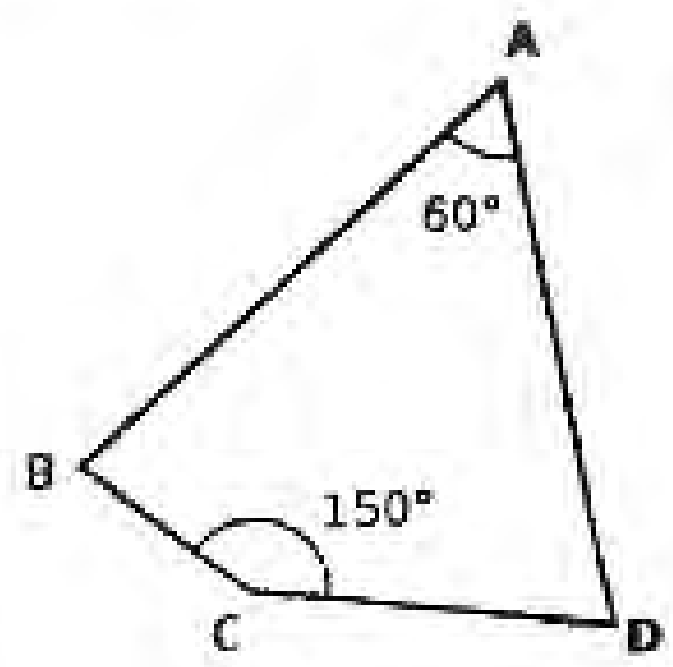
Chapter 2. Circles

Question 1.

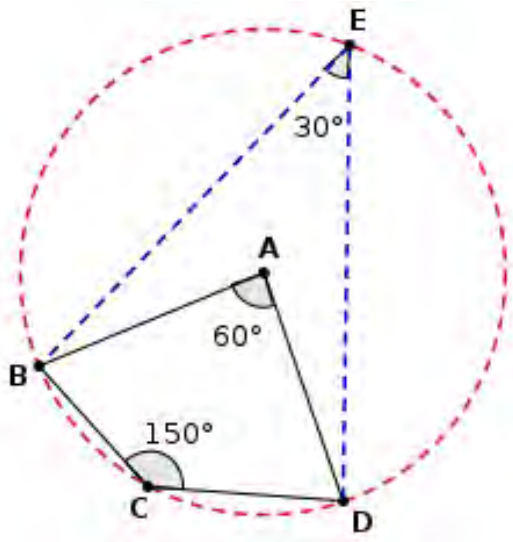


In the figure AB = AD, $\angle A = 60^0$, $\angle C = 150^0$. Show that the circle , centred at A and radius AB.

a) passes through the point D

b) passes through the point C.

Answer:-



a) Given AB = AD (radii) and $\angle A = 60^0$.

Joint BD , then Δ ABD is an isosceles triangle.

ie., $\angle A = \angle D = 60^0$.

Draw a circle at center A and radius AD will passes through D .

b) Accoding to arc, opposite are and center angle theorems,

arc BED = $\frac{1}{2} \cdot \angle A = \frac{1}{2} \times 60 = 30^0$

From the figure BCDE be a cyclic quadrilateral and their opposite angle are supplementary.

ie., $\angle C + \angle = 180^0$.

Hence the circle passes through at C .

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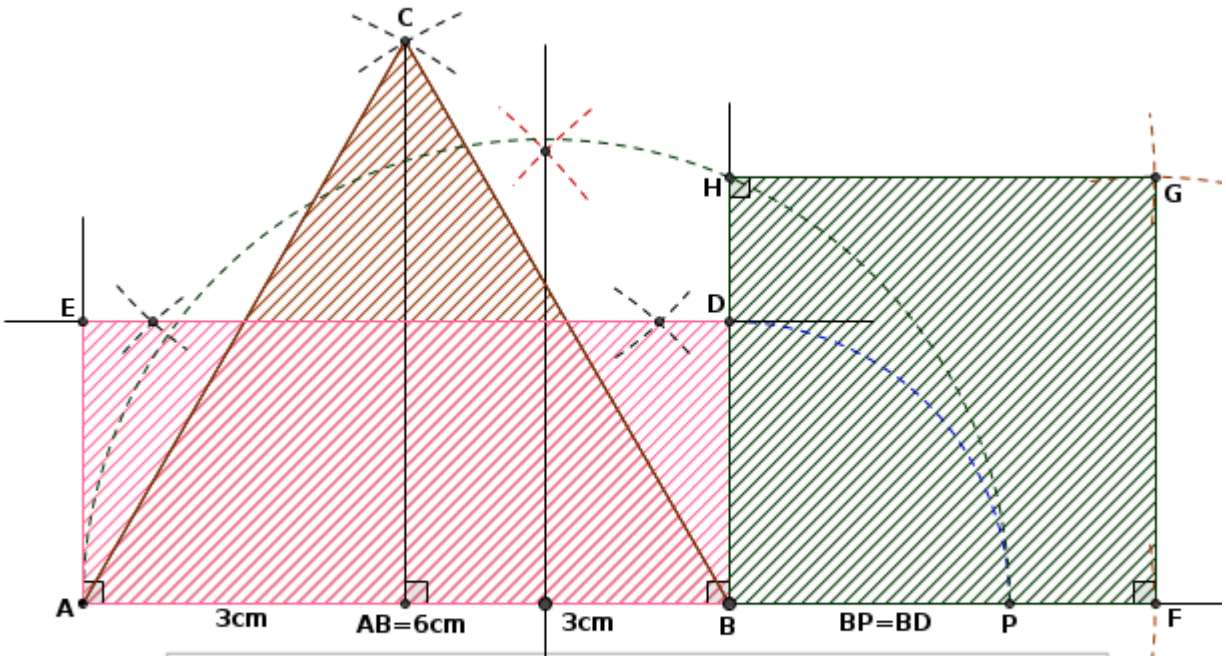
Question 2.

Answer:-



Area of the equilateral triangle ABC = Area of ABE

Area of square ≈ 15.6 cm^2 .

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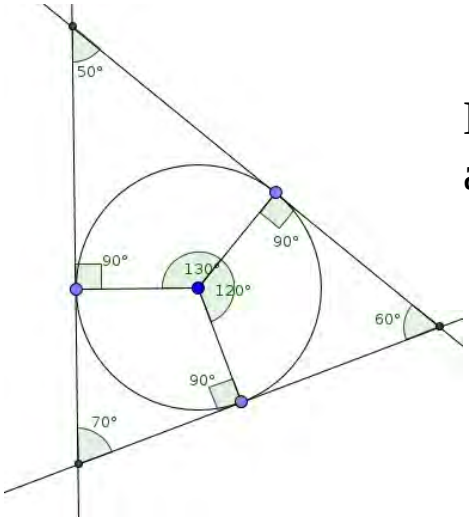
Question 3

Answer:-



Draw a triangle of angles 50^0 , 60^0 , 70^0 and radius of whose in-circle is 3cm.

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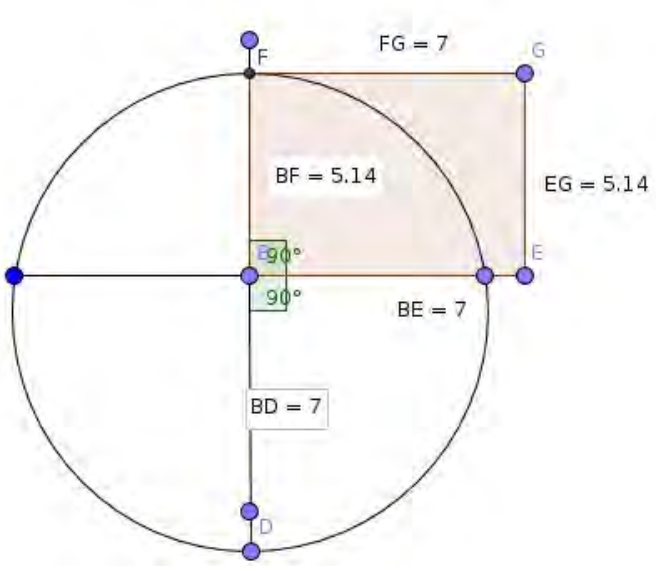
Question 4.

Draw a rectangle of one side 7 cm and area 36 cm^2 .

Answer:-



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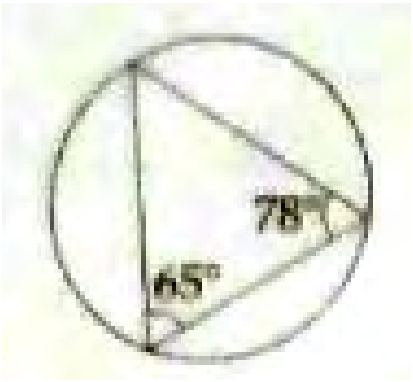
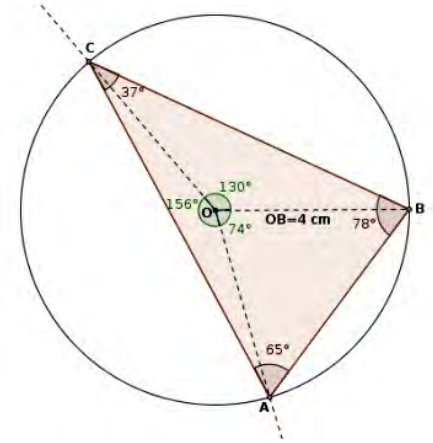
Question 5.



Draw a circle with radius 4 cm. Draw a triangle with two of its angles 65^0 and 78^0 and all vertices on the circle.

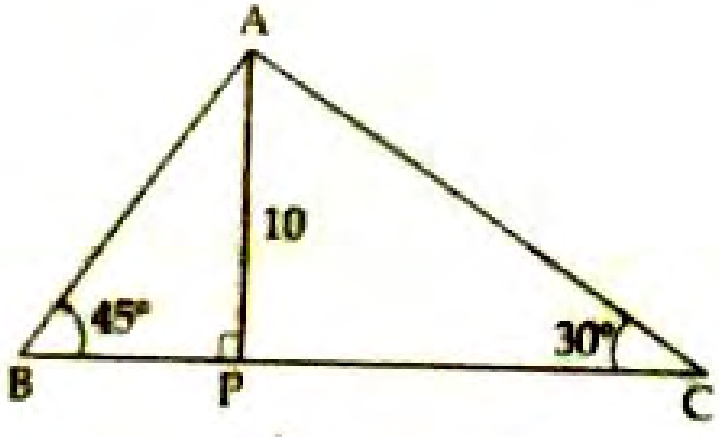
Answer:-



Construction :-

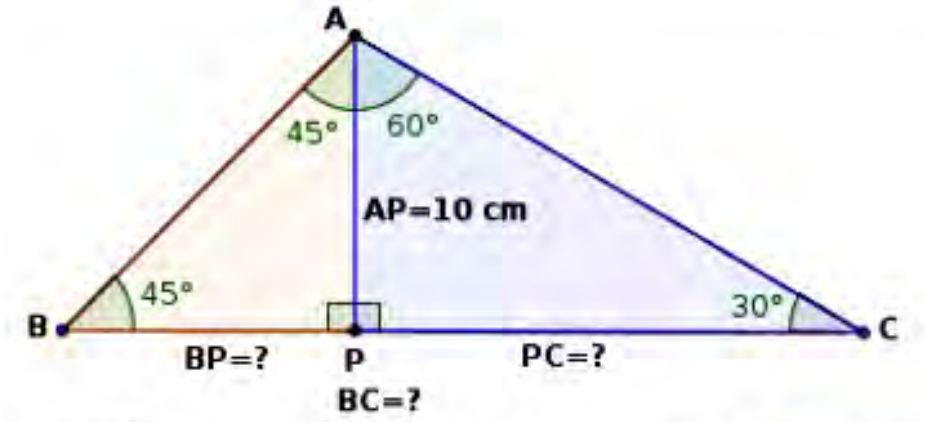
Draw a circle radius OB = 4cm. Make an angle $\angle BOC = 130^0$ ($2\angle B = 2 \times 65 = 130$) and $\angle AOC = 156^0$ and marks B and A respectively. Joint AB, BC, CA is the required construction.

Question 6.



In triangle ABC , the length of AP is 10cm. What is the length of BP ? What is the length of PC? Calculate the length of BC?

Answer:-



In the given figure we can understand that triangle APB be an lossless triangle. So the base angle $\angle B = \angle A = 45^0$ each.

$\therefore$ AP = BP = 10cm (Given AP = 10cm)

ie., BP = 10cm

In the figure right angled triangle APL , $\angle C = 30^0$.

ie., $\tan 30^0 = \frac{AP}{PC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{PC}$ $\quad (\tan 30^0 = \frac{1}{\sqrt{3}}$)

Hence PC = $10\sqrt{3}$

From the figure, BC = BP + PC

$= 10 + 10\sqrt{3}$ cm.

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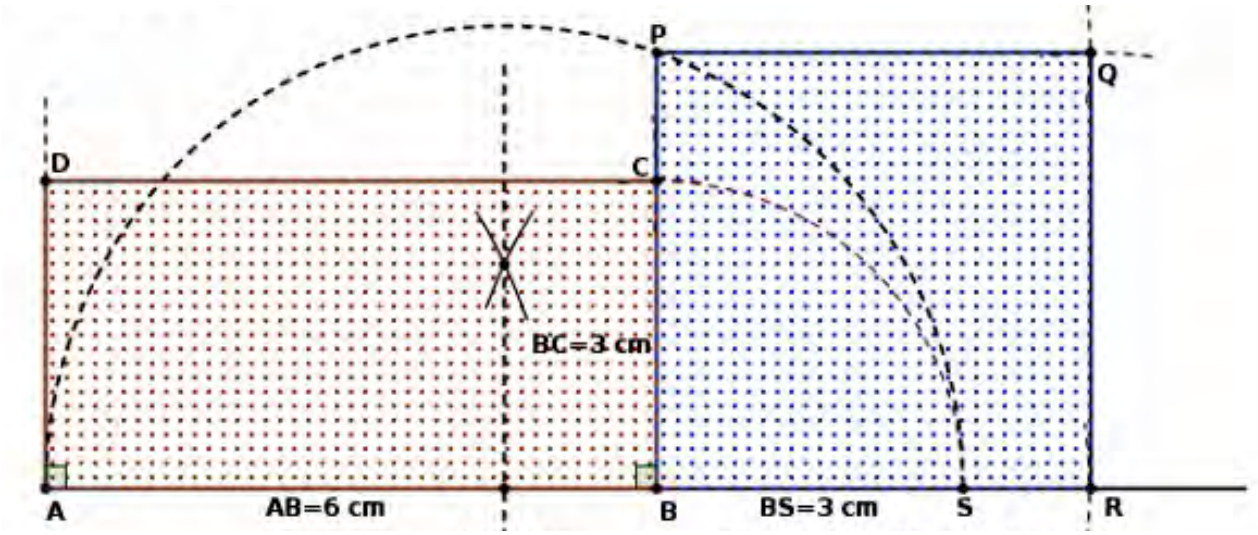
Question 7.

Draw rectangle of area $18cm^2$. Draw a square of the same area.

Answer:-



Given area = 18 cm^2 .

So sides be 6cm and 3cm.

Construction

Draw a rectangle ABCD length be 6cm and breadth be 3cm.To extant the line AB and mark S as BS= 3cm. Draw a perpendicular bisector of AS and mark E on AS. Draw a semi circle , center be E and radius is AE. BC extant and meet the semi circle at P. Construct a square Sides are BP = PQ = QR = BR. BPQRB be the required square.

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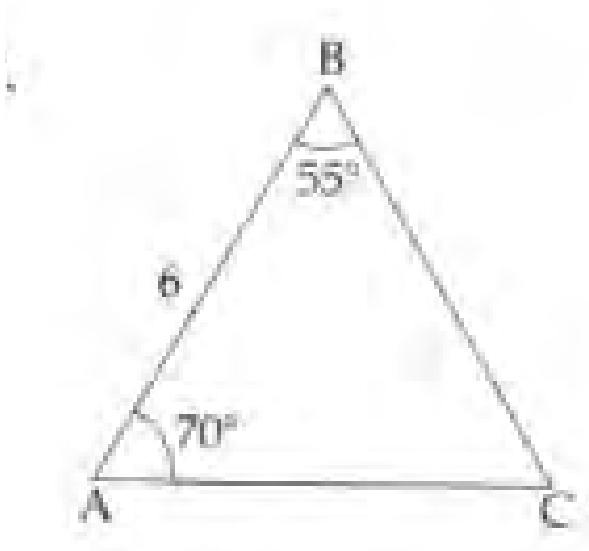
Question 8.



In triangle ABC, length of AB = 6 cm, $\angle A = 70^0$, $\angle B = 55^0$.

(a) Find $\angle C$

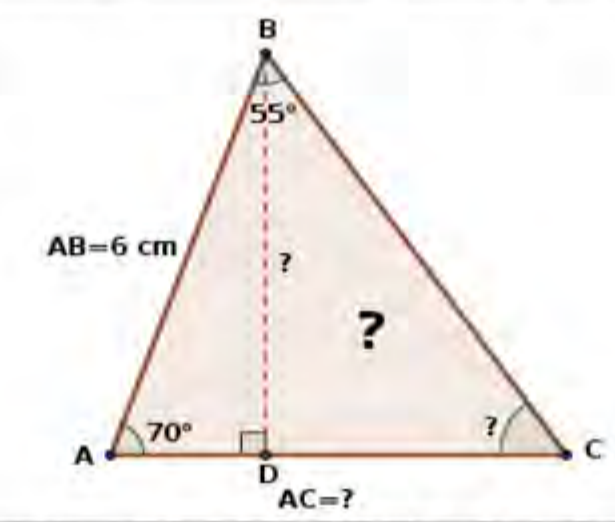
(b) Find AC

(c) Find the area of triangle ABC

($\sin 70^0 = 0.93$)

Answer:-

Given AB = 6cm. , $\angle A = 70^0$, $\angle B = 55^0$



a) $\angle C = 180 - (70 + 55) = 180 - 125 = 55^0$.

b) ΔABC is an isosceles triangle

$\therefore AB = AC = 6$ cm

c) Area of the triangle = $\frac{1}{2} \times AC \times AB \times \sin 70^0$.

(ΔADB is right angled triangle. Sin 70 is the included angle of sides AB and AC.)

$\therefore$ Area of the triangle ¿ $\frac{1}{2} \times 6 \times 6 \times 0.93 = 16.74$ cm^2 .

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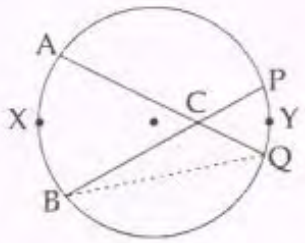
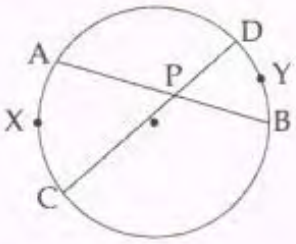
Question 9



In the circle shown, the chords AQ and BP passes through C.

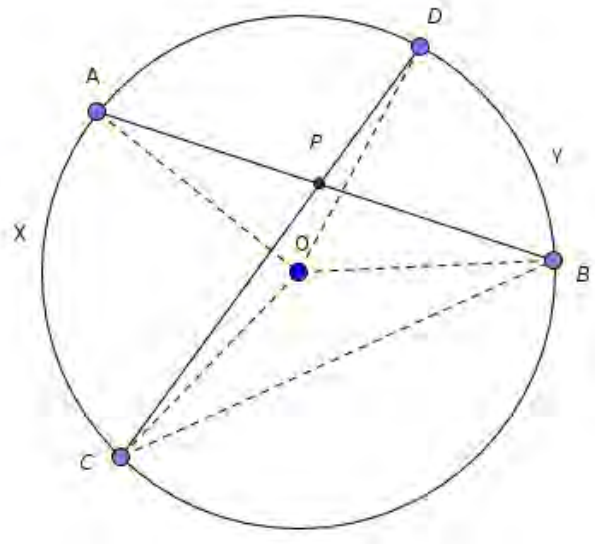
(a) The central angle of arc AXB is 100° calculate ∠Q. The central angle of arc PYQ is 60°. Find all angles of the triangle BQC.

(b)



In the picture, prove that ∠APC is half the sum of the central angle of arc AXC and arc BYD.

Answer:-



Center angle of the arc AXB =100^0

$\angle Q = \frac{1}{2} \times \angle AOB$

$= \frac{1}{2} \times 100 = 50^0$. (Center angle relation, arc and opposite arc relation)

Center angle of arc QYP = 60^0 .

ie., $\angle B = \frac{60}{2} = 30^0$

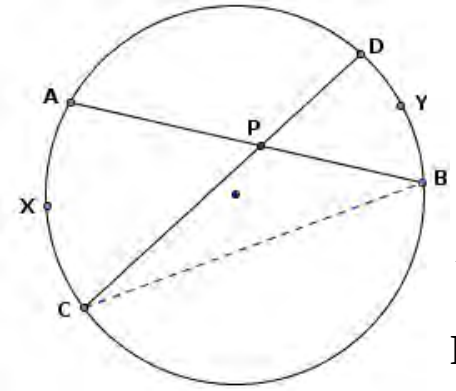
$\angle BCQ = 180 - (\angle Q + \angle B)$

$= 180 - (50 + 30)$

$= 180 - 80 = 100^0$.

The angles of the ΔBQC , $\angle B = 30^0$, $\angle Q = 50^0$, $\angle C = 100^0$.

b)



Join C and B .

In ΔPBC , the exterior angle

∠APC = ∠B + ∠C (sum of the interior opposite angles)

∠APC = ½ [Center angle of the arc AXC + Center angle of the arc BYD]

Henc the ∠APC is the half sum of the center angles of the arc AXC and BYD.

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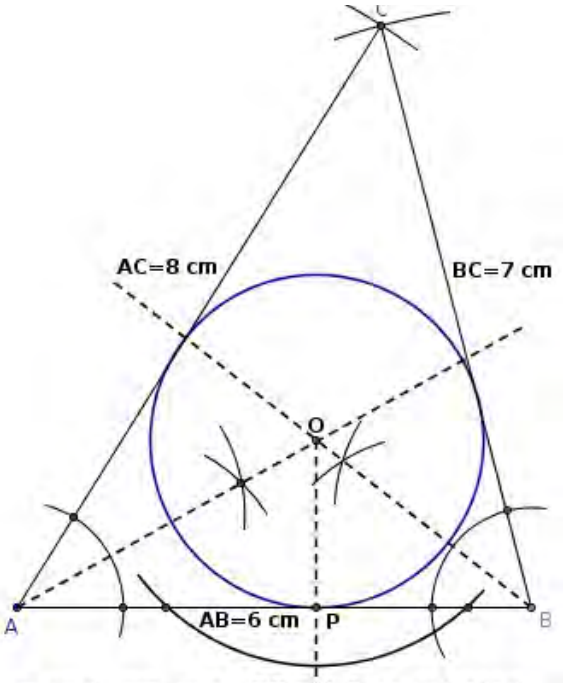
Question. 10.

Draw a triangle of sides 6 centimetre, 7 centimetre and 8 centimetre. Draw a circle which touches all sides of the triangle and measure its radius.

Answer:-



Construction:

Construct the triangle in the given measurement.

Draw any two angle bisector and intersect it at a point O . Draw the circle OP as the radius .

Draw a triangle of sides 6 centimetre, 7 centimetre and 8 centimetre. Draw a circle which touches all sides of the triangle and measure its radius.

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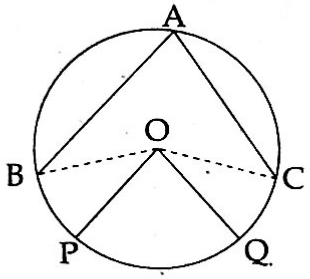
Question. 11.

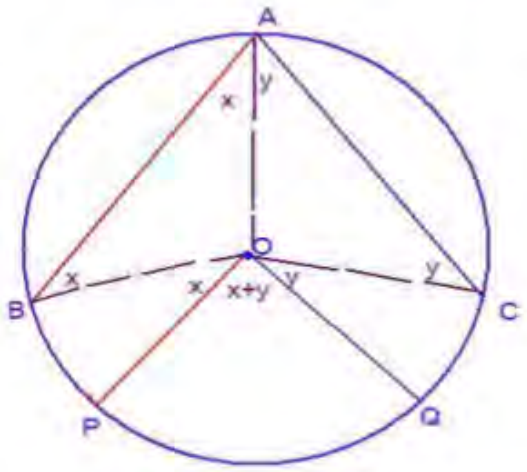
In the figure below, AB and AC are chords of the circle and OP and OQ are radii parallel to them :



(a) What is the relation between ∠BOC and ∠POQ ?

(b) What is the relation between the small arc joining B and C and the small arc joining P and Q ?

Answer:-



Let <ABO = x and <ACO = y then <BAC = (x+y)

By central angle theorem if <BAC = (x+y) we have <BOC = 2(x+y)

AB//OP hence <BOP =x (alternate angles) similarly <COQ= y

Here <BOC = 2(x+y) hence <POQ=(x+y)

So we can seen that length of arc BC double the length of arc PQ

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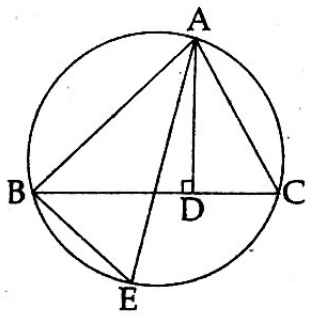
Question. 12.

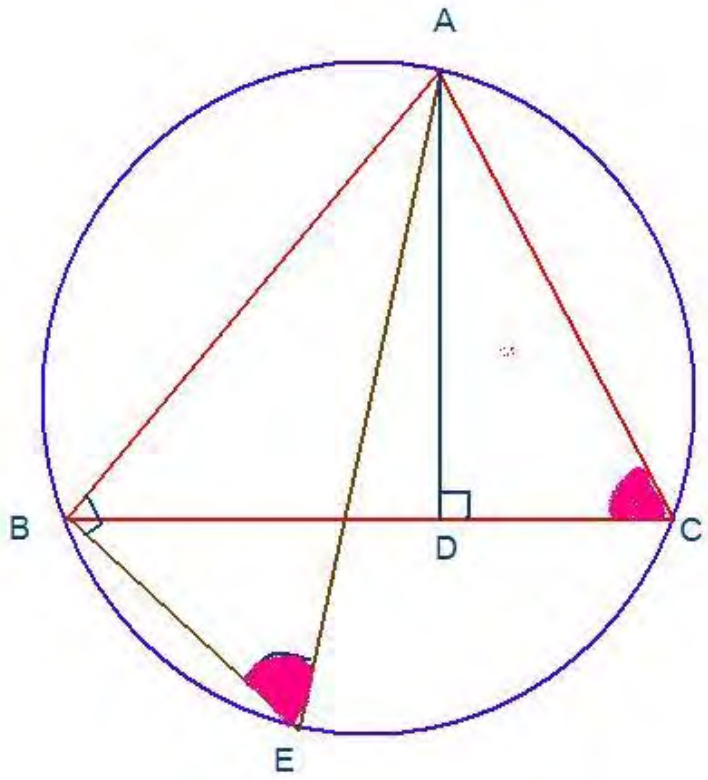
In the figure below, AD is the perpendicular from A to BC and AE is the diameter through A of the circumcircle of ΔABC :



(a) Prove that ΔADC and ΔABE are similar.

(b) Prove that the area of ΔABC is $\frac{AB \times BC \times CA}{2AE}$.

Answer:-



a) Consider Triangle ABE and ADC

Since AE is the diameter $<ABE = 90^0$

$<ADC = 900$ (given)

$<AEB = ACB = x$ (angles in same part of the circle)

Hence $<ABE = 90^0 = <ADC$ and $<AEB = ACD = x$

Therefore Triangle ABE and ADC are similar

b) Since ABE and ADC are similar

$$\frac{AD}{AB} = \frac{AC}{AE}$$

From this we have $AD = \frac{ABxAC}{AE}$

Area of triangle ABC = ½ x BC x AD

$$= ½ \text{ x } \frac{ABxBCxCA}{AE}$$

Hence the proof

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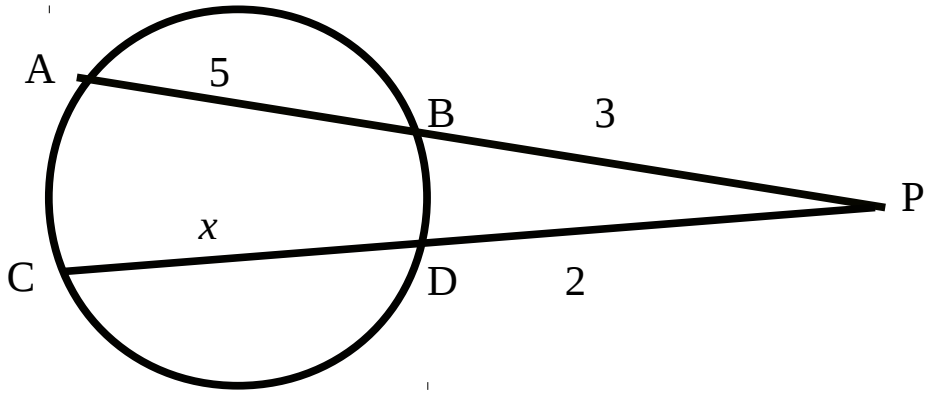
Question. 13.

Two chords AB and CD of a circle intersect each other at P outside in the circle. If AB = 5cm , BP = 3cm and PD = 2cm, find CD.



Answer:-

Since two chords AB and CD of a circle are intersecting At P , when produced

$\therefore$ PA . PB = PC . PD

$\Rightarrow$ (AB + PB) . (PB) = (PD + DC) PD

$\Rightarrow$ (5 + 3) (3) = (2 + x) 2

$\Rightarrow$ 24 = (2 + x) 2

$\Rightarrow$ 12 = (2 + x)

x = 10 cm.

Thus , CD = 10cm.

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Question 14.

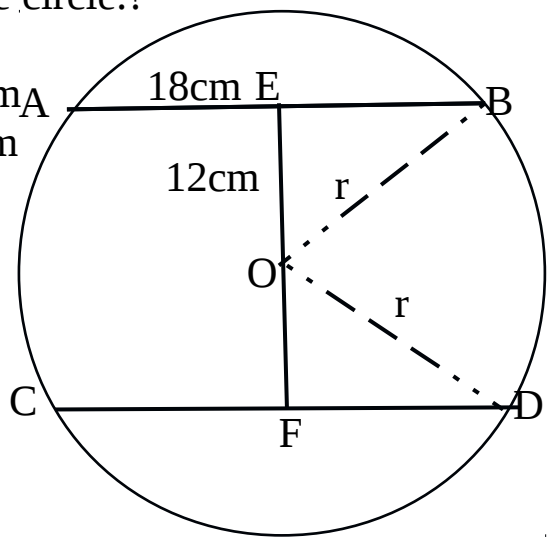
A chord of a circle which is at a distance of 12cm from the center is 18cm long. What will be the length of another chord of the same circle which is at a distance of 9cm from the center of the circle.?

Answer:-

Given, AB length of first chord (c) = 18cm

OEdistance (d) = 12cm

ER (c/2) = $\frac{18}{2}$ = 9cm ; r = ?



ΔOEB is a right angled triangle

By Pythagoras,

$r = \sqrt{9^2 + 12^2} = \sqrt{81 + 144}$

$= \sqrt{225} = 15$ cm.

BO (r) = 15cm.

From ΔOFD , is a right angled triangle

By Pythagoras,

FD (c/2) = $\sqrt{15^2 - 9^2} = \sqrt{225 - 81} = \sqrt{144} = 12$ cm

Hence the length of the other chord = $12 \times 2 = 24$ cm.

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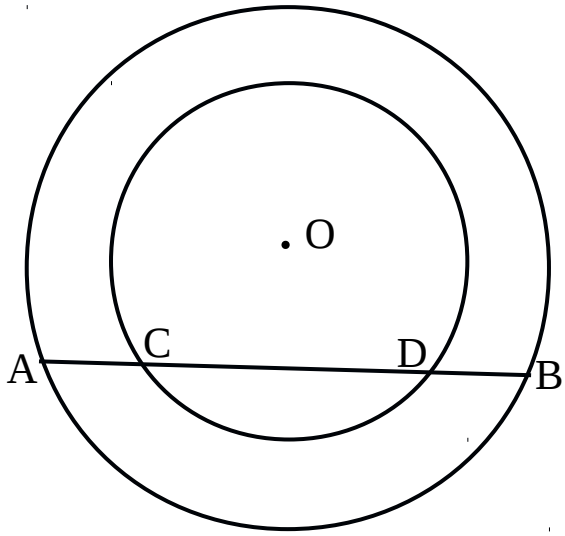
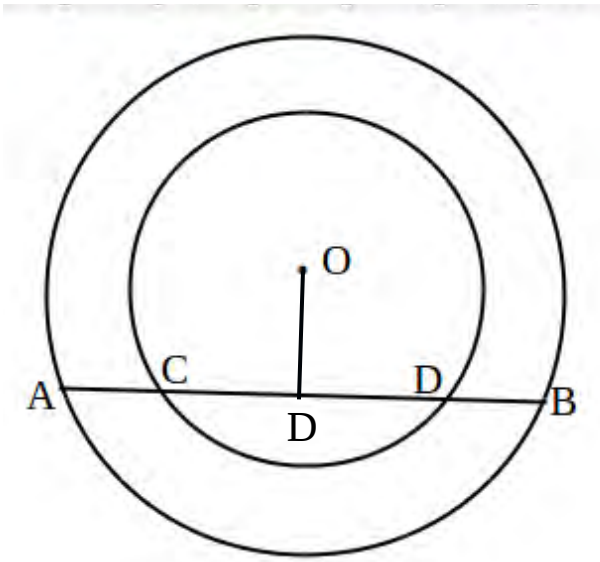
Question 15.

In figure, there are two concentric circles with center O . Chord AB of the larger circle intersects the smaller circle at C and D. Prove that AC = BD.



Answer:-



Joint OM such thst OM perpendicular to AB.

Then AM = MB and also CM = MD.

(Because perpendicular from the center of the circle to a chord with bisect the chord.)

AM = MB

$\Rightarrow$ AC + CM = MD+ DB

$\Rightarrow$ AC = MD + DB – CM

$\Rightarrow$ AC = BD + MD – CM

$\therefore$ AC = BC (CM = MD)

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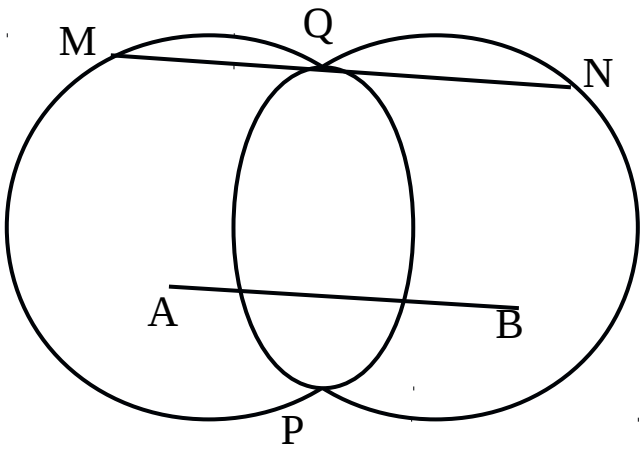
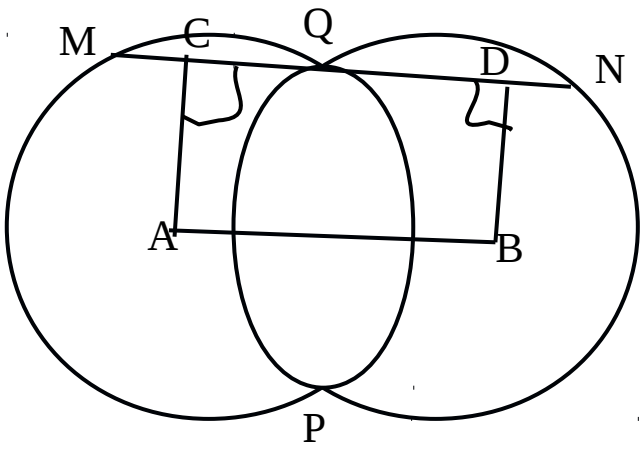
Question 16.

In the figure , A and B are centers of two circles with intersect at O and P. If MN is parallel to AB, prove that MN = $2 \times$ AB .



Answer:-



Join AC and BD perpendicular to MN

Then MC = CO , OD = DN …….. (1)

Also, AB = CD ……...(2)

MN = MC + CO + OD + DN

= CO + CO + OD (from (1)

= 2CO + 2OD

= 2 (CO + OD) = 2 .CD

= 2.AB (from (2)

∴ MN = 2.AB.

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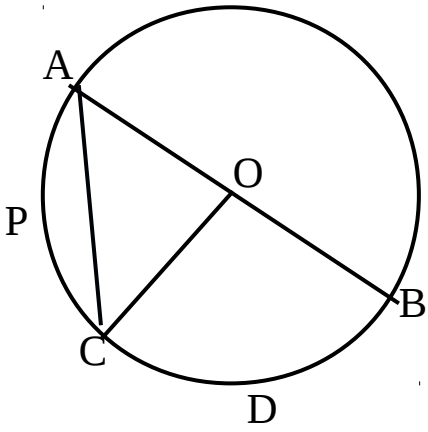
Question 17.

O is the cener of the circle , if the center angle of arc BDC = 60^0 . Find the measure of ∠OAC ?.



Answer:-

Given the center angle of the arc BDC = 60^0 .

$\therefore$ $\angle BOC = 60^0$.

Since O is the center, arc ACB is a semi circle .

Center angle of are APC = 180 – 60 = 120.

$\therefore$ $\angle AOC = 120^0$.

(or, $\therefore$ $\angle BOC = 60^0$, $\angle AOC$ and $\angle BOC$ are linear pair

$\angle AOC = 180 - 60 = 120^0$).

OA and OC are radii of the same circle , thus OA = OC.

$\therefore$ OAC is an isosceles triangle .

$\angle OAC = \angle OCA$.

$\angle OAC + \angle OCA = 180 - 120 = 60^0$. (sum of the measure of the three angles of a triangle is 180^0)

$\angle OAC = 30^0$.

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Question 18.

Find the value of *x* if the measure of the center angle of an arc is (x + 60) and the measure of the center angle of its complementary arc is (2x + 30).

Answer: -

Given condition ,

(x + 60) + (2x + 30) = 360.

3x + 90 = 360

3x = 360 – 90 = 270

x = 270 / 3 = 90^0 .

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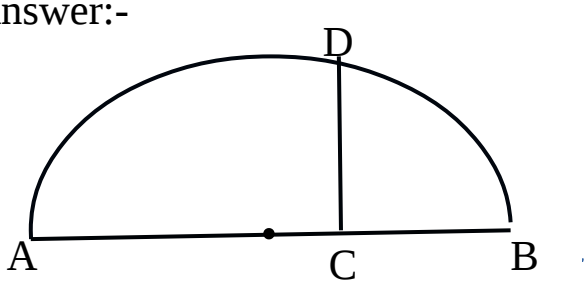
Question 19.

D is a point on the semicircle . C is a point on the diameter AB of the semicircle 2cm away from the center, and if CD perpendicular AB and radius of the semicircle is 6cm, what is the length of CD?.

Answer:-



Radius of the semicircle = 6cm

$\therefore$ In the figure AO = 6cm

(Radius= 6cm)

OC = 2cm

$\therefore$ AC = 8cm

$AC \times CB = CD^2$. ; $8 \times 4 = CD^2$; $CD = \sqrt{32}$

Hence, CD = $4\sqrt{2}$ cm.

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The End of the Chapter Circles .

Prepared by Dr.V.S.RaveendraNath

Mobile 9447206495 ,

7012030930 .