



JAIN COLLEGE

463/465, 18th Main Road, SS Royal, 80 Feet Road, Rajarajeshwari Nagar,

Bangalore - 560 098

Timings Allowed: 3hr

II PUC MOCK I

SUBJECT: Mathematics

Timings Allowed: 3 Hrs.

Total Marks: 100

PART-A

I Answer all the questions.

1X10=10

1. Define an injective function.
2. Find the principle value of $\cos^{-1}\left(-\frac{1}{2}\right)$
3. What is symmetric matrix.
4. Find the value of $\begin{vmatrix} 200 & 201 \\ 202 & 203 \end{vmatrix}$ without actual expansion.
5. If $2x + 3y = \cos x$ find $\frac{dy}{dx}$
6. Evaluate $\int x \sqrt{x} dx$
7. Find the scalar and vector components of the vector with initial points (2,1) and terminal point (5,8).
8. Find the direction cosines of the line segment joining the points (3,5,-4), (-1,1,2)
9. Define Feasible solution.
10. A fair die is rolled $E = \{1,5\}$ $F = \{1,4\}$ Find $P\left(\frac{E}{F}\right)$.

PART-B

II Answer any TEN questions.

10X2=20

11. Find gof and fog if $f(x) = 8x^2$ and $g(x) = x^{\frac{1}{3}}$
12. Prove that $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $x \in [-1,1]$
13. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$
14. Find the area of the triangle with vertices (1,0), (6,0), (4,3) using determinant.
15. Differentiate $x^{\sin x}$, $x > 0$ w.r.t x
16. If $x = at^2$ $y = 2at$ find $\frac{dy}{dx}$
17. If radius of sphere is measured as 7m with error 0.02m. Find the approximate error in calculating its volume.
18. Evaluate $\int e^x(\sin x + \cos x) dx$
19. Evaluate $\int_0^{\frac{\pi}{2}} \cos^2 x dx$
20. Form the differential equation representing the family of curves $y = mx$ where m is arbitrary constant.
21. Show that the vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.
22. Find the area of the parallelogram whose adjacent sides are determined by
$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}, \quad \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$
23. Find the distance of the plane $3x - 3y + 4z - 6 = 0$ from the origin.
24. Two cards are drawn at random without replacement from a deck of 52 cards. Find the probability that both cards are red

Part-C

III Answer any TEN questions.

10X3=30

25. Show that the relation R in the set of all integers Z defined by

$R\{(a, b)/2 \text{ divides } a - b\}$ is an equivalence relation.

26. Prove that $\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $|x| < \frac{1}{\sqrt{3}}$

27. Using the elementary transformation find the inverse of $\begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix}$

28. If $x^y + y^x = a^b$, Find $\frac{dy}{dx}$

29. Find the equation of the tangent and normal to the circle $x^2 + y^2 = 1$ at the point (x_0, y_0)

30. Verify Rolles theorem for $f(x) = x^2 + 2x - 8$, $x \in [-4, 2]$

31. Evaluate $\int_0^\pi \log(1 + \sin x) dx$

32. Express $\int_0^1 e^{2x} dx$ as limit of a sum

33. Find the area of the region bounded by the curve $x^2 = 4y$ and its latus rectum

34. In a cultural test, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours the count reaches 2,00,000 if the rate of growth of the bacteria is proportional to the number present.

35. Find a unit vector perpendicular to each of $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

36. Show that the position vector of the point p which divides the line joining the points A and B internally in the ratio m:n is $\frac{m\vec{b} + n\vec{a}}{m+n}$

37. Find the vector and Cartesian equation of the line that passes through the points (3,-2,-5) and (3,-2,6)

38. A fair coin is tossed 8 times. Find the probability of at least six tails.

PART-D

IV Answer any SIX questions.

6X5=30

39. Consider the function $f: R_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$ show that f is invertible and find the inverse of f.

40. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ Calculate AC, BC, $(A + B)C$. Also verify that $(A + B)C = AC + BC$

41. Solve the following system of equations by matrix method $x - y + 2z = 7$, $3x + 4y - 5z = 5$, $2x - y + 3z = 2$.

42. If $y = Ae^{mx} + Be^{nx}$ show that $y'' - (m + n)y' + mny = 0$.

43. A particle moving along the curve $6Y = X^3 + 2$, Find the points on the curve at which the y co-ordinate is changing 8 times as the x-coordinate

44. Find the integral of $\int \frac{dx}{\sqrt{a^2 + x^2}}$ hence evaluate $\int \frac{dx}{\sqrt{x^2 + 9}}$

45. Find the area of the region bounded by the curve $y^2 = 4x$ and $x^2 = 4y$

46. Find the particular solution of $(1 + x^2)^2 \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$

47. Derive the equation of a plane in normal form both in vector and Cartesian form

48. Find the probability of getting 5 exactly twice in 7 throws of a die.

PART-E

1X10=10

V Answer any ONE question

49 a). Prove that $\int_0^a f(x)dx = \int_0^a f(a-x) dx$

Hence evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x}} dx$

b). Find the value of k if

$f(x) = \begin{cases} kx + 1 & \text{if } x \leq 5 \\ 3x - 5 & \text{if } x > 5 \end{cases}$ is continuous at $x=5$.

50 a). A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs.10,500 and Rs.900 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximize the total profit of the society?

b). Prove that $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (x^3 - 1)^2$