JAIN COLLEGE

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SUBJECT: Mathematics

Timimgs Allowed: 3hr ***II PUC MOCK II*** ***Total Marks: 100***

PART-A

I Answer ALL the questions. 10X1=10

1. Define an Equivalence relation
2. Find the principal value of $cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$
3. If a matrix has five elements, What are the possible orders it can have?
4. If A is a symmetric matrix with $|A| = 6$ find the value of $|AA'|$
5. Find the derivative of $\sqrt{e^{\sqrt{x}}}$ w.r.t x
6. Evaluate $\int(1-x)\sqrt{x}dx$
7. Find the scalar and vector components of the vector with initial points (2,1) and terminal point (-5,7).
8. Find the equation of the plane with intercepts 2,3 and 4 on x, y and z axis respectively.
9. What is the Objective function
10. If P(A)=0.8 P(B)=0.5 and P(B/A)=0.4 find P(A∩ B).

Part- B

II Answer any ten questions **10x2=20**

11. Is * defined by $a * b = ab + 1$ ∀a, b ∈Q binary operation.
12. Prove that $2tan^{-1}x = tan^{-1}\left(\frac{2x}{1-x^2}\right),\ -1 < x < 1$
13. If $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4} \quad find\ x$
14. Find the area of the triangle with vertices (1,0), (6,0), (4,3) using determinant.
15. If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}\ find\ \frac{dy}{dx}$
16. If $y = (cos^{-1}x)^x \quad find\ \frac{dy}{dx}$
17. Find the local maximum and the local minimum values of $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$
18. Evaluate $\int logx\, dx$
19. Evaluate $\int_0^{\frac{\pi}{2}} Sin^2xdx$
20. Find the order and degree of the Differential equation $\frac{d^3y}{dx^3}$+2 $\frac{d^2y}{dx^2}$+$\frac{dy}{dx}$=0
21. Show that the vector $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and$-4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear.
22. Find the area of the triangle having the points (1,1,1), (1,2,3) and (2,3,1) as vertices.
23. Find the equation of the plane which passes through intersection of the planes 3x-y+2z=4 and x+y+z=2 *and the point* (2,2,1)
24. *An un*biased die is thrown twice. Let the event A be “odd number on the first throw”, event B be “ odd number on the second throw”. Are A and B independent.

PART-C

III Answer any TEN questions. **10X3=30**

25. * be the binary operation on N defined by a*b=$\frac{ab}{4}$ Is * is commutative and associative.

26. Prove that $tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right) = tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \ |x| < \frac{1}{\sqrt{3}}$

27. If A and B are square matrices of the same order show that $(AB)^{-1} = B^{-1}A^{-1}$

28. If $y = Sin^{-1}x \quad show\ that \quad (1-x^2)y'' - xy' = 0$

29. Verify Mean value theorem if f(x)=x^2-4x+3 in the interval x∈ [1,4]

30. Find the approximate change in volume of the cube of side x mtrs caused by increasing the side by 3%

31. $Evaluate\ \int_0^\pi \log(1+cosx)\,dx$

32. $Evaluate\ \int e^{x^3}x^2dx$

33. Find the area enclosed by the circle $x^2 + y^2 = 2^2\ and\ the\ line\ \ x + y = 2$

34. In a bank principal increases continuously @ r% per year. Find the value of r if Rs.100 doubles itself in 10 years. (log2=0.6931).

35. Find a unit vector perpendicular to each of $\vec{a}+\vec{b}\ and\ \vec{a}-\vec{b}$ where $\vec{a} = \hat{\imath}+\hat{\jmath}+\hat{k}\ and\ \vec{b} = \hat{\imath}+2\hat{\jmath}+\widehat{3k}$

36. Show that the position vector of the point p which divides the line joining the points A and B internally in the ratio m:n is $\frac{m\vec{b}+n\vec{a}}{m+n}$

37. Find the distance between the lines $\vec{r} = (\hat{\imath}+2\hat{\jmath}+\hat{k}) + \alpha(\hat{\imath}+2\hat{\jmath}+3\hat{k})$
And $\vec{r} = (2\hat{\imath}-\hat{\jmath}-\hat{k}) + \beta(\widehat{2\imath}+\hat{\jmath}+2\hat{k})$

38. An insurance company insured 2000 scooter drivers , 4000 car drivers and 6000 truck drivers. The probability of an accident are 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident what is the probability that he is a scooter driver.

$PART - D$

IV $\boldsymbol{Answer\ any\ SIX\ questions.}$ $\boldsymbol{6X5 = 30}$

39. Consider f:$R_+ \to (5,\infty)$ given by $f(x) = 9x^2+6x-5$ Show that f is invertible with $f^{-1}(y) = \left\{\frac{\sqrt{y+6}-1}{3}\right\}$

40. Verify (B+C)A= BA+CA if $A = \begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 8 \\ 11 & 21 \end{bmatrix}$ $C = \begin{bmatrix} 7 & 13 \\ 5 & 19 \end{bmatrix}$

41. Solve the equation by matrix method $3x-2y+3z = 8,\ 2x+y-z = 1, 4x-3y+2z = 4$

42. If y =3cos(logx) +4 sin (logx), Show that $x^2y_2+xy_1+y=0$

43. Find the integral of $\frac{1}{x^2-a^2}$ with respect to x and evaluate $\int \frac{dx}{x^2-8x+5}$

44. A balloon, which remains spherical on inflation , is being inflated by pumping in 900 ccm of gas per sec. Find the rate at which the radius of the balloon increases when the radius is 15cm

45. Find the area of the region enclosed by the Parabola $x^2 = 4y$ and the line $x = 4y-2\ and\ the\ x-axis.$

46. Derive the formula to find the shortest distance between the two skew lines $\vec{r} = \overrightarrow{a_1} + \mu\overrightarrow{b_1}\ \ and\ \vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ in the vector and Cartesian form.

47. Find the particular solution of the differential equation $\frac{dy}{dx} + \frac{2xy}{1+x^2} = 1$ when y=0 and x=1

48. Five cards are drawn successively with replacement from well shuffled deck of 52 cards. What is the probability that
a) All the cards are spades
b) None is spade

PART-E

V Answer any ONE question. **1X10=10**

49. a) One kind of cake requires 200gm of flour and 25gm of fat and another kind of cake requires 100gm of flour and 50 gm of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1kg of fat assuming that there is no shortage of the other ingredient used in making the cakes.

b). Find the value of k if

$$f(x) =) = \begin{cases} k(x^2-2) & x \le 0 \\ 4x+1 & x > 0 \end{cases}$$ is continuous at x=0

50. a)Prove that $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ when $f(2a-x) = f(x)$ and hence evaluate $\int_0^{\pi} |cosx|dx$

b). Prove that $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$=2(x+y+z)^2