

MATHEMATICS^{EM}

1. ARITHMETIC SEQUENCES

First term: f

Common difference: d

n^{th} term:

$$X_n = f + (n - 1)d$$

$$X_n = dn + (f - d)$$

$$X_n = an + b$$

(a – common difference,
 $a + b$ – first term)

Common difference

$$d = \frac{X_m - X_n}{m - n}$$

Sum of sequence

$$S_n = \frac{n}{2}[2f + (n - 1)d]$$

$$S_n = \frac{n}{2}[x_1 + X_n]$$

$$S_n = \frac{n}{2}[f + 1]$$

$$S_n = \frac{d}{2}n^2 + n\left[f - \frac{d}{2}\right]$$

1. If the 17th term of an arithmetic progression is 23 and the 23rd term is 17, then what is its first term and what is the common difference?
2. If the 5th term of an arithmetic progression is 32 and the 11th term is 74, then what is its first term and what is the common difference?
3. Seats are arranged in rows in an auditorium. Each row has 5 more seats than the row in front. The 15th row from the front has 82 seats. Which row has 97 seats?
4. Is 0 a term of the arithmetic progression - 123, -120, -117...?
5. Is 203 a term of the arithmetic progression 7, 12, 17?
6. Find the 17th of the A.P 4, 9, 14
7. The sum of three consecutive terms in an A.P is 6 and their product is -120. Find the three consecutive numbers?

8. Find the sum of the arithmetic series 5+11+17+...+95
9. In an arithmetic series, the sum of first 14 terms is -203 and the sum of the next 11 terms is -572. Find the arithmetic series?
10. Find the sum of all the multiples of 9 between 100 and 300?
11. If the sum of the first 25 terms of arithmetic progression is 350, what is its 8th term?
12. The 10th and 18th terms of an A.P are 41 and 73 respectively. Find the 27th term?

2. CIRCLES

↻ Arc and alternate arc: the portion of a circle b/w two points on it is called an arc. The remaining portion of the circle is called alternate arc.

↻ The sum of the central angles of an arc and its alternate arc in a circle is 360°

↻ The central angle of an arc of a circle to be double to angle made by the arc at any point on the alternate arc.

↻ All angles in the same segments of a circle are equal.

↻ Angles in the alternate segments of a circle are supplementary.

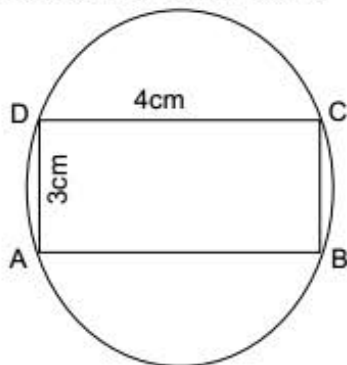
Quadrilateral and Circle

A quadrilateral with all its vertices on a circle is called a cyclic quadrilateral. In this, opposite angles are supplementary. Rectangle, square, isosceles, trapeziums are all cyclic quadrilaterals.

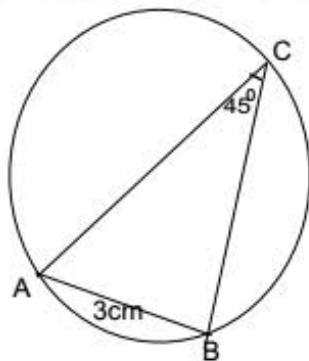
Intersection of chords

The intersection may be within or outside the circle Chords AB&CD intersect at P. then $AP \times PB = CP \times PD$

1. Draw a triangle of sides 4, 5, 6cm and draw a square of the same area?
2. AB and CD are two chords of a circle which intersect each other internally at P.
 - a) If $CP=4\text{cm}$, $AP=8\text{cm}$, $PB=2\text{cm}$, then find PD.
 - b) If $AP=12\text{cm}$, $AB=15\text{cm}$, $CP=PD$, then find CD?
3. In $\triangle ABC$, we have $\angle A=60^\circ$ and $\angle B=70^\circ$. Is the vertex C inside or outside the circle with diameter AB?
4. What is the area of the given circle?



5. Prove that if a pair of opposite angles of a quadrilateral is right, then a circle can be drawn through all four of its vertices?
6. What is the radius of the given circle?



7. Draw a rectangle of sides 5cm and 4cm and draw a square of the same area?

3. MATHEMATICS OF CHANCES

Mathematics of chance is a measure of uncertainty.

Probability of an event is the ratio of number of favourable outcome to the total number of outcome.

Geometrical Probability: The probability of putting dot without looking to be within the given one part of the circle is $\frac{1}{6}$.

For another picture, the area of triangle is $\frac{1}{2}$ of the area of rectangle. So the probability of the dot falling within the triangle is also $\frac{1}{2}$.

[In geometry the probability is the ratio b/w areas.

Pairs: For pairs, calculate the total number of all possible pairs of numbers and then find how many of them have required probabilities. The second number divided by the first gives the probability.

1. A die is rolled. Find the probability of getting
 - a) The number 4
 - b) An even number
 - c) A prime factor of 6
2. A box contains 4 white balls and 6 black balls and another one, 3 white and 5 black. We can choose one box and take a ball. If we want a black ball, which box is the better choice?
3. An integer is chosen from the first twenty natural numbers. What is the probability that it is a prime number?
4. A die is thrown twice. Find the probability of getting a total of 9?
5. You ask someone to say a natural number less than 10.
 - a) What is the probability that the number is a prime?
 - b) What if the number asked is to be less than 100?
6. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is thrice that of drawing a red ball, then find the number of blue balls in the bag?

7. A ticket is drawn from a bag containing 100 tickets. The tickets are numbered from one to hundred. What is the probability of getting a ticket with a number divisible by 10?
8. 20 lots numbered 1 to 20 are taken in a box. Another box contains lots of prime numbers less than 20. One lot each is drawn from each box.
 - a) What is the probability of getting two prime numbers?
 - b) What is the probability of getting two even numbers?

4. SECOND DEGREE EQUATIONS

☞ A quadratic equation in the variable x is of the form

$ax^2+bx+c = 0$, where a, b, c are real numbers and $a \neq 0$

☞ The solutions of second degree equation $ax^2+bx+c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

☞ If $b^2-4ac = 0$, the equation has only one solution.

☞ If $b^2-4ac < 0$, the equation has no solution.

☞ If $b^2-4ac > 0$, the equation has two different solutions

1. The sum of a number and 5 multiplied by 4 gives 36. What is the number?
2. The common difference of an arithmetic progression is 2. If we add 1 to the product of the first and second terms we get 9. Find the progression?
3. When each side of a square is reduced by 5m, the area became $225m^2$. What was the length of a side of the original square?
4. Sharmin is 4 years older than Noshin. If 4 is added to the product of their ages is 169. What are their ages?
5. The square of a natural number and four times the next natural number is added and the result is 36. What is the number?

6. The length of a rectangle is 2m more than its width and its area is $15m^2$. Find its dimensions?
7. IF from the square of a number six times the number is subtracted, we get 40. What is the number?
8. If the product of a number with 8 less than the number is 65. What is the number?
9. A wire 60cm long is bent into a right angled triangle of hypotenuse 26cm. Compute the lengths of the other two sides of the triangle?
10. When the length of each side of a square was increased by 5cm, its perimeter became 36cm. What was the length of a side of the original square?
11. The sum of the ages of two children is 30 and the product of their ages is 216. How old are they?
12. If the difference of two negative numbers is 5 and their product is 336. What are the numbers?

5. TRIGONOMETRY

Trigonometry means, the science which deals with the measurements of triangles.

Trigonometrical ratios

In triangle ABC

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}}$$

Pythagoras Relation

1. If the angles of a triangle are $60^\circ, 60^\circ, 60^\circ$; then its sides will be in the ratio 1:1:1.
2. If the angles of a triangle are $45^\circ, 45^\circ, 90^\circ$; then its sides will be in the ratio 1:1: $\sqrt{2}$.

3. If the angles of a triangle are 30° , 60° , 90° ; then its sides will be in the ratio 1: $\sqrt{3}$:2.

☞ Using trigonometry the angles of a triangle determine the ratio of its sides.

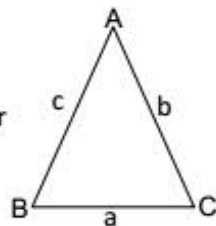
Angles	sin	cos	tan
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	∞

Chord and radius

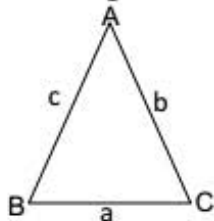
Length of chord = $2r\sin(c/2)$, where r = radius of the circle, c = central angle of chord

Law of sine

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$



The length of the sides of a triangle



$$a = 2r\sin A$$

$$b = 2r\sin B$$

$$c = 2r\sin C$$

Where r is the radius. If any angle is greater than a right angle, the sine of its supplementary angles should be taken.

- Two adjacent sides of a parallelogram are 5cm and 3cm long and the angle between them is 60° . Find the length of their diagonals?
- In a circle a chord of length 8cm makes an angle of 100° at the centre. What is the radius of the circle?
- One angle of a right angled triangle is 30° and its hypotenuse is 4cm. What is its area?

- The hypotenuse of a right angled triangle is 6cm and one of its angles is 40° . What are the lengths of its other two sides?
- One angle of a triangle is 70° and the length of its opposite side is 4cm. What is its circumradius?
- Two sides of a triangle are 7cm and 6cm long and the angle between them is 140° . What is the length of the third side?
- A ladder leans against a wall with its foot 2 metres away from the wall and it makes a 40° angle with the ground. How high is the top of the ladder from the ground?
- The length of the shadow of a tree is 18m, when the sun is at an elevation of 40° . Find the height of the tree?
- A man 1.75m tall, standing at the foot of a tower sees the top of a hill 40m away at an elevation of 60° . On climbing to the top of the tower, he sees the top of the hill at an elevation of 50° . Compute the heights of the hill and the tower?
- The area of a parallelogram is 30cm^2 . One of its sides is 6cm and one of its angles is 60° . What is the length of its other sides?

6. COORDINATES

The two mutually perpendicular number lines intersecting each other at their zeros are called rectangular axes or co-ordinate axes.

☞ Horizontal line is called x-axis and vertical line is called y-axis.

☞ Once axes are drawn, we can denote the position of any point using a pair of numbers. These numbers are called the coordinates of the points.

☞ In (x,y) , x is the x-coordinate and y is the y coordinate.

☞ Point of intersection of the x-axis and y-axis is the origin $(0,0)$

Distance b/w points

1. Two points $(x_1,0)$, $(x_2,0)$ are on the x-axis, distance = $|x_1-x_2|=|x_2-x_1|$
2. Two points $(0,y_1)$, $(0,y_2)$ are on the y-axis, distance = $|y_1-y_2|=|y_2-y_1|$
3. The distance b/w two points with coordinates $(x_1, y_2), (x_2, y_1)$ is

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

4. The distance b/w the point with coordinates (x,y) and the origin is

$$\sqrt{x^2+y^2}$$

1. The centre of a circle is $(5,4)$ and its radius is 8cm. Find which of the points $(5,5)$, $(-4,7)$, $(-3,-2)$, $(4,5)$, $(3,4)$ lie within the circle?
2. The coordinates of vertices of a triangle are $(2,4)$, $(4,5)$, $(3,7)$. Find the coordinates of the centre of its circumcircle and the circumradius?
3. Find the distance from x-axis $(4,4)$, $(4,3)$, $(5,7)$, $(4,-3)$?
4. Draw x and y axis then mark the following points.
 $(4,3)$, $(-4,7)$, $(-4,-5)$, $(5,8)$, $(6, -5)$
5. With the axes of coordinates chosen along two adjacent sides of a rectangle, two opposite vertices have coordinates $(0,0)$ and $(4,3)$. What are the coordinates of the other two vertices?

7. TANGENTS

A line drawn through any point of a circle, which touches only one point of a circle is called tangent of the circle.

It may be

- a) Tangent just touches at one point on a circle and is perpendicular to the radius.

One and only one tangent can be drawn through a point on the circumference of the circle. The angle b/w a tangent and a chord through a point of contact is equal to an angle in the alternate segment.

- b) Two tangents are drawn from an external point then; they subtend equal angles at the centre.

Opposite angles of a cyclic quadrilateral are supplementary.

Sum of opposite sides of a quadrilateral formed by joining the tangents on four points of a circle are equal.

Incircle: circle can be drawn by touching the three sides of a triangle. The centre of this circle will be the point of intersection of the angle bisectors.

Radius of an inscribed circle in the triangle is equal to the area divided by half of its perimeter.

Many circles can be drawn by touching the both sides of the cone. The centres of all these circles will be on its angle bisector.

1. Draw a circle of radius 3cm and mark a point P which is 7cm away from its centre. Draw the tangents from P to the circle?
2. Draw a triangle of sides 4cm, 5cm, 6cm and draw its incircle?
3. The length of the tangent from a point to a circle of radius 12cm is 16cm. How far away is this point from the centre of the circle?

4. The length of the tangent from a point 61cm away from the centre of a circle is 60cm. What is the radius of the circle?
5. Prove that in any circle, the tangents at two points make equal angles with the chord joining the points of contact?
6. Draw an equilateral triangle of sides 6cm and draw its incircle and circumcircle?
7. Prove that in an equilateral triangle, the circumcentre and incentre are the same. What is the ratio of the circumradius and inradius?
8. Draw a circle of radius 5cm and draw regular pentagon with all its sides touching the circle?

8. SOLIDS

Square Pyramid

Base edge = a Slant edge = e,

Slant height = l Height = h

$$1. l^2 = \left(\frac{a}{2}\right)^2 + h^2$$

$$2. e^2 = \left(\frac{a}{2}\right)^2 + l^2$$

$$3. e^2 = h^2 + \frac{a^2}{2}$$

Base area = a^2

Base perimeter = $4a$

Slant surface area = $2al$

Surface area = $a^2 + 2al$

Volume = $\frac{1}{3} a^2 h$

Cone

Radius = r, Slant height = l

Height = h Then $l^2 = r^2 + h^2$

Base perimeter = $2\pi r$

Base area = πr^2

Curved surface area = $\pi r l$

Surface area = $\pi r (r + l)$

Volume = $\frac{1}{3} \pi r^2 h$

Sphere

Only one face
(curved surface)

Surface area = $4\pi r^2$

Volume = $\frac{4}{3} \pi r^3$

Hemisphere

Two faces

Circular face area = πr^2

Curved surface area = $2\pi r^2$

Surface area = $3\pi r^2$

Volume = $\frac{2}{3} \pi r^3$

The ratio b/w surface area of two spheres with radius r_1 and $r_2 = r_1^2 : r_2^2$

The ratio of volume is $r_1^3 : r_2^3$

1. What is the volume of a square pyramid of base edge 10cm and slant height 15cm?
2. If the curved surface area of a solid hemisphere is 2772cm^2 . Find its total surface area?
3. What is the curved surface area of a cone of base diameter 30cm and height 40cm?
4. The base radius of a cylindrical block of wood is 15cm and its height is 40cm. What is the volume of the largest cone that can be carved out from this?
5. The ratio of the base-radii of two cones is 3:5 and their heights are in the ratio 2:3. What is the ratio of their volumes?
6. Two cones have the same volume and their base-radii are in the ratio 4:5. What is the ratio of their heights?
7. What is the ratio of the base-radius and slant height of a cone made by rolling up a semicircle?
8. What is the central angle of the sector needed to make a cone of base-radius 10cm and slant height 25cm?

9. In two square pyramids of the same volume, the base edge of one is half the base edge of the other. How many times the height of the pyramid with larger base is the height of the other?
10. If the curved surface area of a solid sphere is 98.56cm^2 . Find the radius of the sphere?

9. GEOMETRY AND ALGEBRA

Distance b/w two points

☞ If (x_1, y_1) , (x_2, y_2) are the points.

$$\text{Distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Distance of point (x, y) from the origin $(0, 0)$ is $\sqrt{x^2 + y^2}$

☞ Let p be the length of a part of a line, and w be the length of whole line, then

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{p}{w}$$

☞ Co-ordinates of midpoint of a line, which joins the points (x_1, y_1) and (x_2, y_2)

$$\text{is } \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right]$$

Slope of a line

☞ The difference of y co-ordinates divided by the difference of the x co-ordinates is called slope of line.

☞ If (x_1, y_1) and (x_2, y_2) are the two given points then slope = $\frac{y_2 - y_1}{x_2 - x_1}$

1. The centre of a circle is $(3, 4)$ and it passes through the point $(2, 5)$. What is its radius?
2. What is the slope of the line given by the equation $2x - 3y + 4 = 0$?
3. Prove that the points $(2, 3)$, $(7, 5)$, $(9, 8)$, $(4, 6)$ are the vertices of a parallelogram?
4. A circle of radius 3 is drawn with centre at $(-2, 1)$. Find out whether the point $(4, 1)$ lies on the circle, within the circle or outside the circle?

5. Prove that we get a right angled triangle by joining the points $(2, 1)$, $(3, 4)$, $(-3, 6)$?
6. The vertices of a triangle are the points $(1, 2)$, $(2, 3)$, $(3, 1)$. Find the centre and radius of its circumcircle?
7. What is the point at which the line joining $(3, 1)$ and $(2, -1)$ meets the x -axis? And the y -axis?
8. Prove that the line joining $(3, 5)$ and $(1, 7)$ passes through the point $(5, 3)$?

10. POLYNOMIALS

In mathematics, a polynomial is an expression consisting of variables and coefficients which only employs the operations of addition, subtraction, multiplication and non-negative integer exponents.

☞ Let polynomial $p(x)$ is the product of the two polynomial $q(x)$, $r(x)$, then we can say that $q(x)$, $r(x)$ are the factors of $p(x)$.

☞ Remainder Theorem: When a polynomial $p(x)$ is divided by $(x - a)$, if the remainder is r and the quotient is $q(x)$, then we write $p(x) = (x - a)q(x) + r$ and $p(a) = r$

☞ Factor Theorem: $(x - a)$ is a factor of the polynomial $p(x)$ if and only if $p(a) = 0$

☞ If $(ax + b)$ is a factor of the polynomial $p(x)$, then $p\left(\frac{-b}{a}\right) = 0$

☞ Polynomial $p(x)$ can be written as the product of one degree polynomial $(x - a_1)(x - a_2) \dots (x - a_n)$ then

a_1, a_2, \dots, a_n are the solutions of the equation $p(x) = 0$

☞ Third degree polynomial has three factors.

1. Is the polynomial $x-2$ is a factor of the polynomial $x^4-x^3-x^2-x-2$?
2. Is $x+3$ is a factor of $2x^2+3x-5$?
3. Is the polynomial $2x-3$ is a factor of the polynomial $2x^2-x-3$?
4. What number added to $3x^3-2x^2+5x$ gives a polynomial for which $x-1$ is a factor?
5. Write x^2+6x+8 as a product of two first degree polynomials?
6. When x^3-3x^2-5x+n is divided by $(x+3)$ remainder is 5. Find n ?
7. If $(x+1)$ and $(x-1)$ are factors of x^3+2x^2+px+q . Find p and q ?
8. Prove that $(x+2)$ is a factor of $x^3+3x^2-4x-12$?

11. STATISTICS

Frequency table and mean

Arithmetic mean is the sum divided by the numbers. In order to find the sum, we represent each class by its mid values (mean) and multiply it by corresponding frequencies.

$$\text{Mean} = \frac{\text{sum of terms}}{\text{Number of terms}}$$

Frequency table and median

When numbers are arranged in an ascending order, then the middle term is the median.

Median of a data having odd number of observation (ie, n is odd) =

$$\text{value of } \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

Media of a data having even number of observation (ie, n is even) =

$$\frac{\left(\frac{n}{2}\right)^{\text{th}} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ observations}}{2}$$

For frequencies distribution

$$\text{Median} = L + \frac{i}{f} \left(\frac{n}{2} - c\right) \text{ for } n \text{ is even}$$

$$L + \frac{i}{f} \left(\frac{n+1}{2} - c\right) \text{ for } n \text{ is odd}$$

Where L = lower limit of median class

i = class size

f = frequency of median class

c = cumulative frequency just before the median class

1. Find the mean of the following distribution?

x	10	30	50	70	89
f	7	8	10	15	10

2. If the mean of the following data is 5, find p ?

x	2	3	5	p	9
f	9	4	6	3	8

3. The table below classifies the number of days of a month according to the amount of rainfall received in a certain locality. Find the mean?

Rainfall	Number of days
54	3
56	5
58	6
55	3
50	2
47	4
44	5
41	2

4. The table below classifies the members of a committee according to their ages:

Age	Members
25-30	6
30-35	14
35-40	16
40-45	22
45-50	5
50-55	4
55-60	3

Calculate the mean age of the members?

5. Calculate the median for the given data?

x	10	20	30	40	50	60
f	6	15	29	41	60	70

6. The table below shows the number of employees of an office, classified according to the income-tax paid by them:

Income Tax	Employees
1000-2000	8
2000-3000	10
3000-4000	15
4000-5000	18
5000-6000	22
6000-7000	8
7000-8000	6
8000-9000	3

Compute the median of income-tax.